Strong Interaction of Vortices with Attractive Point Defects, and Application to Neutron Star Rotation

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The rate of energy transfer to Kelvin waves is obtained for a slowly moving vortex interacting strongly with a point defect distribution. Vortices are trapped by attractive point defects for relatively long time intervals: a critical vortex velocity below which pinning occurs can be defined in terms of the interaction time. An expression is found for the dissipative force at higher velocities and applied here to superfluid neutron vortices in a neutron star, with reference to the recovery of the spin-down rate change in pulsar glitches. [S0031-9007(98)07752-7]

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Ordered motion of the vortex lattice in an electronic type-II superconductor has been observed recently [1]. A transition from a state of vortex creep to one of coherent motion of the lattice was induced by increasing the Lorentz force acting on the vortices. The possible existence of a similar transition of the neutron-superfluid vortex lattice in a neutron star is important for the interpretation of pulsar glitch relaxation. Pulsar glitches are sudden increases in rotation angular frequency Ω, observed in the periodic radio emission, caused by large-scale vortex unpinning and angular momentum transfer from the neutron superfluid to the charged components of the neutron star. At all times, the large-scale motion of the vortices satisfies the constraint $\mathbf{f}_M + \mathbf{f}_R = 0$, where $\mathbf{f}_R(v_L)$ is the dissipative force per unit length and $\mathbf{f}_M = \rho\boldsymbol{\kappa} \times (\mathbf{v}_L - \mathbf{v}_n)$ is the Magnus force acting on unit length of a vortex moving with velocity $\mathbf{v}_L$ through superfluid of bulk velocity $\mathbf{v}_n$. (The velocities are those in the rest frame of the Coulomb lattice of nuclei forming the solid crust of the star: ρ is the superfluid density and $\boldsymbol{\kappa}$ the quantum of circulation.) The recovery of the spin-down rate change $\delta\dot{\Omega}$ is associated with relaxation of v_L and v_n, in certain regions of superfluid, to new steady-state values consistent with the postglitch rotation rate. The relaxation time is of the order of several days and is one of the few observables which may be sensitive to the internal state and temperature of the neutron star. Its magnitude is consistent with either $\mathbf{f}_R$ given by the creep of pinned vortices [2] or almost exact co-rotation of vortices and superfluid in which the force $\mathbf{f}_R$ is a result of vortex-defect interactions in the Coulomb lattice of nuclei [3]. From the Magnus relation we see that the azimuthal component of $\mathbf{f}_R$ is associated with radial movement of vortices and change of superfluid rotation velocity.

Excitation of the helical displacements of the vortex axis known as Kelvin waves is the most important dissipative process in vortex motion relative to the solid crust [4,5]. The contribution from the planar defects of polycrystalline structure is too small, in the limit of very small v_L, to give the pinning required for pulsar glitches [6,7]. Qualitatively, the reason is that vortices are only very weakly pinned in a large defect-free single crystal. Its regularity of structure allows vortex displacement, under a Magnus force, to a continuous sequence of new positions with extremely small changes in energy. (Consideration of a vortex pinned to a line of nuclei in a direction such as {100} or {111} is not apt because a generally oriented vortex will at some point cross to an adjacent line; see the moving kinks shown in Fig. 1 of Ref. [7].) Significant pinning can occur only through interaction with lattice defects which, in neutron stars, are expected to be those in local thermodynamic equilibrium at solidification and having the lowest free energy of formation [7]: Consequently, this Letter considers the general problem of Kelvin wave excitation on a vortex moving through a random distribution of point defects (monovacancies or interstitals). The resulting dissipative force is known for Kelvin one-phonon processes, where the vortex displacement is negligible compared with the range of the vortex-nucleus interaction [4,5]. (The interaction range, the superfluid coherence length ξ, and the bcc lattice constant a are all of the same order of magnitude, 10^{-11}–10^{-12} cm.) However, we find that the interaction, if attractive, is always strong at small v_L, because the action of the defect on the vortex moving past it produces large long-wavelength displacements. The emphasis here on individual point defects differs from that of collective pinning [8,9] which would be valid in neutron superfluids only for unphysically large point defect concentrations.

This Letter gives expressions for the energy transfer to Kelvin waves $\mathcal{E}_a$ and for the interaction time t_m which are valid for an attractive interaction in the nonperturbative limit at very small v_L. They are obtained from the free-vortex Green function for the excitation of undamped Kelvin waves at zero temperature. The interaction time t_m is that for which the vortex is trapped by an attractive point defect. It is important in the definition of a vortex velocity threshold v_{Lp} such that, for a given system of attractive point defects, pinning always occurs at $v_L < v_{Lp}$.

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An expression is obtained for the dissipative force $\mathbf{f}_R$ at velocities $v_L > v_{Lp}$ and is applied here to neutron superfluids and pulsar glitch relaxation. The relationship between dissipative force and pinning velocity v_{Lp} is directly relevant to the interpretation of glitch phenomena.

The Cartesian coordinates adopted here are fixed in the rest frame of a point defect: The z axis is parallel with the axis of an undisturbed vortex which moves with coordinates $\mathbf{x}(t)$ in the x-y plane such that $\dot{x}_1 = 0$, $\dot{x}_2 = -v_L$. Displacement of a vortex axis from its undisturbed path in the x-y plane caused by interaction with point defects is denoted by the vector $\mathbf{u}(z,t)$. The interaction is represented by a potential $V(\tilde{r})$, where $\tilde{\mathbf{r}} = \mathbf{x} + \mathbf{u}$ (a vortex can be assumed locally rectilinear over intervals of z of the order of the interaction range). The potential assumed is a truncated version of the long-range hydrodynamic term given by the change in kinetic energy of the circulating superfluid [10]:

$$V(\tilde{r}) = \frac{\xi^2}{\xi^2 + \tilde{r}^2} V(0), \qquad \tilde{r} < r_0$$
$$= 0, \qquad \tilde{r} > r_0, \tag{1}$$

in which the superfluid coherence length ξ is assumed larger than the nuclear radius. The Green function [6,7] giving the displacement $\mathbf{u}(z,t)$ produced by unit impulse in each of the $\alpha = 1,2$ directions is

$$G_{\alpha\beta}(z - z', t - t') = -\frac{(\sin\chi + i\sigma_2 \cos\chi)}{2\rho\kappa[\pi c_K(t - t')]^{1/2}}, \tag{2}$$

where

$$\chi = \frac{(z - z')^2}{4c_K(t - t')} - \frac{\pi}{4}, \tag{3}$$

and σ_2 is the Pauli matrix. Unperturbed Kelvin waves of angular frequency ω and wave number p have a dispersion relation $\omega = c_K p^2$, in which the logarithmic dependences of the parameter $c_K = -(\kappa/4\pi)\ln p\xi$ can be neglected. The displacement given by the external forces $\mathbf{F}_i$ derived from interaction with a distribution of point defects at coordinates z_i is

$$u_\alpha(z,t) = \sum_i \int_{-\infty}^{t} dt'\, G_{\alpha\beta}(z - z_i, t - t') F_{i\beta}[\mathbf{u}(z_i, t')]. \tag{4}$$

[Equation (4) assumes the vortex to be rectilinear over intervals of z smaller than r_0; this is valid except for time intervals $t - t' < r_0^2/c_K$ which are many orders of magnitude less than the basic unit of interaction time r_0/v_L.] For the case of a single force at $z_i = 0$, Eq. (4) can be reduced to a pair of nonlinear Volterra equations of the first kind giving the displacement at $z = 0$,

$$u_{\substack{1\\2}}(t) = \tilde{C}\int_0^t dt'\, \frac{F_1(\mathbf{u}; t') \mp F_2(\mathbf{u}; t')}{(t - t')^{1/2}}, \tag{5}$$

in which the vortex enters the force field at $t = 0$ with impact parameter x_1 and force, length, and time are expressed, respectively, in units of $V(0)/r_0$, r_0, and r_0/v_L. All parameters are subsumed into a single dimensionless quantity defined by

$$\tilde{C} = \left(\frac{V(0)}{\rho\kappa c_K r_0}\right)\left(\frac{c_K}{8\pi r_0 v_L}\right)^{1/2}. \tag{6}$$

For $\tilde{C}$ large and negative, it is easy to see that the solution of Eq. (5) has the form $|u_1| < r_0$ and $u_2 \approx u_1 + x_1 - x_2(0) + v_L t$, with both force components $F_{1/2} \approx t^{1/2}/\pi\tilde{C}$. The vortex remains trapped in the attractive point defect potential with $|x_2 + u_2| < r_0$ for a long time interval $0 < t < t_m$. In this limit, the energy transfer to Kelvin waves is of the form

$$\mathcal{E}_a = \int_0^{t_m} F_\alpha \dot{u}_\alpha dt = \gamma V(0) C^2, \tag{7}$$

and the interaction time is

$$t_m = \zeta C^2 \xi / v_L, \tag{8}$$

where $C = \tilde{C}(r_0/\xi)^{3/2}$ is independent of the cutoff r_0. The numerical solution of Eq. (5) with the initial conditions $u_\alpha(0) = 0$ has confirmed the form of Eqs. (7) and (8) with the parameters $\gamma = -0.65$ and $\zeta = 2.1$ (for methods, see Ref. [11]). The position of a vortex in an attractive potential with $\tilde{C} = -1.0$ is shown, for successions of equal time intervals, in Fig. 1. The extent to which the vortex remains trapped for a relatively long time is obvious, as is the similarity with the displacement $\mathbf{u}(t)$ expected for large negative $\tilde{C}$. Energy transfer and interaction time are shown in Fig. 2 as functions of C^2. For $C < 0$, these are almost x_1 independent for $-C > 1.0$. Near $C = 0$, they tend to the one-phonon

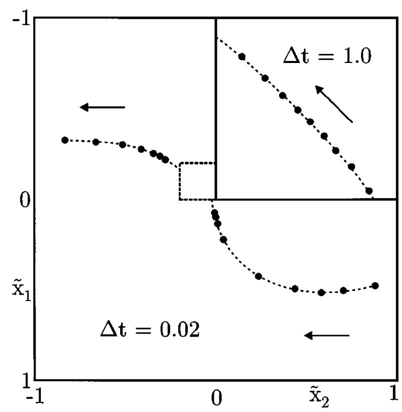

FIG. 1. Vortex position $\tilde{x}_{1,2}$ in the attractive potential is shown at equal time intervals $\Delta t = 0.02 r_0/v_L$ for the case $\tilde{C} = -1.0$, $\xi = 0.5 r_0$ and impact parameter $x_1(0) = 0.479 r_0$. Positions near the origin are too closely spaced to be resolved at this interval. Positions in the box $-0.2 < \tilde{x}_{1,2} < 0.0$ are shown ($\times 5$) in the upper right-hand sector at long time intervals $\Delta t = r_0/v_L$. A vortex moves rapidly into this region, remains trapped for a time t_m, and then exits rapidly.

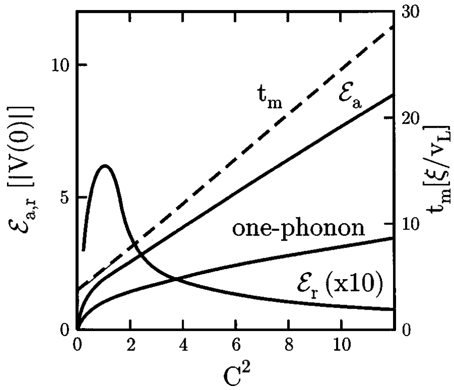

FIG. 2. Computed energy transfers to Kelvin waves are shown for the case $\xi = 0.5r_0$ and impact parameter $x_1(0) = 0.479r_0$ as functions of C^2 [left-hand scale, units of $|V(0)|$; solid curves]. For attractive point defects and $|C| \gg 1$ the energy transfer is $\mathcal{E}_a \propto C^2$ and the trapping time (right-hand scale, units of ξv_L^{-1}; broken curve) is $t_m \propto C^2$. The one-phonon energy transfer is linear in C. The repulsive case energy transfer is shown ($\times 10$); it is $\mathcal{E}_r \propto C^{-1}$ for $C \gg 1$.

values, an energy transfer of $CV(0)$, apart from an x_1-dependent factor of order unity, and an interaction time of $2x_2(0)/v_L$. The energy transfer $\mathcal{E}_r$ is shown for the case $C > 0$. It is $\mathcal{E}_r \approx 0.3V(0)/C$ for $C \gtrsim 1$, which is independent of $V(0)$ and shows the large difference, at small v_L, between interactions with attractive and repulsive point defects. (For $C > 0$, the discontinuity at $\tilde{r} = r_0$ had to be removed from $V(\tilde{r})$ by the addition of a fringe potential allowing the vortex to enter the region $\tilde{r} < r_0$.)

Before investigating what can be learned from these results for a single point defect about interaction with a random distribution, we consider some limitations. The initial condition $u_\alpha(0) = 0$ will not be satisfied exactly in reality owing to thermal excitation or interaction with other point defects. At temperature T, the mean square displacement of a free vortex is

$$\langle u_\alpha^2 \rangle = \frac{k_B T}{2\pi\rho\kappa c_K p_0} + \frac{\hbar}{2\pi\rho\kappa\xi}, \qquad (9)$$

where p_0 is a small wave number cutoff. For $T \approx 10^7$ K and typical neutron superfluid parameters, thermal displacements larger than the bcc lattice constant a have very small wave numbers $\lesssim 10^5$ cm^{-1}. Although thermal motion will produce large deviations from the initial state of vortex rectilinearity assumed here, the corresponding thermal velocity components, $|\omega u_\alpha| \lesssim 10^{-4}$ cm s^{-1}, are small compared with v_L and will not lead to any increase in the number of attractive point defects with which a vortex interacts. The effect of Kelvin waves produced by the interaction with distant point defects at earlier times $t < 0$ should be negligible for C large and negative. This has been confirmed by assuming an initial displacement $u_1^e = u^e \cos(\omega t - pz + \phi)$, $u_2^e = -u^e \sin(\omega t - pz + \phi)$ with $\omega = t_m^{-1}$ and $u^e \leq r_0$. Given that $u_\alpha(t > 0)$ is the total displacement, that part of the displacement obtained from Eq. (4) with the Green function Eq. (2) is $u_\alpha - u_\alpha^e$ (this subtraction is possible because u_α^e satisfies the free-vortex equations of motion). Calculation of $\mathcal{E}_a$ has shown that the effect of $u_\alpha^e \neq 0$ decreases as $-C$ increases. This is consistent with the expectation, derived from the solution of Eq. (5), that both $\mathcal{E}_a$ and t_m become independent of the initial conditions for C large and negative. For intermediate C, averaging over x_1 and ϕ cancels, to at least the first order in $\omega u^e v_L^{-1}$, the effect on $\mathcal{E}_a$ and on the collision width.

It is possible to see how a moving vortex pins in the case of interaction, initial condition $u_\alpha(0) = 0$, with an array of attractive point defects spaced at regular intervals Δz along the z axis. Assume that n point defects $j \neq i$ contribute substantially at some instant t to the displacement $u(z_i, t)$ given by Eq. (4). The form of the Green function Eq. (2) shows that a sufficient condition for this is that $|z_i - z_j| < (4c_K t)^{1/2}$ for n point defects $j \neq i$, which can be reexpressed as $n\Delta z < (4c_K t)^{1/2}$. The effect of contributions from n point defects can be approximated to within an order of magnitude by the replacement $\tilde{C} \rightarrow n\tilde{C}$ in Eq. (5), giving a trapping time of $n^2 t_m$, with t_m defined by Eq. (8). A sufficient condition for indefinite trapping is that n should increase with time so that $t < n^2 t_m$ for all t, which is satisfied provided $\Delta z < (4c_K t_m)^{1/2}$. More generally, a single vortex interacts simultaneously with a large number of point defects in the random distribution through which it moves, the displacement being given by Eq. (4). The interactions are independent provided that, within the time interval $0 < t < t_m$ of interaction with a given point defect at z_i, point defects $j \neq i$ contribute negligibly to the displacement $u(z_i, t)$. A sufficient condition is that their mean separation satisfies $\Delta z \gg (4c_K t_m)^{1/2}$. If this condition is not satisfied we can infer, by reference to the linear array, that pinning occurs if $\Delta z \lesssim (4c_K t_m)^{1/2}$. For a random distribution of attractive point defects, the necessary fractional concentration is

$$c_a \gtrsim \frac{a^2}{v_L t_m (4c_K t_m)^{1/2}}. \qquad (10)$$

This can be reexpressed in terms of the velocity v_L. Pinning occurs for $v_L \lesssim v_{Lp}$, where

$$\rho\kappa v_{Lp} = \left(\frac{\zeta}{2\pi}\right)^{3/4} \left(\frac{|V(0)|^3 c_a}{4\rho\kappa c_K a^2 \xi^3}\right)^{1/2}. \qquad (11)$$

Equation (11) is identical, except for a numerical factor close to unity, with the very elementary static estimate of v_{Lp} obtained previously [Eq. (28) of Ref. [7]] and so provides a more firm basis for the statement [7] that an unphysically large concentration of point defects would be necessary to give the pinning strength required for pulsar glitches. The dissipative force per unit length of vortex at velocities $v_L \gg v_{Lp}$, where the interactions are independent, and for which Eq. (7) and (8) are valid, is

$$f_R \approx \frac{c_a \mathcal{E}_a}{a^2} = \left(\frac{2\pi\gamma^2}{\zeta^3}\right)^{1/2}\left(\frac{\rho\kappa v_{Lp}^2}{v_L}\right). \qquad (12)$$

(The equivalent expression for $C > 0$ would be $c_r \mathcal{E}_r / a^2 \propto v_L^{1/2}$.) For $|C| \lesssim 0.1$, where the one-phonon energy transfer expression is valid (see Fig. 1), the v_L dependence becomes $f_R \propto v_L^{-1/2}$ [4,5].

On the basis of these results, a transition from vortex creep to motion of the vortex lattice occurs as v_L increases to $v_L > v_{Lp}$. It is analogous with that in electronic type-II superconductors [1], the Lorentz force being replaced by the Magnus force, except that the intervortex spacing in the neutron superfluid ($10^{-2} - 10^{-3}$ cm) is so large compared with vortex displacements of the order of $v_L t_m$ that the intervortex interaction is not an important factor.

The actual $V(\tilde{r})$ may be much less simple than that assumed in Eq. (1). It may change sign at some $\tilde{r}$ (the long-range hydrodynamic interaction [10] is attractive for monovacancies and repulsive for interstitials [7]). But Eqs. (7) and (8) should remain approximately valid provided the maximum attractive force is substituted for that given by Eq. (1) $[0.65V(0)/\xi]$. The concentration c_a of attractive point defects could equal that of either monovacancies or interstitials, and modern computational speeds may allow its estimation by calculations of mobility and of enthalpy of formation. The phases of rod and slab nuclei predicted [12] at densities intermediate between the bcc lattice of spherical nuclei and the liquid core do not contain point defects.

Equations (11) and (12) are particularly significant for models in which the relaxation giving spin-down rate recovery $\delta\dot{\Omega}(t)$ occurs in superfluid regions where the vortices are not pinned but co-rotate with v_L almost exactly equal to v_n [3]. A large glitch produces initial deviations $\delta v_{L,n} \lesssim 10^2$ cm s^{-1} from the steady-state values of the new postglitch rotation rate. With a coherence length derived from the neutron energy gap of Ainsworth *et al.* [13] and $V(\tilde{r})$ given by Bildsten and Epstein [10] the value of C (for $v_L = 1\,\mathrm{cm\,s^{-1}}$) has a maximum $|C| \approx 350$ at a matter density of $3.4 \times 10^{13}\,\mathrm{g\,cm^{-3}}$ but decreases rapidly at both lower and higher densities ($|C| \approx 14$ at $8 \times 10^{13}\,\mathrm{g\,cm^{-3}}$). Although there are considerable uncertainties in both the superfluid parameters and $V(\tilde{r})$ [14,15], these values indicate that the attractive point defect concentration c_a is much more important. But f_R could be a very complicated function of v_L if there were appreciable contributions from both attractive and repulsive point defects.

If the pinning causing pulsar glitches is in the liquid core of the star, the spin-down rate recovery $\delta\dot{\Omega}$ could be the relaxation of the subset of vortices which pass only through the crust. If this is so and assuming the above value of $|C|$ for 8×10^{13} g cm^{-3}, the spin-down rate recovery time of the order of 10^5 s observed in the Vela pulsar imposes the following constraints on point defect concentrations: $c_a \lesssim 10^{-11}$, $c_r \lesssim 10^{-10}$. If it were established that such concentrations are unphysically low, spin-down rate recovery would have to be associated with unknown phenomena in the liquid core of the star and, with no crust involvement, it would be necessary to reassess what could be learned from glitch observations about the neutron star interior.

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