Lifetime of a Disoriented Chiral Condensate

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The lifetime of a disoriented chiral condensate formed within a heat bath of pions is calculated assuming temperatures and densities attainable at present and future heavy-ion colliders. A generalization of the reduction formula to include coherent states allows us to derive a formula for the decay rate. We predict the half-life to be between 4 and 7 fm/c, depending on the assumed pion density. We also calculate the lifetime in the presence of higher resonances and baryons, which shortens the lifetime by at most 20%. [S0031-9007(98)07558-9]

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Formation of hot and dense matter in heavy-ion collisions has the possibility of creating a phase where chiral symmetry is restored. As this matter cools and expands, the vacuum could relax into the "wrong" zero temperature ground state. Subsequent shifting of the vacuum back into alignment with the outside world could then lead to an excess of low momentum pions [1] in a single direction in isospin space [2]. This excess is called a disoriented chiral condensate (DCC), and many studies have looked into whether its presence could be a signal of chiral symmetry restoration in heavy-ion collisions.

Most of the discussion has centered around the details of formation of such regions [1–6]. There has been a general consensus that these regions could be numerous and large enough with the conditions at the Relativistic Heavy-Ion Collider (RHIC) and the CERN Large Hadron Collider (LHC) to be detected. While the idea of DCC formation is appealing, the physics governing the chiral phase transition is not known well enough to make reliable predictions about the possible formation of these condensates. Experiment is needed to establish their existence.

The ability to detect a DCC from hadronic observables in heavy-ion collisions depends on the condensate lifetime. An early estimate [4] gave a half-life of $\tau \sim 3$ fm/c. A more recent calculation [5] has found a damping rate of $\gamma \sim 1$ (fm/c)$^{-1}$. This roughly corresponds to a half-life of $\tau \sim 1/\gamma \sim 1$ fm/c, which would be short enough to jeopardize the definiteness of the signal. This calculation was based on the $O(4)$ sigma model in the symmetric phase. However, after formation, the DCC lives in the phase of spontaneously broken chiral symmetry. Characteristic of this phase is the suppression of S-wave scattering among pions, which should protect the low momentum pion modes in the DCC. Indeed, an extension of Ref. [5] to the broken phase produces smaller damping rates [6].

The purpose of this Letter is to provide a reliable estimate of the lifetime of a DCC state in the hadronic (chirally broken) phase. Defining the DCC to be a coherent state of pions with low momenta, we derive a general formula for the decay rate in the presence of other hadrons. The effect of higher resonances as well as that of baryons to the DCC lifetime is also estimated. Our calculation is constrained by data at every point possible.

Assuming the formation of a DCC, interactions with the thermal heat bath can enhance or deplete the number of pions in the condensate. In a heavy-ion collision, pions are the most abundant thermal particles, and so their interactions with the DCC are expected to give the dominant contribution to the decay rate. The DCC can be written as a coherent state [2,4]

$$|\eta\rangle = e^{-N/2} e^{\int d\tilde{k}\, \eta_k a_k^\dagger} |0\rangle \,, \tag{1}$$

normalized to unity and for definiteness taken to consist of neutral pions, π^0, created by $a_k^\dagger$. The Lorentz invariant measure $d\tilde{k} = d^3k/(2\pi)^3 2E_k$ is weighted by the normalizable function η_k to give a momentum distribution which could evolve over time [7]. The number of particles in the DCC is then given by $N = \int d\tilde{k} |\eta_k|^2$.

We can get an estimate of the decay rate by considering two body interactions of the DCC pions with the surrounding thermal pions. Contributions come from $\pi\pi$ scattering with either two thermal pions knocking one pion into the condensate or a DCC pion interacting with a thermal pion to escape from the condensate. There could also be individual scatterings with two pions knocked out or put into the condensate, but these should be suppressed due to the restricted phase space of the DCC. Using

$$\langle \eta | \pi^0(x) | \eta \rangle = \int d\tilde{k}\, e^{-ikx} \langle \eta | a_k | \eta \rangle = \int d\tilde{k}\, e^{-ikx} \eta_k \,, \tag{2}$$

a rederivation of the Lehmann-Symanzik-Zimmerman (LSZ) reduction formula taking into account the coherent state gives

$$\frac{dN}{dt} = \int d\tilde{k}_1\, d\tilde{k}_2\, d\tilde{k}_3\, F_{123} \langle |\mathcal{T}_{\pi\pi}|^2 \rangle \times 2\pi \delta(E_0 + E_1 - E_2 - E_3) \frac{|\eta_{k_2+k_3-k_1}|^2}{(2E_0)^2} \,. \tag{3}$$

The matrix element for $\pi\pi$ scattering $\langle|\mathcal{T}_{\pi\pi}|^2\rangle = \frac{1}{2}\sum_I \frac{1}{3}(2I+1)|T^I_{\pi\pi}|^2$ is isospin averaged to account for only π^0's coming from the DCC and the factor of $\frac{1}{2}$ accounts for identical particles in the final state after isospin considerations. The thermal weighting is given by

$$F_{123} = f_2 f_3(1+f_1) - f_1(1+f_2)(1+f_3), \quad (4)$$

with $f_i = [\exp(E_i/T) - 1]^{-1}$ representing the Bose-Einstein momentum distribution. The DCC pion energy is $E_0 = [(k_2 + k_3 - k_1)^2 + m_\pi^2]^{1/2}$.

The phase space correlations in Eq. (3) can be simplified for DCC pions narrowly peaked near a single momentum k_0. Then the square of the momentum distribution can be replaced by a momentum-conserving delta function $|\eta_{k_2+k_3-k_1}|^2 \to N(2\pi)^3 2E_0\delta^3(k_0 + k_1 - k_2 - k_3)$, giving the decay rate

$$\frac{1}{N}\frac{dN}{dt} = \frac{1}{2E_0}\int d\tilde{k}_1\, d\tilde{k}_2\, d\tilde{k}_3\, F_{123}\langle|\mathcal{T}_{\pi\pi}|^2\rangle(2\pi)^4 \times \delta^4(\Sigma k_i). \quad (5)$$

Although this formula pertains to two-particle interactions,it can easily be generalized to include more particles. To understand the various contributions to the decay rate, we first take the DCC pions to be at rest, $k_0 = (m_\pi, \mathbf{0})$. We relax this condition in Eq. (13), allowing for a finite spread in the momenta in accordance with Eq. (3).

This result was derived assuming the removal or addition of a pion to the condensate does not affect the DCC coherent state. This approximation is valid to better than 10% until the number of particles in the condensate decreases below $N \sim 5$, at which point the existence of a macroscopic condensate is questionable anyway. Equation (5) can also be derived starting from the Boltzmann equation by assuming the DCC momentum distribution function obeys $f_0 \sim 1 + f_0$, which again is appropriate for large enough N and distributions peaked at zero momentum.

We can now estimate the decay rate by replacing the $\pi\pi$ scattering matrix element by its experimental value in terms of the center of mass energy $\sqrt{s}$, three-momentum q, and scattering angle θ,

$$T^I_{\pi\pi} = 32\pi\sum_l (2l+1)P_l(\cos\theta)\frac{\sqrt{s}}{2q}e^{i\delta^I_l}\sin\delta^I_l. \quad (6)$$

Accounting for the three dominant phase shifts δ^0_0, δ^1_1, and δ^2_0 by using a parametrization of the data [8], the momentum integral in the decay rate can be readily evaluated. Defining the half-life τ by

$$\frac{1}{N}\frac{dN}{dt} \equiv -\frac{1}{\tau}, \quad (7)$$

we find $\tau = 8.9$ fm$/c$ for $T = 150$ MeV and $\tau = 5.6$ fm$/c$ for $T = 170$ MeV. Considering the entire hadronic phase exists for about 10–20 fm$/c$, the DCC could live for a substantial fraction of the hadronic phase, allowing for a strong signal of its existence, e.g., in the dilepton channel [9]. Even assuming 3 times the number of thermal pions, as predicted by some event generators [10], the half-life is still $\tau = 3.8$ fm$/c$ at $T = 150$ MeV.

It should be noted that the momentum dependence of the phase shifts is important to the result. Approximating the $\pi\pi$ amplitude purely by threshold ($q = 0$) scattering lengths gives a half-life 5 times longer. Only if the range terms, which have a q^2 dependence, are also kept does the estimate come within 10% of the full phase shift result. The ρ channel (P-wave) contribution is merely a few percent effect, in agreement with estimates using a simple Breit-Wigner form. The main contribution to the decay rate comes from the S wave. This is because the other phase shifts are very small in the region where the Bose-Einstein distribution functions are appreciable.

The ρ resonance could possibly play a role as a thermal particle though. We therefore look at $\pi\rho$ scattering. The dominant contribution comes from the formation of an $a_1(1240)$ meson in the s channel, which we can model by a relativistic S-wave Breit-Wigner matrix element

$$\langle|\mathcal{T}_{\pi\rho}|^2\rangle = \frac{(8\pi s)^2}{q^2}\frac{3\Gamma^2_{a1}(s)}{(s - m^2_{a1})^2 + s\Gamma^2_{a1}(s)} \quad (8)$$

with center of mass three-momentum q and a momentum dependent width whose form is dictated by the general decay of an axial-vector particle into a vector and pseudoscalar particle ($q_{a1} = q|_{s=m^2_{a1}}$)

$$\Gamma_{a1}(s) = 400\,\frac{q(3m^2_\rho + q^2)}{q_{a1}(3m^2_\rho + q^2_{a1})}\text{ MeV}. \quad (9)$$

Adding this matrix element to Eq. (5) gives a negligible effect, not changing the original $\pi\pi$ scattering half-life to the accuracy quoted. This can be seen in Fig. 1, where the matrix elements for $\pi\pi$ and $\pi\rho$ scattering are compared for various temperatures.

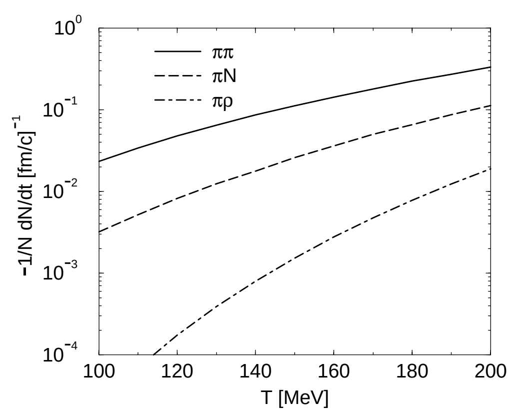

FIG. 1. The contribution to the decay rate, Eq. (5), in inverse fm$/c$ from $\pi\pi$, $\pi\rho$, and πN scattering keeping the ratio of pions to nucleons fixed at five.

The Boltzmann suppression of the a_1 pole along with threshold suppression from the width conspire to make $\pi\rho$ scattering a small effect. Other meson-meson processes not considered here, such as πa_1 scattering, are therefore expected to be smaller.

Baryons could also play a substantial role in reducing the lifetime of a DCC. Even at CERN Super Proton Synchrotron (SPS) energies ($E_{\mathrm{c.m.}} = 20$ GeV/nucleon) a large number of baryons are found in the central rapidity region. We therefore study the dependence of the half-life on nucleon density. The matrix element for πN scattering is [11]

$$\langle|\mathcal{T}_{\pi N}|^2\rangle = 64\pi^2 s \sum_I \frac{1}{3}(2I+1)(|g_I(s,\theta)|^2 + |h_I(s,\theta)|^2) \qquad (10)$$

with $g_I(s,\theta)$ and $h_I(s,\theta)$ denoting the spin nonflip and spin-flip amplitudes for each of the two possible isospins $I = \frac{1}{2}, \frac{3}{2}$. These can be written in terms of phase shifts labeled by $\alpha = l, 2I, 2J$

$$g_{1/2}(s,\theta) = \mathcal{F}_{S11} + \cos\theta(2\mathcal{F}_{P13} + \mathcal{F}_{P11}) + \ldots,$$
$$h_{1/2}(s,\theta) = \mathcal{F}_{P13} - \mathcal{F}_{P11} + \ldots,$$

with similar expressions for the $I = \frac{3}{2}$ channel by direct substitution of the isospin indices. The ellipses refer to higher partial waves we will not consider here and

$$\mathcal{F}_\alpha = [q\cot\delta_\alpha(q) - iq]^{-1}. \qquad (11)$$

Adding this matrix element to Eq. (5), we must also replace the thermal weighting factor of this term by

$$F_{123}^N = f_2 f_3^N (1 - f_1^N) - f_1^N (1 + f_2)(1 - f_3^N) \qquad (12)$$

with $f_i^N = \{\exp[(E_i - \mu)/T] + 1\}^{-1}$. A chemical potential μ has been introduced to enforce a particular nucleon density.

For a parametrization of the πN phase shifts, we use the full relativistically improved $\Delta(1232)$ isobar model [11], which reproduces the experimental phase shifts very well up to the inelastic threshold. Adding this decay rate to those of $\pi\pi$ and $\pi\rho$ scattering without a chemical potential gives only a few percent decrease in the half-life. But taking $\mu = 260$ MeV at $T = 150$ MeV to enforce a 5:1 ratio between pions and nucleons as observed at the SPS [12], we end up with a half-life of $\tau = 7.2$ fm/c. The effect of the nucleons is about a 20% reduction in τ. Still, this effect is small enough to imply experiments already taking place at the SPS should be capable of detecting remnants of DCC formation, provided, of course, that the conditions for DCC formation are met at these energies.

A summary of our results is shown in Fig. 1. At each temperature the chemical potential of the nucleons is chosen to enforce the 5:1 ratio between pions and nucleons. Since the effect of the nucleons is small, this result also applies for RHIC energies ($E_{\mathrm{c.m.}} = 200$ GeV/nucleon) where the central rapidity pion to baryon ratio is probably even larger.

We also show a plot of the dependence of half-life on nucleon density for $T = 150$ MeV in Fig. 2. The πN contribution becomes comparable with the $\pi\pi$ contribution when there is about one nucleon for every pion. However, we have so far assumed completely thermalized pions and nucleons for this estimate, whereas some event generators indicate an enhancement of the occupation numbers by a factor of 3 [10]. Taking this into account in our calculation gives the dashed line in Fig. 2, which shows the half-life is between 3 and 3.5 fm/c for all the pion to nucleon ratios considered. If the overpopulation of particle spectra predicted by event generators is correct, it implies the DCC signal would be difficult to detect from hadronic observables, requiring an indirect means such as dileptons [9].

Rather than taking the condensate to consist of only pions at rest, we can investigate the change in the lifetime for a finite spread in the DCC momenta. This will open up the phase space of the DCC and allow for further interactions with the heat bath. By introducing an integration over the Mandelstam variables $s = (k_0 + k_1)^2$ and $t = (k_0 - k_2)^2$ [13], we can simplify Eq. (5) for πP scattering (with $P = \pi$, ρ, or N) to

$$\frac{1}{N}\frac{dN}{dt} = \frac{1}{(2\pi)^4 16 E_0^3} \int ds\, dt\, \langle|\mathcal{T}(s,t)|^2\rangle \times \int dE_1\, dE_2 \frac{F_{123}}{\Phi^{1/2}} \qquad (13)$$

with $\Phi = 4K_1^2K_2^2 - (K_1^2 + K_2^2 - K_3^2)^2$, $k_0 = (E_0, \mathbf{k}_0)$,

$$K_i^2 = -\frac{m_\pi^2}{E_0^2}(E_i^2 - \bar{m}_i^2) - m_i^2 + \frac{E_i}{E_0} q_i^2 - \frac{q_i^4}{4E_0^2},$$
$$q_1^2 = s - m_\pi^2 - m_P^2, \qquad q_2^2 = 2m_\pi^2 - t,$$
$$q_3^2 = q_1^2 - q_2^2, \qquad \bar{m}_1^2 = m_{1,3}^2 = m_P^2,$$
$$\bar{m}_2^2 = m_2^2 = m_\pi^2, \qquad \bar{m}_3^2 = 2E_0E_3 + u,$$

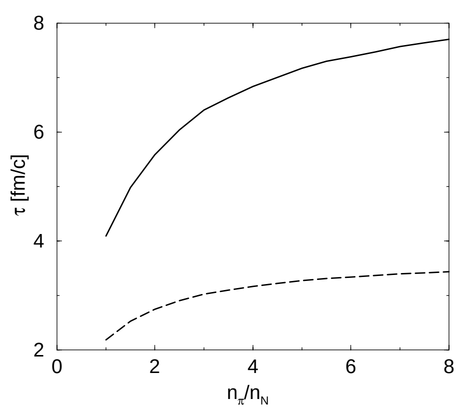

FIG. 2. The half-life τ at $T = 150$ MeV for various pion to nucleon ratios. The dashed line is for 3 times the number of thermal pions and nucleons.

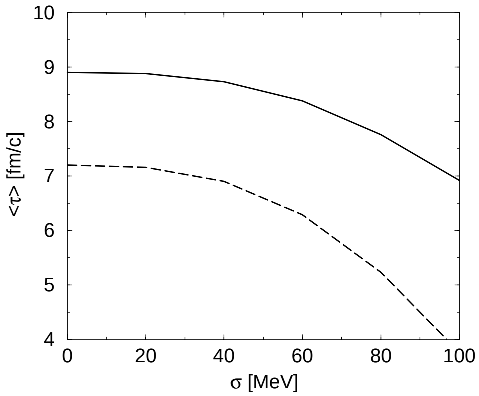

FIG. 3. The average half-life $\langle \tau \rangle$ for $T = 150$ MeV as a function of the width of the DCC pion momentum distribution σ from $\pi\pi$ scattering alone (solid) and also including $\pi\rho$ and πN scattering (dashed).

with integration limits dictated by $s > (m_\pi + m_P)^2$, $t < 0$, $K_i^2 > 0$, $\Phi > 0$, and $E_0 + E_1 > E_2 + m_P$. The thermal pion energy should also be restricted to be above the maximum DCC energy E_0.

We then average the half-life obtained from Eq. (13) over the momentum distribution in the DCC as dictated by Eq. (3). Taking $|\eta_k|^2 \propto \exp(-|\mathbf{k}_0|^2/\sigma^2)$, the average half-life is plotted as a function of the width σ for $T = 150$ MeV in Fig. 3. The solid line is for the $\pi\pi$ scattering contribution alone, and the dashed line also includes the $\pi\rho$ and πN contributions. The pion to nucleon ratio is kept fixed at 5:1. The decrease of the half-life from 8.9 fm$/c$ to 7.2 fm$/c$ for $|\mathbf{k}_0| = 0$ was due to the nucleon contribution. As σ increases, P-wave $\pi\pi$ scattering and, to a lesser extent, $\pi\rho$ scattering become more active, decreasing $\langle \tau \rangle$. Consequently, smaller DCC domains (characterized by a larger σ) decay faster.

In conclusion, we have derived a simple decay rate for a DCC and applied it assuming initial conditions at present and future heavy-ion colliders. In all stages of the calculation we have used input from data to constrain our results. The lifetime obtained is longer than previously estimated.

We find interactions with the surrounding heat bath are dominated by $\pi\pi$ scattering leading to $\tau = 8.9$ fm$/c$. Even with large nucleon densities, such as those that occur at the SPS, inclusion of πN scattering reduces the half-life by at most 20%. Therefore models that account for only pion degrees of freedom should still give reasonable results. However, the factor of 3 estimated enhancement of thermal distributions assumed at the SPS is what most dramatically effects the half-life, reducing it to about 3–4 fm$/c$. This is probably not long enough to observe DCCs in existing hadronic data.

The decay of small DCC domains is accelerated by the contribution of P-wave $\pi\pi$ scattering as well as $\pi\rho$ scattering away from threshold, decreasing the average lifetime. With large domains formed in the absence of nucleons, as could occur at future colliders, the detection of DCCs looks more promising.

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[1] K. Rajagopal and F. Wilczek, Nucl. Phys. **B404**, 577 (1993).
[2] J. D. Bjorken, Int. J. Mod. Phys. A **7**, 4189 (1992); K. L. Kowalski and C. C. Taylor, hep-ph/9211282.
[3] J.-P. Blaizot and A. Krzywicki, Phys. Rev. D **46**, 246 (1992); J. D. Bjorken, Acta Phys. Pol. B **23**, 561 (1992); S. Gavin, A. Goksch, and R. D. Pisarski, Phys. Rev. Lett. **72**, 2143 (1994); S. Gavin and B. Müller, Phys. Lett. B **329**, 486 (1994); M. Asakawa, Z. Huang, and X.-N. Wang, Phys. Rev. Lett. **74**, 3126 (1995); D. Boyanovsky, H. J. de Vega, and R. Holman, Phys. Rev. D **51**, 734 (1995); F. Cooper, Y. Kluger, and E. Mottola, and J. P. Paz, Phys. Rev. D **51**, 2377 (1995); J. Randrup, Phys. Rev. Lett. **77**, 1226 (1996).
[4] A. A. Anselm and M. G. Ryskin, Phys. Lett. B **266**, 482 (1991).
[5] T. S. Biro and C. Greiner, Phys. Rev. Lett. **79**, 3138 (1997).
[6] D. Rischke, nucl-th/9806045.
[7] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).
[8] G. Bertsch, M. Gong, L. McLerran, V. Ruuskanen, and E. Sarkkinen, Phys. Rev. D **37**, 1202 (1988).
[9] Y. Kluger, V. Koch, J. Randrup, and X. N. Wang, Phys. Rev. C **57**, 280 (1998).
[10] H. Sorge (private communication).
[11] T. Ericson and W. Weise, *Pions and Nuclei* (Clarendon Press, Oxford, 1988).
[12] NA49 Collaboration, S. V. Afanasiev *et al.*, Nucl. Phys. **A610**, 188c (1996); P. Braun-Munzinger and J. Stachel, nucl-ex/9803015.
[13] J. Kapusta, P. Lichard, and D. Seibert, Phys. Rev. D **44**, 2774 (1991).