Plasmons in Coupled Bilayer Structures

S. Das Sarma and E. H. Hwang
Department of Physics, University of Maryland, College Park, Maryland 20742-4111
(Received 23 July 1998)

We calculate the collective charge density excitation dispersion and spectral weight in bilayer semiconductor structures *including effects of interlayer tunneling.* The out-of-phase plasmon mode (the "acoustic" plasmon) develops a long wavelength gap in the presence of tunneling with the gap being proportional to the square root (linear power) of the tunneling amplitude in the weak (strong) tunneling limit. The in-phase plasmon mode is qualitatively unaffected by tunneling. The predicted plasmon gap should be a useful tool for studying many-body effects. [S0031-9007(98)07645-5]

PACS numbers: 73.20.Mf, 71.45.Gm, 73.20.Dx

Collective charge density excitations (or, equivalently plasmon modes) in bilayer structures have attracted a great deal of theoretical and experimental attention over the last sixteen years ever since the existence of an *undamped* acoustic (i.e., with a long wavelength dispersion linear in wave vector) plasmon mode was predicted [1] in semiconductor double quantum well systems. In an uncoupled bilayer system, ignoring any *interlayer* Coulomb interaction, each layer can support a two dimensional (2D) plasma mode [2] with a long wavelength ($q \rightarrow 0$) dispersion $\omega \sim q^{1/2}$, where $q \equiv |\mathbf{q}|$, and $\mathbf{q}$ is the 2D wave vector. When the two layers are near each other (separated by a distance d in the z direction with the 2D layers in the x-y plane), the 2D plasmons are coupled by the interlayer Coulomb interaction leading to the formation [1] of in-phase and out-of-phase interlayer density fluctuation modes $\omega_\pm$: an out-of-phase acoustic plasmon mode, $\omega_- \sim q$, where the densities in the two layers fluctuate out of phase with a linear wave vector dispersion and an in-phase optical plasmon mode, $\omega_+ \sim q^{1/2}$, where the densities in the two layers fluctuate in phase with the usual 2D plasma dispersion. These $\omega_\pm$ modes have been observed [3,4] in double layer semiconductor quantum well systems via inelastic light scattering spectroscopic experiments, and the observation of the $\omega_- \sim q$ mode in GaAs-AlGaAs multilayer systems is, in fact, the only unambiguous direct experimental observation of an acoustic plasmon mode in solid state plasmas in spite of the theoretical literature on the subject going back more than 40 years [5].

In this Letter we consider the experimentally relevant issue of the collective mode dispersion in bilayer structures *in the presence of significant interlayer quantum tunneling.* It is well known that tunneling introduces qualitatively new physics [6] by introducing a new energy scale, the *interlayer* tunneling energy, in addition to the Coulomb energy and the *intralayer* kinetic energy. We find that tunneling *significantly* affects the out-of-phase ω_- mode, qualitatively modifying its long wavelength dispersion to $\omega_- \sim (\Delta^2 + C_1 q + C_2 q^2)^{1/2}$, where Δ defines the plasmon gap $\Delta \equiv \omega_-(q=0)$ which depends nontrivially on the 2D electron density n and the interlayer tunneling amplitude t. In particular we obtain the interesting result that $\Delta \sim t$ or $t^{1/2}$ depending on whether the interlayer quantum tunneling is strong or weak, respectively. We also find that, in contrast to the no-tunneling situation when the ω_+ in-phase mode exhausts the plasmon spectral weight in the long wavelength limit (and the ω_- acoustic mode carries significant spectral weight only at finite wave vectors), the out-of-phase mode ω_- may carry significant spectral weight in the presence of tunneling even in the long wavelength limit and may be easily observable via inelastic light scattering spectroscopy [3,4] or frequency-domain far infrared (or microwave) spectroscopy [2]. We note the somewhat nonintuitive result that finite tunneling in fact converts the out-of-phase "acoustic" plasmon mode (in the $t=0$ situation) to an "optical" plasmon mode (in the $t \neq 0$ situation) by producing a finite plasmon gap $\omega_-(q=0) = \Delta$ whereas the original optical plasmon (for $t=0$) in-phase mode ω_+ becomes the acoustic plasmon mode, albeit with a $q^{1/2}$ long wavelength dispersion, in the sense that $\omega_-(q=0)$ vanishes in the presence of finite tunneling. The situation in the presence of tunneling is therefore similar to the familiar phonon terminology where the optical phonon (which has a finite energy at zero wave vector) corresponds to the out of phase intracell ionic dynamics and the acoustic phonon (with vanishing long wavelength energy) corresponds to the in-phase intracell ionic dynamics.

The collective mode spectrum is given by the zero of the dynamical dielectric function of the system, which for a bilayer system in the presence of a finite interlayer tunneling amplitude t becomes a tensor ϵ_{ijlm} of the fourth rank where $i,j,l,m = 1$ or 2 is the layer index with 1, 2 denoting the two layers. The dielectric function $\epsilon_{ijlm}(q,\omega) \equiv \delta_{il}\delta_{jm} - v_{ijlm}(q)\Pi_{lm}(q,\omega)$ is obtained within the mean field random phase approximation (RPA) in our theory where the deltas are Kronecker delta functions and v_{ijlm} is the intralayer/interlayer Coulomb interaction matrix element with Π_{lm} as the irreducible noninteracting electron polarizability function. It is convenient to use one electron energy eigenstates $E_\pm$ (we take $\hbar = 1$ throughout this paper) as the basis set rather than the layer index since the latter is *not* a good quantum number in the presence of tunneling. The energy levels

$E_\pm = \varepsilon(k) \pm t$, where $\varepsilon(k) = k^2/2m$ is the parabolic one electron 2D kinetic energy in each layer and t is the tunneling strength, are the usual symmetric and antisymmetric states in the presence of tunneling with a single particle symmetric-antisymmetric (SAS) gap given by $\Delta_{\mathrm{SAS}} = E_+ - E_- = 2t$. In the SAS representation the collective mode spectra become decoupled by virtue of the symmetric nature of our double quantum well system (i.e., both layers identical with equal electron density), and the collective density fluctuation spectra are given by the following two equations for the in-phase and the out-phase plasmon modes $\omega_\pm$, respectively:

$$\epsilon_+(q,\omega) = 1 - v_+(q)[\Pi_{++}(q,\omega) + \Pi_{--}(q,\omega)] = 0\,, \tag{1}$$

and

$$\epsilon_-(q,\omega) = 1 - v_-(q)[\Pi_{+-}(q,\omega) + \Pi_{-+}(q,\omega)] = 0\,. \tag{2}$$

In Eqs. (1) and (2) the Coulomb interaction matrix-elements are given by $v_\pm(q) = v_1(q) \pm v_2(q)$, where $v_{1,2}(q)$ are, respectively, the intralayer and interlayer Coulomb interaction matrix elements. We use the simplest model for the Coulomb interaction (at no loss of generality) assuming the intralayer Coulomb interaction to be purely a 2D Coulomb interaction (and thus neglecting subband effects in each layer, which is entirely justified in most experimental situations where the intralayer intersubband energy is much larger than Δ_{SAS})—subband effects can be trivially incorporated by using the appropriate subband form factors [7]. For this simple model, $v_1(q) = 2\pi e^2/(\kappa q)$; $v_2(q) = v_1(q)\exp(-qd)$, with κ as the (high frequency) background lattice dielectric constant. Finally, $\Pi_{\alpha\beta}(q,\omega)$ in Eqs. (1) and (2), together with $(\alpha,\beta) = (+,-)$, is the noninteracting SAS polarizability functions within our RPA theory:

$$\Pi_{\alpha\beta}(q,\omega) = 2\int \frac{d^2k}{(2\pi)^2}\,\frac{f_\alpha(\mathbf{k}+\mathbf{q}) - f_\beta(\mathbf{k})}{\omega + E_\alpha(\mathbf{k}+\mathbf{q}) - E_\beta(\mathbf{k})}\,, \tag{3}$$

where $f_{\alpha,\beta}$ are Fermi occupancy factors (we restrict ourselves to $T = 0$ K in this paper), and the factor of 2 in the front arises from spin.

Solving Eqs. (1)–(3) we obtain the collective density fluctuation spectra of the coupled bilayer system. In the absence of tunneling, $t = 0$, one has $E_+ = E_- = \varepsilon(\mathbf{k})$, and one then recovers in a straightforward fashion the well-known optical ($\omega_+ \sim q^{1/2}$) and acoustic ($\omega_- \sim q$) plasmons of a bilayer system without any electron tunneling. It is, in fact, straightforward (but quite tedious) to obtain analytically [from Eqs. (1)–(3)] the long wavelength ($q \to 0$) plasma modes of the coupled bilayer system *including effects of interlayer tunneling.* We obtain in the long wavelength limit the following results:

$$\omega_+^2(q \to 0) = \frac{2\pi e^2 N}{\kappa m}\,q\,, \tag{4}$$

$$\omega_-^2(q \to 0) = \Delta^2 + C_1 q + C_2 q^2, \tag{5}$$

where $N = 2n$ is the total 2D electron density (n being the electron density per layer), $C_2 > 0$, and

$$\Delta^2 = \Delta_{\mathrm{SAS}}^2 + \frac{\pi}{m}(n_+ - n_-)(q_{\mathrm{TF}}d)\Delta_{\mathrm{SAS}}$$
$$C_1 = -(\Delta^2 - \Delta_{\mathrm{SAS}}^2)\,\frac{d}{2}\,, \tag{6}$$

where $q_{\mathrm{TF}} = 2me^2/\kappa$ is the 2D Thomas-Fermi wave vector and $n_\pm = n \pm n_c$ is the electron density in the symmetric/antisymmetric $\pm$ level. Here, $n_c = (m/2\pi)\Delta_{\mathrm{SAS}}$ for $n > n_c$ when both symmetric and antisymmetric levels are occupied (i.e., the 2D Fermi energy $E_F > \Delta_{\mathrm{SAS}}$), and $n_c = n$ for $n \le n_c$ when only the symmetric level is occupied (i.e., $E_F < \Delta_{\mathrm{SAS}}$). Note that the in-phase mode ω_+ is unaffected in the long wavelength limit either by finite tunneling or by level occupancy and depends only on the total 2D electron density N, following the standard 2D plasmon dispersion. The positive coefficient $C_2(q_{\mathrm{TF}}, d, E_F, \Delta_{\mathrm{SAS}})$ is not shown for brevity.

The most important qualitative feature of the plasmon dispersion in the presence of interlayer tunneling is that the out-of-phase mode ω_-, which is purely acoustic in the absence of tunneling ($\omega_- \sim q$ if $\Delta_{\mathrm{SAS}} = 0$), develops a plasmon gap at $q = 0$ in the presence of nonzero tunneling. It is easy to see from Eqs. (5) and (6) that this plasmon gap Δ, $\omega_-(q = 0) = \Delta$, has the following behavior:

$$\Delta \sim \Delta_{\mathrm{SAS}} \quad \text{or} \quad \sqrt{\Delta_{\mathrm{SAS}}}\,, \tag{7}$$

depending on whether the interlayer tunneling is strong ($\Delta_{\mathrm{SAS}}/E_F \gg q_{\mathrm{TF}}d$) or weak ($\Delta_{\mathrm{SAS}}/E_F \ll q_{\mathrm{TF}}d$). It should be emphasized that the strikingly nonintuitive $\Delta_{\mathrm{SAS}}^{1/2}$ dependence of the collective mode gap (on the square root of the single particle gap) is purely a Coulomb interaction effect, which dominates the collective excitation spectra in the weak tunneling situation. Finally, it is interesting to note that the first order dispersion correction to the out-of-phase plasmon gap is negative.

In Figs. 1–3 we present our numerical results for the collective excitation spectra of the coupled bilayer system without restricting to the long wavelength limit. In our calculations we have used RPA and also the so-called Hubbard approximation (HA) which includes a model static local field correction [8] to the noninteracting RPA irreducible polarizability [Eq. (3)]. In general, HA should be a better approximation than RPA at lower electron densities (and higher wave vectors) although, being an uncontrolled approximation, the quantitative improvement in HA over RPA is unknown. In Figs. 1 and 2 we show our calculated collective mode dispersions for different electron densities (with a fixed interlayer separation of $d = 200$ Å) in both RPA and HA, and for both zero and nonzero tunneling ($\Delta_{\mathrm{SAS}} = 0$, $\Delta_{\mathrm{SAS}} = 1$ meV). The important features to note in these results are (i) in general both tunneling and local field correction have little effect on the in-phase mode ω_+, which even at low densities seems to be well

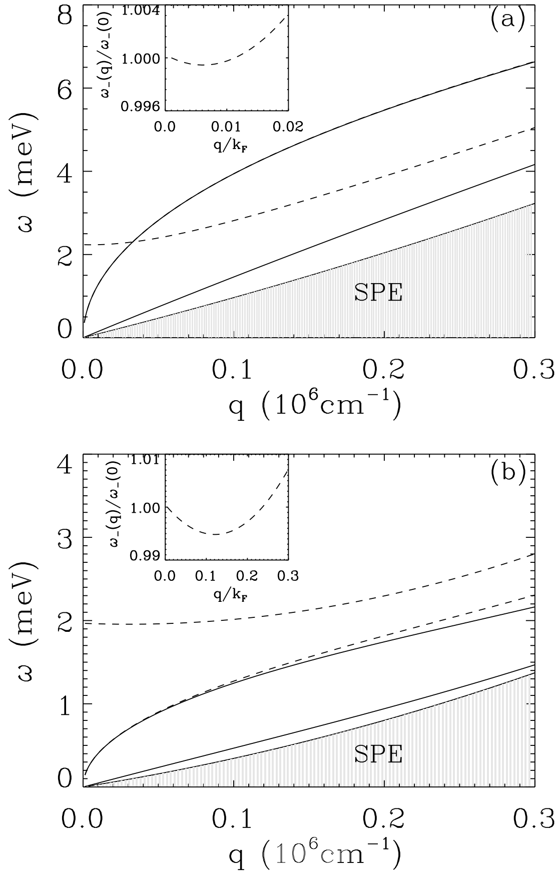

FIG. 1. RPA plasmon dispersion for (a) $n = 10^{11}$ cm^{-2} and (b) $n = 10^{10}$ cm^{-2}. Solid (dashed) lines represent plasmon dispersion in the absence (presence) of interlayer tunneling. Inset shows that the finite wave vector minimum of the out-of-phase mode ω_- in the presence of tunneling. We use parameters corresponding to GaAs quantum wells: $m = 0.067m_e$, $\kappa = 10.9$, and $d = 200$ Å; $\Delta_{\mathrm{SAS}} = 1.0$ meV for all our figures. Shaded region indicates the single particle excitation (SPE) Landau damping continuum.

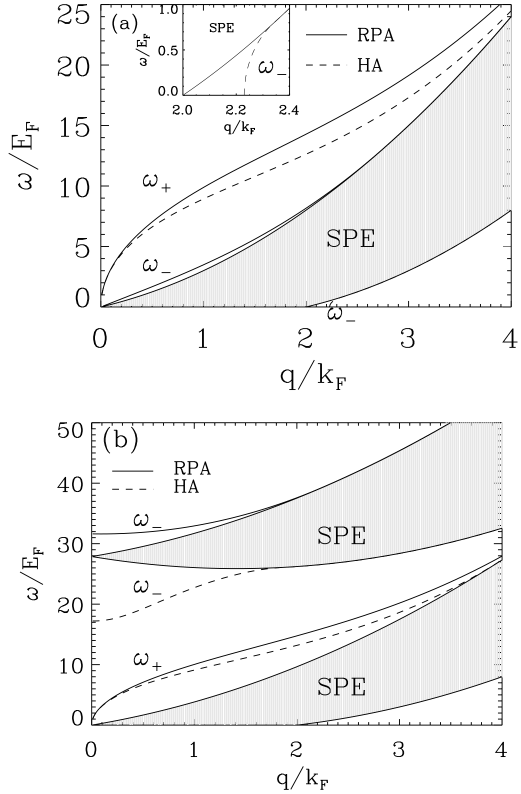

FIG. 2. Local field effects on the plasmon dispersion for density $n = 1.0 \times 10^9$ cm^{-2}: (a) in the absence of tunneling and (b) in the presence of tunneling ($\Delta_{\mathrm{SAS}} = 1.0$ meV). Inset shows the reappearance of the out-of-phase mode ω_- near $q = 2k_F$.

described by the long wavelength RPA formula [Eq. (4)]. This is a direct consequence of the f-sum rule—the ω_+ mode being the effective 2D plasma mode of the system is robust and insensitive to many-body and/or tunneling effects. This somewhat disappointing result is however important because it states emphatically that all efforts to study many-body effects by studying the usual in-phase 2D plasma dispersion are doomed to failure, a fact already empirically known to experimentalists. (ii) The out-of-phase ω_- mode is strongly influenced by tunneling and local field effects and could in principle be a sensitive experimental tool in studying many-body effects [9]. (iii) Local field effects in general reduce the ω_- frequency, and at low densities this could even lead to a complete suppression of the ω_- mode. In the absence of tunneling this complete suppression of the ω_- mode occurs below the critical density $n_c = q_{\mathrm{TF}}^2/[8\pi(1 + q_{\mathrm{TF}}d)^2]$ in the HA. Tunneling, however, has the dramatic effect of making the suppressed ω_- mode reappear when the layer separation d increases above $d_c = 1/2m\Delta_{\mathrm{SAS}} - 1/q_{\mathrm{TF}}$ even if $n < n_c$. (Note that, within the RPA, this Landau damping induced suppression of ω_- mode does not happen, and the ω_- mode exists for small wave vectors at all densities.) (iv) Tunneling opens up a zero frequency plasmon gap in the ω_- mode. In RPA the calculated gap $\Delta \equiv \omega_-(q = 0)$ is always greater than Δ_{SAS} [Eq. (6)]. In the HA, however, Δ could be above or below Δ_{SAS} and is given by

$$\Delta^2 = \Delta_{\mathrm{SAS}}^2 + \frac{\pi}{m}(n_+ - n_-)(q_{\mathrm{TF}}d) \times \left(1 - \frac{1}{2k_F d}\right)\Delta_{\mathrm{SAS}}. \quad (8)$$

If the total density $N < N_c = 1/(8\pi d^2)$, the plasmon gap Δ is less than the single particle gap Δ_{SAS}, and we have the interesting situation of a collective plasmon mode being lower in energy than the corresponding single particle energy. (v) Tunneling leads to a weak negative dispersion of the ω_- mode at long wavelengths (i.e., $C_1 < 0$). (vi) Finally, a very interesting finding (Fig. 2) is that at low densities the ω_- mode is, in fact, more stable in the HA (than in RPA) because it lies below the single particle continuum in the presence of tunneling. In the absence of tunneling the ω_- mode may reappear at *large wave vectors* beyond the single particle continuum although it is completely suppressed (by Landau damping) at long

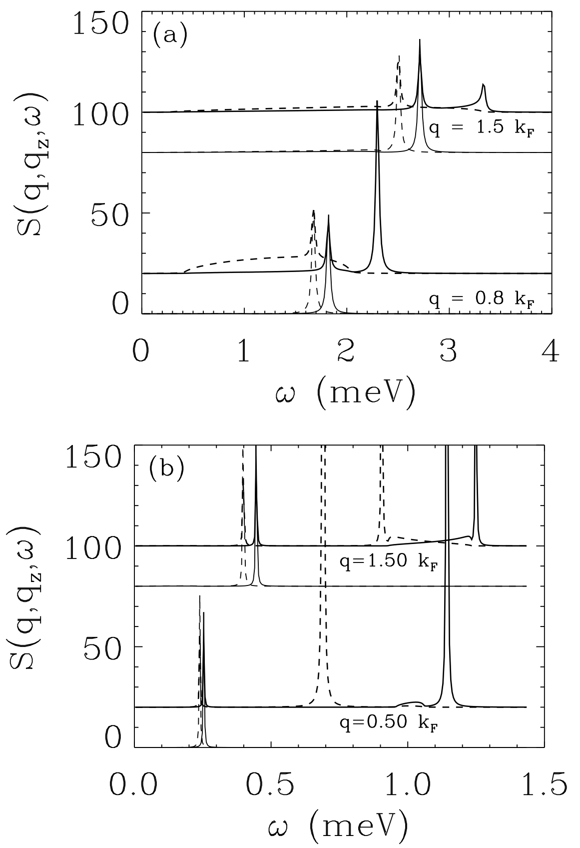

FIG. 3. The dynamical structure factor for (a) $n = 10^{10}$ cm^{-2} and (b) $n = 10^9$ cm^{-2} in the RPA (solid lines) and HA (dashed lines) for finite tunneling ($\Delta_{SAS} = 1.0$ meV). Here, thin (thick) lines correspond to $q_z d = 0.0$ ($q_z d = \pi/4$).

wavelengths. This reappearance of the ω_- mode at large wave vectors (for $\Delta_{SAS} = 0$) is purely a local field effect and has earlier been attributed [10] to a charge density wave instability in bilayer structures. Tunneling, according to our theory, stabilizes the long wavelength ω_- mode which now lies between the two (symmetric and antisymmetric) single particle continua, and is in fact more stable in HA than in RPA. Tunneling, therefore, opposes the presumptive charge density wave instability [10].

In Fig. 3 we show our calculated spectral weight or the dynamical structure factor $S(q, q_z, \omega)$, which is given [7] by the imaginary part of the density-density correlation function where q_z is the probe wave vector normal to the layers [3,4,7], for the collective modes in coupled bilayer systems both in RPA and HA for finite tunneling. For $q_z d = 0.0$ only the in-phase mode ω_+ carries any weight, but for finite $q_z d$ the out-of-phase mode carries substantial spectral weight even at long wavelengths ($q \rightarrow 0$). The most important message here is that both $\omega_\pm$ modes in general carry finite spectral weights and should be observable in resonant inelastic light scattering [3,4] and far infrared optical [2] spectroscopies.

We point out that the ω_- mode, which presumably becomes the Goldstone mode in the symmetry-broken phase, could be used as an experimental probe to search for the theoretically predicted [11] interlayer-spontaneous-phase-coherent quantum state in low density bilayer structures. It is interesting in this context to point out that it has been claimed [12] that the local field correction by itself within the so-called quasilocalized charge approximation [12], *even in the absence of any interlayer tunneling,* could lead to the opening of a long wavelength gap in the out-of-phase mode in bilayer systems.

The main approximations of our theory, namely, treating the tunneling amplitude t as a phenomenological parameter and using the Hubbard approximation for the local field correction, should not affect our qualitative conclusions. In particular, t decreases with increasing layer separation d in a calculable way, and therefore for large d one always recovers the no-tunneling limit of a vanishing plasma gap. While there is no general consensus on the best possible local field corrections in electron liquids, it is well known [8] that the Hubbard approximation gives quantitatively similar results to more sophisticated local field corrections involving self-consistent approximations [8].

In conclusion, we establish in this paper that bilayer semiconductor double quantum well structures should be a useful tool for studying the interplay between tunneling and many-body effects on collective mode dispersion—in particular, the dispersion of the out-of-phase collective mode in the presence of tunneling should show interesting observable many-body effects.

This work is supported by the U.S.-ARO and the U.S.-ONR. One of us (S. D. S.) thanks J. Eisenstein for asking an important question.

[1] S. Das Sarma and A. Madhukar, Phys. Rev. B **23**, 805 (1981).
[2] S. J. Allen, Jr. *et al.*, Phys. Rev. Lett. **38**, 980 (1977); T. N. Theis, Surf. Sci. **98**, 515 (1980).
[3] D. S. Kainth *et al.*, Phys. Rev. B **57**, R2065 (1998); A. S. Bhatti *et al.*, in *Proceedings of the 23rd International Conference on the Physics of Semiconductors,* edited by M. Scheffler and R. Zimmerman (World Scientific, Singapore, 1996).
[4] A. Pinczuk *et al.*, Phys. Rev. Lett. **56**, 2092 (1986); G. Fasol *et al., ibid.* **56**, 2517 (1986); D. Olego *et al.*, Phys. Rev. B **25**, 7867 (1982).
[5] D. Pines, Can. J. Phys. **34**, 1379 (1956); D. Pines and J. R. Schrieffer, Phys. Rev. **124**, 1387 (1961).
[6] S. Das Sarma *et al.*, Phys. Rev. Lett. **79**, 917 (1997); Phys. Rev. B **58**, 4672 (1998), and references therein.
[7] J. K. Jain and S. Das Sarma, Phys. Rev. B **36**, 5949 (1987).
[8] M. Jonson, J. Phys. C **9**, 3055 (1976); N. Iwamoto, Phys. Rev. B **43**, 2174 (1991).
[9] K. Flensberg and B. Y. K. Hu, Phys. Rev. Lett **73**, 3572 (1994).
[10] D. Neilson *et al.*, Phys. Rev. Lett. **71**, 4035 (1993).
[11] L. Zheng *et al.*, Phys. Rev. B **55**, 4506 (1997).
[12] G. Kalman and K. I. Golden, Phys. Rev. B **57**, 8834 (1998).