Vortex Heating in Superfluid Helium at Low Temperatures

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We address the fundamental problem of dissipation of kinetic energy in superfluid turbulence at temperatures low enough that friction against the normal fluid is negligible. We find that the kinetic energy of a turbulent vortex tangle, transformed into compressional energy, can result in a significant heating of the fluid. We suggest that *vortex heating* can be used to investigate turbulence at these low temperatures, where the traditional second-sound technique fails. [S0031-9007(98)07636-4]

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Current experiments on the turbulence of helium II have produced some surprising results and opened a number of interesting questions about the relation between traditional classical Navier-Stokes turbulence and the less studied turbulence of a quantum fluid. For example, Donnelly and co-workers [1] measured the decay of quantized vorticity generated by a towed grid, and found that it obeys the same power law of the decay of vorticity in classical turbulence, independently of temperature T at all values of T investigated in the range $1.4\ \mathrm{K} < T < T_\lambda$, where $T_\lambda = 2.17$ K is the superfluid transition. Similarly, Tabeling [2] found that a sample of helium II made turbulent by rotating blades exhibits the classical $-\frac{5}{3}$ Kolmogorov spectrum down to the lowest temperature measured, which was also $T = 1.4$ K.

From the point of view of Landau's two-fluid theory, these results are at first surprising. Landau's theory describes helium II as the intimate mixture of a viscous normal fluid component of density ρ_n and an inviscid superfluid component of density ρ_s; helium's total density is $\rho = \rho_n + \rho_s$, and the relative proportions of normal fluid and superfluid are very strong functions of temperature: $\rho_n/\rho \rightarrow 1$ and $\rho_s/\rho \rightarrow 0$ in the high temperature limit $T \rightarrow T_\lambda$, and $\rho_n/\rho \rightarrow 0$ and $\rho_s/\rho \rightarrow 1$ in the low temperature limit $T \rightarrow 0$. In particular, the normal fluid ratio ρ_n/ρ drops rapidly as T is lowered below T_λ and is only 7.5% at $T = 1.4$ K, too small to be held responsible for the apparently classical behavior which has been observed, hence the surprise.

The experiments of Donnelly [1], Tabeling [2], McClintock [3], and others [4] have stimulated theoretical investigations [5] about the role played by the quantized vortex lines in the kinetic energy cascade and energy dissipation. In particular, an elegant numerical simulation has been performed recently by Nore, Abid, and Brachet [6] who address for the first time the fundamental issue of kinetic energy dissipation at zero temperature, in the absence of normal fluid. In this limit neither viscous dissipation nor mutual friction [7] are present. However, there are mechanisms even at $T = 0$ that can convert kinetic energy to heat energy (phonons); vortices can radiate phonons and vortex reconnections can produce phonons and rotons. Nore *et al.* [6] used the nonlinear Schroedinger equation (NLSE) model, which naturally describes a pure superflow ($T = 0$) and nonsingular quantized vortex lines. Starting from a Taylor-Green vortex flow at time $t = 0$, they integrated the NLSE as an intense vortex tangle was generated and then decayed, and found that the energy spectrum of the superflow had an inertial range compatible with Kolmogorov's, even at a temperature of $T = 0$. This result is in qualitative agreement with Tabeling's observation of Kolmorogorov's law at $T = 1.4$ K, since the NLSE is a valid model of superfluidity at temperatures which are low enough that the normal fluid density is negligible.

Over the course of that simulation, Nore *et al.* observed that a significant fraction of the initial kinetic energy of the vortex tangle was transformed into compressional energy, that is to say, into energy of sound (phonons). The aim of this Letter is to draw attention to the thermodynamics consequence of Nore's simulation: *vortex heating* of the sample of helium II. We can calculate this heating for the vortex tangle of Nore *et al.* and for the dissipation of a general vortex tangle.

Let ΔE be the kinetic energy of the vortex configuration transformed into compressional energy, i.e., phonons. After thermodynamic equilibration, this energy results in an increase of the temperature of the helium sample from the initial T_I ($T_I = 0$ in Nore's case) to a final, nonzero T_F. We calculate the temperature rise $T_F - T_I$ due to the creation of phonons by integrating $dE = C_v dT$. Below $T = 0.6$ K we need only to consider the specific heat of phonons (at constant volume $\mathcal{V}$), which is $C_v = aT^3$, with $a = 2\pi^2 k_B^4 \mathcal{V}/15\hbar^3 c^3$, where k_B is Boltzmann's constant, $\hbar = h/2\pi$, h is Planck's constant, and $c \approx 2.4 \times 10^4$ cm/s is the speed of (first) sound. Solving for the final temperature T_F, we have

$$T_F^4 = T_I^4 + \frac{30\hbar^3 c^3}{\pi^2 k_B^4} \frac{\Delta E}{\mathcal{V}} . \quad (1)$$

The temperature increase $T_F - T_I$ is plotted in the figure for two sample kinetic energy densities; 0.05 erg/cm^3, which is a typical value of the vortex energy density in towed grid experiments [1], and 5 erg/cm^3, representing a very dense vortex tangle. These two values correspond to vortex line densities $L \approx 2.5 \times 10^5$ cm^{-2} and $L \approx 2.5 \times 10^7$ cm^{-2}, respectively, using the kinetic energy per unit length $\epsilon = (\rho_s \kappa^2/4\pi)\log(b/a_0)$ where $\kappa = 9.97 \times 10^{-4}$ cm^2/s is the quantum of circulation, $a_0 \approx 10^{-8}$ cm is the vortex core parameter, $\rho_s \approx \rho \approx 0.14$ g/cm^3 at the low temperatures under consideration, and $b \approx L^{-1/2}$ is the average intervortex spacing. Vortex line densities as high as $L \approx 10^7$ cm^{-2} have, in fact, been observed by McClintock's group [3].

Since T_F is raised to the fourth power, the temperature increase $T_F - T_I$ is relatively insensitive to the magnitude of the kinetic energy density. From the figure we see that the vortex heating is significant only at fairly low initial temperatures, consistent with the approximation involved in the use of the NLSE. These required low temperatures are not too low to be experimentally unaccessible; for example, a great number of experiments were performed in this range to study the motion of ions and the interaction of ions and quantized vortices [8].

Another way to look at our result is the following: From Eq. (1) we can define a critical temperature

$$T_{\mathrm{crit}} = \left(\frac{30\hbar^3 c^3}{\pi^2 k_B^4} \frac{\Delta E}{\mathcal{V}} \right)^{1/4}. \qquad (2)$$

If the initial temperature is below this critical temperature then the rise in temperature is large compared with the initial temperature. If the initial temperature is zero, then the final temperature will be T_{crit}. For the typical examples given in the figure, the critical temperatures are 0.05 K for $\Delta E/\mathcal{V} = 0.05$ erg/cm^3 and 0.16 K for $\Delta E/\mathcal{V} = 5$ erg/cm^3.

To return to the simulations of Nore *et al.*, the energy transfered to compressional energy over the duration of one simulation [6] was typically 0.02 in their nondimensional units. Converting this to dimensional units, we have an energy density of $\Delta E/\mathcal{V} = 1.4 \times 10^4$ erg/cm^3. This large (though not unphysically large) value is due to the practical constraints of their simulation; they had to have length scales small enough to resolve the vortex cores and they also needed a significant number of vortex lines within the computational volume. By Eq. (1), assuming $T_I = 0$, the final temperature would be $T_F = 1.2$ K. At this high a temperature the contribution of the rotons to the specific heat would have to be included, so this value can be considered only a rough estimate. In any case, the conversion of the kinetic energy to compressional energy clearly results in a very large heating of the helium sample in that simulation.

Our simple argument is not concerned with the dynamics of the vortices, so it does not give information about the time scale for the temperature rise. This time scale depends on the details of the decay mechanisms which convert vortex line energy into phonons and rotons. Besides the work of Nore *et al.* [6], decay events are apparent in the calculations of Jones and Roberts [9] and of Koplick and Levine [10]. The decay dynamics of an analogous system of line singularities have been investigated by Mondello and Goldenfeld [11]. Phonons can be generated both by the motion of vortex lines and by vortex reconnection events, perhaps via the emission of vortex waves which then emit phonons. We do not know which mechanism is more important, but it is likely that the generation of phonons by reconnections may dominate the early stages of decay, when the vortex line density is large and reconnections are common. Clearly, these are important issues which must be investigated further.

We conclude that at low temperatures the vortex heating effect is large enough to be measured without difficulty. This suggests a new way to probe experimentally the vortex tangle in the turbulence experiments carried out at low enough temperatures that the NLSE model applies directly, i.e., when the normal fluid ratio is small and the rotons' contribution to the thermodynamics properties of helium is not important ($T < 0.6$ K). The possibility of using thermometers as detectors is particularly important because the second sound technique, which is the traditional probe of quantized vorticity, fails in the low temperature regime. Experiments in which quantized vortices are generated by a vibrating grid at temperatures as low as $T = 0.07$ K are being conducted by Davis *et al.* [12]. At this low temperature the effect which we predict should be noticeable (Fig. 1).

Experimental attempts to detect the heat signal of vorticity are being carried out in the different physical context of superfluid ^{3}He-B at ultralow temperatures to investigate the Kibble-Zurek hypothesis for the formation of defects in the early Universe. As yet, however, there is no evidence [13] that the detected signal is related to vorticity.

Finally, it is hoped that the calculation of Nore *et al.* and our remarks will stimulate more experimental investigations of superfluid turbulence at low temperatures, where one faces the fundamental issue of turbulence and disorder in the absence of mutual friction and viscous dissipation,

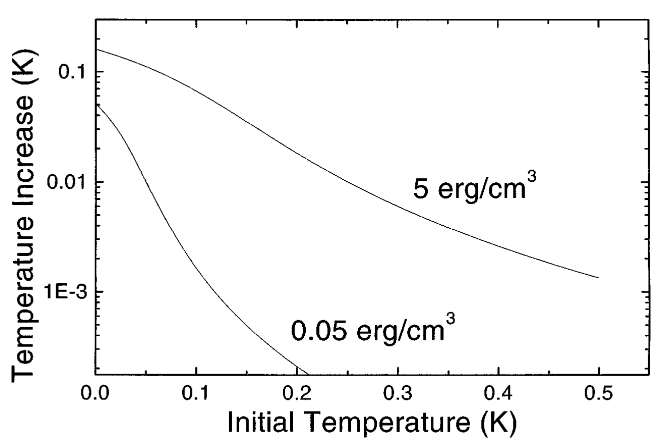

FIG. 1. Temperature increase $T_F - T_I$ as a function of initial temperature T_I for two typical energies of the vortex tangle.

which dominate the dynamics of superfluid turbulence at higher temperatures [14].

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