

Wexler *et al.* Reply: In the preceding Comment [1], Sonin claims that some publications by us [2–4] reach an incorrect result for the transverse force on a vortex in a superfluid. Sonin argues that the cause for this error is that we do not consider the effects of quasiparticle excitations and impurities.

Let us start from the latter point. Some of the results presented before [2,3] do not explicitly include disorder effects, and this, strictly speaking, may restrict their validity to bulk superfluids or ultraclean superconductors. Yet preliminary calculations indicate a certain degree of insensitivity to disorder in the determination of the transverse forces [5]. We fail to understand why this point is raised in the context of the preceding Comment, which deals entirely with superfluids, and which does not seem to offer any further insight into this problem. In fact, Sonin does not get into this issue any further, either here or in previous publications [6].

The effects of quasiparticle excitations are, however, included in our formulation. The fact that the normal fluid does not exert a transverse force on the vortex should not be confused with the effect of quasiparticle excitations. In fact, from the original two-fluid model of Landau [7], it is understood that quasiparticle effects are responsible for the renormalization of the fluid density ρ to the *superfluid* density ρ_s in most transport properties. This is explicit in Wexler's calculation of the superfluid velocity part of the Magnus force [4]. Thouless, Ao, and Niu [3] also take this into consideration when the normal fluid is considered irrotational (we will come back to this later).

Sonin bases the criticism of Refs. [2–4] on the fact that a detailed demonstration of the asymmetric scattering of phonons by a vortex is missing in Ref. [8]. The primary goal of Ref. [8] was the determination of dissipative effects caused by phonons. Whether or not the approximate and phenomenological method used there is suitable for the transverse effect was a secondary issue. The exact expression for the transverse forces at finite temperatures was *later* obtained from microscopic [3] and thermodynamic arguments [4]. Sonin also claims that a partial-wave analysis does not provide a definitive answer either due to problems with infinite series. This is surprising since he has used the method himself [6]. We wish to point out that a careful calculation of this has been already performed by us [9] in the phonon-dominated regime, and results in a transverse force due to phonon-vortex scattering which is much smaller than that claimed by Sonin. We choose to direct the reader to Ref. [9] for further discussion.

In addition, note that the direct calculations [6,8,9] of the phonon-vortex force mentioned above are possibly missing the most important effect of the vortex on the quasiparticles: the coupling to the vortex velocity field produces, on any finite system, an energy difference between circulating and countercirculating excitations. Therefore there is a different equilibrium population of these modes [9,10].

This is the renormalization of circulation by quasiparticle excitations that leads to the result of Ref. [3], and which reduces the effective circulation of the fluid to the circulation of the superfluid component only.

Finally, we open the question on whether the origin of the puzzle is related to a well-known ambiguity in hydrodynamics: a description of a sound pulse is not unique [11]; the Lagrangian and Eulerian descriptions are related by a nonlinear transformation that introduce a certain amount of backflow to second order in the perturbations. We believe that a more detailed analysis may further illuminate the discussion.

The microscopic global approaches [2,3] and the thermodynamic demonstration [4] have ruled out the additional transverse force. Those approaches have bypassed the ambiguity in hydrodynamics, therefore giving a detail-insensitive prediction on the transverse force.

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