Comment on "Berry's Phase and the Magnus Force for a Vortex Line in a Superconductor," "Transverse Force on a Quantized Vortex in a Superfluid," and "Magnus and Iordanskii Forces in Superfluids"

I comment on three aforementioned papers [1–3] together, since they do not differ in their results but vary only in methods of derivation. I call it TANW (Thouless-Ao-Niu-Wexler) theory. The conclusion of TANW is that normal quasiparticles and impurities have no effect on the force transverse to the vortex velocity $\vec{v}_L$. Here I discuss only normal quasiparticles.

In general the transverse force in a Galilean invariant liquid may be presented in the form given by Wexler [3]:

$$\vec{F} = A[\vec{\kappa} \times (\vec{v}_L - \vec{v}_s)] + B[\vec{\kappa} \times (\vec{v}_L - \vec{v}_n)]. \quad (1)$$

There is a complete consensus that $A = \rho_s$ (the superfluid Magnus force). Three TANW papers suggest new interesting derivations of this result, but they do not provide any new information compared to the first paper by Hall and Vinen [4] on vortex dynamics in superfluids: in the expression for the Magnus force known from classical hydrodynamics one must replace the mass density ρ by the superfluid mass density ρ_s. But TANW claim that $B = 0$ *exactly*. Meanwhile a transverse force linear in $\vec{v}_L - \vec{v}_n$ must arise because of asymmetry of scattering of quasiparticles by a vortex, as was shown for rotons and for phonons [5] (see further relevant references in [6]).

Why then did TANW fail to reveal this force? Simply because their derivations did not consider quasiparticle scattering. They claim that the quasiparticle scattering yields only a longitudinal dissipative force and no transverse force. Two from TANW (Ao and Niu) participated in writing the paper [7] which supported this conclusion. I shall comment why it is incorrect for scattering of the sound wave (phonon) by the vortex.

Because of the long-range scattering potential $\propto 1/r$, the scattering amplitude is divergent at small angles and one may not determine the transverse force from the sound wave on the vortex from the differential cross section as was shown first by Iordanskii [5]. He found the transverse force (the Iordanskii force) summing the infinite series in the method of partial waves. Later on his result was confirmed by another method using a more accurate asymptotic solution for the sound wave in the configurational space at small scattering angles where the usual asymptotics "incident wave + scattered wave" is invalid [8]. This method is free from the problem of summing divergent series in the partial-wave method [6].

Scattering of the sound wave by a vortex is analogous to the Aharonov-Bohm effect for the electron wave scattered by the magnetic-flux tube [8]. So there is the transverse force on the magnetic flux similar to the Iordanskii force on the vortex (see [9], and references therein). Asymmetry of wave scattering by a vortex was revealed theoretically and experimentally in classical hydrodynamics for water surface waves (the acoustic Aharonov-Bohm effect [10,11]).

The TANW theory is in conflict with all aforementioned works since TANW lost the transverse force from scattering asymmetry. I emphasize that disagreement is not conceptual, but appears in calculations. Solving the sound wave equation in the Born approximation at small scattering angles Demircan *et al.* [7] obtained the same integrals as in Ref. [8]. Disagreement arose when they calculated these integrals: they used the usual wave asymptotics incident wave + scattered wave which is incorrect at small angles where the Aharonov-Bohm interference takes place and incident and scattered waves cannot be separated [6,8–10].

The experimental aspects of the present problem were recently discussed in Refs. [12,13]. TANW believe [13] that their result ("no Iordanskii force") is confirmed by experiments with vortices trapped by wires. In this case the vortex has a *macroscopical* hard core (a wire). But the Iordanskii force was derived for a *microscopical* core when the mean-free path essentially exceeds the vortex-core radius [8]. Thus TANW [13] reject the Iordanskii force in the case in which it has not been predicted.

E. B. Sonin
Low Temperature Laboratory
Helsinki University of Technology
02150 Espoo, Finland
and Racah Institute of Physics
Hebrew University of Jerusalem
Jerusalem 91904, Israel

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