

Procedure for Direct Measurement of the Cabibbo-Kobayashi-Maskawa Angle γ

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A natural procedure is presented to measure the angle γ from the decay $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$. It is based in the Dalitz plot fitting analysis. Neither amplitudes nor strong phases have to be known *a priori*. We present simulations of this decay computing both statistical and theoretical uncertainties and analyze the experimental feasibility. We found that γ could be measured with a combined error of the order of 20° with 90% of C.L. after about a couple of years of running of the first generation of B factories. [S0031-9007(98)07586-3]

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The study of hadronic decays in the B system seems to be a powerful tool for the understanding of CP violation. To check standard model (SM) predictions it is particularly important to measure the three Cabibbo-Kobayashi-Maskawa (CKM) angles $\alpha \equiv \arg(-V_{td}V_{tb}^*/V_{ud}V_{ub}^*)$, $\beta \equiv \arg(-V_{cd}V_{cb}^*/V_{td}V_{tb}^*)$ and $\gamma \equiv \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$. Only β is expected to be clearly measured from the gold plated $B_0 \rightarrow J/\psi K_s$ decay which is almost free from theoretical uncertainties [1] and benefits from the large (10^{-3}) branching ratio. The extraction of the two other angles requires the measurement of decays with branching ratios of the order of 10^{-5} or less. Many interesting methods using various decays have been proposed so far in the literature [2-4] but the matter is still open.

More precisely, the angle γ seems to be hard to measure. The method presented in Ref. [2] provides a theoretically clean procedure to extract γ combining decays with D^0 in the final state. Unfortunately, both the original method and clever extensions [3] of it demand a large statistics. As a result, one expects to need about 10 years [5] of data taking in the first generation of B factories to attain a reasonable error—at least 15° —in the measurement of γ . It is then interesting to look for other methods that could provide a constraint for the value of γ in much less time—their eventual theoretical errors should be as well estimated as possible [6].

In this Letter we present a direct and simple method that could provide a nice first measurement of the angle γ after about a couple of years of data taking in the first generation of B factories. We use the decay $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ where the necessary interference is given by the intermediate resonant channel $\chi_{c0} \pi^\pm$. This channel has been first pointed out in Ref. [7]; nevertheless, in that reference the method used to extract the angle was very model dependent and demanded large statistics. Here, we present a totally different approach: we show the viability of performing a full Dalitz plot analysis of this decay. It can provide a *direct* measurement of the angle γ free from model dependencies.

Other methods existing in the literature to measure γ , independently of the considered channel are based in the measurement of *branching ratios* and asymmetries [1]. The relationship between these measured *numbers* and the angle γ is not direct. Moreover, these methods generally present discrete ambiguities.

The main feature of our method is that it exploits the fact that in three body decays one can have a direct measurement of the *amplitude* of a decay—instead of branching ratios, that is amplitude squared. This means that one can have a *direct experimental access to the phase* of a given decay. This fact has already been used [8] in connection with CP violation, in a quite different context. The method presented in this Letter can eventually be also used to extract CP violating angles from other three body decays of charged B 's.

Let us present our ideas using the channel $B^+ \rightarrow \pi^+ \pi^+ \pi^-$. Many intermediate channels contribute. Indeed, resonant channels— $\rho^0 \pi^+$, $f_0 \pi^+$, $\chi_{c0} \pi^+$, etc.—together with the direct nonresonant decay produce the same experimentally detected final state. This final state is thus the product of the *interference* of all these intermediate states.

The fact that in three body decays one can measure differential widths—usually displayed in a Dalitz plot (DP)—allows a clean separation of these partial channels. The distribution of measured events in the plot can be fitted using appropriate fitting functions.

The fitting technique has proven to be very successful in describing, for example, three body decays of D mesons [9], *even with only about one hundred reconstructed events* [10].

In order to do so, one considers a fitting function including one term for each intermediate channel contributing to the final state. For example, for the decay $B^+ \rightarrow \pi^+ \pi^+ \pi^-$ the fitting function should be

$$\mathcal{F}_{B^+ \rightarrow \pi^+ \pi^+ \pi^-}(m_1^2, m_2^2) = |\sum_i a_i e^{i\theta_i} F_i(m_1^2, m_2^2)|^2, \quad (1)$$

where $m_1^2 = (p_{\pi_1^+} + p_{\pi^-})^2$ and $m_2^2 = (p_{\pi_2^+} + p_{\pi^-})^2$ are the usual Dalitz plot invariant variables, F_i are the

amplitudes corresponding to each partial channel, and a_i and θ_i are unknown real parameters that will emerge from the fit. The sum is performed over all the intermediate resonances as well as the nonresonant decay. For the resonant channels, the function F_i is very well known: it is simply the usual Breit-Wigner [11] times an angular function according to the spin of the resonance. The nonresonant decay amplitude is discussed in [12]; we will be back to it later.

The Dalitz plot maximum-likelihood technique uses the function of Eq. (1) to fit the measured differential width distribution $d\Gamma/dm_1^2 dm_2^2$ of the total decay. The output is the amplitude fractions a_i and phases θ_i of each partial decay. In other words, it brings a *direct measurement* of all the phases.

In general, these phases can be written as $\theta_i = \delta_i + \phi_i$, where δ_i is a CP conserving and ϕ_i is a CP violating phase, respectively. Obviously, in this way it is not possible to separate δ_i from ϕ_i because only their sum is measured.

Nevertheless, now consider the CP conjugated decay $B^- \rightarrow \pi^- \pi^- \pi^+$. The phase of each partial amplitude changes to $\bar{\theta}_i = \delta_i - \phi_i$.

If one then applies the fitting procedure to the DP's corresponding to both B^+ and B^- decays, *one gets a direct measurement of the CP violating phase*,

$$\phi_i = (\theta_i - \bar{\theta}_i)/2. \quad (2)$$

This procedure does not require one to make any assumption about final-state interactions (FSI) as other methods to measure CP violation demand [1]. In B meson decays, FSI are usually assumed to be small [13], but this is not necessarily correct [14,15]. Here, strong phases δ_i do not have to be known *a priori*. Moreover, using this procedure one can also obtain a direct measurement of the strong phases $\delta_i = (\theta_i + \bar{\theta}_i)/2$. This by-product of our method could be an interesting input to other methods.

For this procedure to apply one needs at least two intermediate channels with different CP violating phases. Indeed, if all the intermediate channels have the same CP violating phase it would factor out; in other words, there would be no interference to peak CP violation.

In the decay $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$, the $\chi_{c0} \pi^\pm$ partial channel produces the necessary interference to extract the angle γ . This channel is driven by the CKM coefficients $V_{bc} V_{cd}^*$ and it thus has no CP violating phase. On the other side,

the direct nonresonant contribution as well as the other resonant channels— $\rho^0 \pi^\pm, f_0 \pi^\pm$, etc.—proceed via the CKM coefficients $V_{ub} V_{ud}^*$ and their amplitudes thus contain the weak phase γ .

Unfortunately, all the partial channels but $\chi_{c0} \pi$ contain also penguin diagrams which are driven by another CP violating angle, β . These penguin contributions are expected to be small but not negligible [1,16].

In the following, we will present our simulations of the B^\pm decays. As a first step, we will not include penguin contributions. The corrections due to their inclusion will be studied later in this Letter. They are the *only* source of theoretical uncertainties within the method presented here.

The experimental simulation consisted in the following. First, we have generated a sample of $B^+ \rightarrow \pi^+ \pi^+ \pi^-$ events using Monte Carlo technique. The dynamics was given by the function \mathcal{F} of Eq. (1), with a given set of input parameters a_i and θ_i . Then, we fitted the generated distribution of events in the DP using the maximum-likelihood fitting technique and MINUIT package [17]. The fitting function was \mathcal{F} but now a_i and θ_i are the floating parameters which are obtained from the fit. These two steps were then repeated for the CP conjugated decay $B^- \rightarrow \pi^- \pi^- \pi^+$.

In fact, we have considered a number of sets of input parameters a_i and θ_i corresponding to various possible scenarios for the unknown quantities involving this decay: the relative weight of each partial channel, their relative strong phases, and the angle γ .

In Tables I and II we show the result of one of our simulations of the decay. It describes a probable scenario according to our present knowledge:

(1) $\mathcal{B}(B^+ \rightarrow \chi_{c0} \pi^+) \sim 5 \times 10^{-5}$ [7]; $\mathcal{B}(\chi_{c0} \rightarrow \pi^+ \pi^-) \sim 0.8\%$ [18]; $\mathcal{B}(B^+ \rightarrow \pi^+ \pi^+ \pi^-)_{\text{NR}}$ (non-resonant) $\sim 10^{-5}$ [19]; $\mathcal{B}(B^+ \rightarrow \rho^0 \pi^+) \sim 8 \times 10^{-6}$ [20]; $\mathcal{B}(\rho^0 \rightarrow \pi^+ \pi^-) \sim 100\%$. We have assumed $\mathcal{B}(B^+ \rightarrow f_0 \pi^+) \sim \mathcal{B}(B^+ \rightarrow \rho^0 \pi^+)$. From the square roots of the numbers above, one simply gets the coefficients a_i of the column “input” in Tables I and II.

(2) The CP violating phase $\phi_2 = \phi_3 = \phi_4 = \gamma$ for B^+ has been chosen as [21] 65° . The unknown strong FSI phases δ_i have been arbitrarily taken as $5^\circ, 15^\circ$, and -10° for NR, $f_0 \pi^\pm$ and $\rho^0 \pi^\pm$, respectively. With these numbers one gets the values θ_i of the column “input” in Tables I and II.

TABLE I. Fitting results for B^+ Monte Carlo sample.

Decay	Input	200 Events	500 Events	1000 Events
$\chi_{c0} \pi^+$	a_1	1.0	Fixed	Fixed
	θ_1	0°	Fixed	Fixed
NR	a_2	4.0	3.1 ± 0.7	4.1 ± 0.6
	θ_2	70°	$64^\circ \pm 25^\circ$	$71^\circ \pm 17^\circ$
$f_0 \pi^+$	a_3	2.5	2.5 ± 0.6	2.8 ± 0.4
	θ_3	80°	$99^\circ \pm 28^\circ$	$88^\circ \pm 18^\circ$
$\rho^0 \pi^+$	a_4	3.0	2.1 ± 0.6	3.4 ± 0.5
	θ_4	55°	$74^\circ \pm 26^\circ$	$39^\circ \pm 18^\circ$

TABLE II. Fitting results for B^- Monte Carlo sample.

Decay	Input	200 Events	500 Events	1000 Events
$\chi_{c0}\pi^-$	a_1	1.0	Fixed	Fixed
	$\bar{\theta}_1$	0°	Fixed	Fixed
NR	a_2	4.0	4.2 ± 1.0	3.6 ± 0.6
	$\bar{\theta}_2$	-60°	$-72^\circ \pm 28^\circ$	$-64^\circ \pm 16^\circ$
$f_0\pi^-$	a_3	2.5	2.6 ± 0.7	2.4 ± 0.6
	$\bar{\theta}_3$	-50°	$-48^\circ \pm 29^\circ$	$-55^\circ \pm 18^\circ$
$\rho^0\pi^-$	a_4	3.0	3.0 ± 0.7	2.8 ± 0.7
	$\bar{\theta}_4$	-75°	$-81^\circ \pm 27^\circ$	$-81^\circ \pm 16^\circ$

The NR distribution has been considered flat. We fixed the $\chi_{c0}\pi$ parameters $a_1 = 1.0$ to have an overall normalization and $\theta_1 = 0$ to fix our phase definition.

We show in Table I the result of the simulation for the B^+ decay for three different numbers of generated events (200, 500, and 1000). In Table II we present the same results for the CP conjugated decay.

One then uses Eq. (2) for the three CP phase changing channels NR, $f_0\pi$, and $\rho\pi$. One gets

$$\begin{aligned} \gamma &= 68 \pm 19, & \gamma &= 68 \pm 12, & \gamma &= 66 \pm 7 & (\text{NR}) \\ \gamma &= 73 \pm 20, & \gamma &= 71 \pm 12, & \gamma &= 62 \pm 8 & (f_0\pi) \\ \gamma &= 77 \pm 19, & \gamma &= 60 \pm 12, & \gamma &= 66 \pm 8 & (\rho\pi) \end{aligned} \quad (3)$$

for 200, 500, and 1000 generated events, respectively. The errors in Eq. (3) have been obtained summing in quadrature the independent errors of B^+ and B^- fits.

These results correspond to the scenario shown in Tables I and II. Nevertheless, as this scenario is based in particular assumptions we have performed a systematic study of these results, allowing a *large variety* of other scenarios.

First, we have varied the branching ratio (BR) of the partial channels—i.e., the square of the input coefficients a_i —by as much as a factor of 5 and we got acceptable fits with similar errors. Second, we have tried other values of the CP conserving phases δ_i and we got the same accuracy for any value of the phases, even when they were all set to zero. Third, we have tried many different values of γ between 0 and 2π and we have always found the *same accuracy* in the results. Finally, we have made simulations releasing the shape of the function F_2 describing the nonresonant channel [12] and found no important variations in the errors of Eq. (3).

We are then confident that in any acceptable scenario for this decay, the error to extract the angle γ would be similar. This procedure thus brings a simple way of predicting *the error* in the measurement of γ ; it would only depend on the number of reconstructed events—this is not the case for many other methods [5].

It is worth mentioning another important point of our simulations. The method has no discrete ambiguities; accordingly, we always get only one value of γ from the fit.

Let us now discuss the real scenario including penguins, studying by how much their inclusion modifies our previous results. For example, in the $f_0\pi^+$ channel the measured quantity $a_3 e^{i\theta_3}$ is in fact^y

$$a_3 e^{i\theta_3} = T e^{i(\delta_T + \gamma)} + P e^{i(\delta_P - \beta)}, \quad (4)$$

where $T e^{i(\delta_T + \gamma)}$ is the tree contribution and $P e^{i(\delta_P - \beta)}$ is the penguin one [1]; δ_T and δ_P are the strong phases.

A pictorial representation of Eq. (4) is shown in Fig. 1. It shows that when measuring the angle θ_3 we are missing the actual tree phase by an angle ϵ_+ . The same argument holds for B^- , leading to an angle ϵ_- . As a result, Eq. (2) becomes

$$\phi_3 = (\theta_3 - \bar{\theta}_3)/2 = \gamma + (\epsilon_+ + \epsilon_-)/2. \quad (5)$$

Thus, $\epsilon \equiv (\epsilon_+ + \epsilon_-)/2$ is the theoretical error of our method. Figure 1 shows that the worst case corresponds to the configuration when the tree and the penguin contributions are orthogonal in the complex plane. As a consequence, we have

$$|\epsilon_{\pm}| \leq \arctan(P/T). \quad (6)$$

The actual value of the ratio P/T is not known at present. An estimate was obtained for the decay $B \rightarrow \pi\pi$; $P/T \sim 0.2$ [16]. In our case, one expects the ratio P/T to be of the same order. Assuming $P/T = 0.2$, the uncertainty on γ extraction due to penguin contribution would be at most $\sim 11^\circ$. Anyway, Eq. (6) shows that as long as P/T remains not very large, the penguin pollution does not invalidate the method; for example, even for the improbable value $P/T \sim 0.5$, $\epsilon \leq 26^\circ$.

The inclusion of final state rescattering does not spoil our analysis of the error. The $B^+ \rightarrow f_0\pi^+$ decay proceeds through a unique isospin amplitude and thus [15]

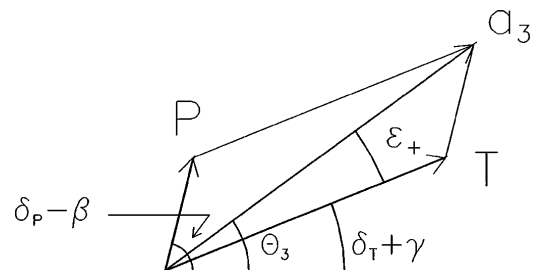


FIG. 1. Tree, penguin, and measured contributions.

Eq. (4) remains unchanged. For partial decays with more than one isospin amplitude—as $B^+ \rightarrow \rho^0 \pi^+$, for example—the form of Eq. (4) does not change either, but the *interpretation* of the two terms in this equation does. Indeed, even with rescattering one still has two types of diagrams: the first type having weak phase γ , the second having weak phase $-\beta$. But now, for example, the term we call T in Eq. (4) would be a more subtle combination of tree, color-suppressed, and annihilation quark diagrams [15]. Thus, the method also applies to intermediate channels with more than one isospin amplitude; nevertheless, a complete isospin analysis would be required to establish the theoretical uncertainty—note, however, that this is necessary only if rescattering effects are found to be large.

Let us now study the experimental feasibility of this method. Using Eqs. (3) and (5),(6) one can immediately get the error in the extraction of γ according to the number of reconstructed events. For example, with 1000 reconstructed events and $P/T = 0.2$, this method would give γ with a statistical plus theoretical error of 22° with 90% of C.L. Moreover, combining the measurement of γ using all the intermediate channels NR , $\rho\pi$, and $f_0\pi$ would certainly decrease the error of this procedure.

Much less statistics is needed to simply *detect* CP violation in this decay. One observes CP violation when, e.g., $\bar{\theta}_2 - \theta_2$ is different from zero. Assuming $\gamma \sim 65^\circ$ then from Eq. (3) one concludes that only 200 events are needed in order to detect a 3σ CP violation effect.

Three main features allow us to be optimistic about the possibility of doing this analysis in a short period of time, either in BaBar, KEK, or CLEOIII. First, tagging is not required because one needs only charged B 's. Second, the three detected particles are charged, thus we expect the efficiency to be high and the background to be not very large. Third, the method itself demands small statistics. For the $B^+ \rightarrow \pi^+ \pi^+ \pi^-$ decay, assuming, for example, a total BR of 3×10^{-5} and a reconstruction efficiency of 60% [5] one would expect to need about 2 years of running of BaBar to reconstruct 1000 events. Of course, a full experimental simulation of this decay is required to have definite conclusions.

In the end, let us summarize the main points of the procedure to extract γ presented in this Letter. This method brings a direct measurement of the CP violating angles. No previous knowledge of BR's or FSI phases is required. No necessity of making complicated triangle constructions is needed. The Dalitz analysis deals directly with amplitudes; thus, it is *linearly* sensitive to suppressed decays, as $\chi_c \pi$. Because of all of this, it is natural that this method does provide a measurement which demands less statistics than other methods. Finally, as one directly gets the angle itself—instead of, e.g., twice its sine or its cosine as in other methods—there are no discrete ambiguities.

The limitations of this method is mainly due to the fact that in order to have a complete knowledge of the errors in the extraction of γ one needs the ratio P/T . One

hopes that in the near future this ratio will be better known both from the theoretical and the experimental sides. As a result, although this method will probably not yield a very accurate measurement of γ , it will certainly allow a simple and effective step to have a quick constraint in the value of this angle.

As a by-product of this method to extract γ , we have presented in this Letter a general procedure to measure CP violating angles in charged three body decays. It is a natural and clear method. The procedure is quite general as it applies to any three body decay. Moreover, the whole procedure applies for *any* CP violating phase. For example, using this method with existing data from D meson decays one could obtain upper limits of less than 1° for many CP violating angles. This could be used to constrain beyond the standard model physics.

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- [1] For a general review, see Y. Nir and H.R. Quinn, in *B Decays*, edited by S. Stone (World Scientific, Singapore, 1994), p. 520; M. Gronau, Nucl. Phys. (Proc. Suppl.) **B38**, 136 (1995); A. Buras, hep-ph/9509329.
 - [2] M. Gronau and D. Wyler, Phys. Lett. B **265**, 172 (1991); I. Dunietz, Phys. Lett. B **270**, 75 (1991).
 - [3] D. Atwood, I. Dunietz, and A. Soni, Phys. Rev. Lett. **78**, 3257 (1997).
 - [4] M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990); M. Gronau, J.L. Rosner, and D. London, Phys. Rev. Lett. **73**, 21 (1994); M. Gronau *et al.*, Phys. Rev. D **50**, 4529 (1994); M. Gronau and J.L. Rosner, Phys. Rev. Lett. **76**, 1200 (1996).
 - [5] L. Lanceri *et al.*, *BaBar Physics Book*, SLAC-R-504 (to be published).
 - [6] R. Fleischer, hep-ph/9802433.
 - [7] G. Eilam, M. Gronau, and R.R. Mendel, Phys. Rev. Lett. **74**, 4984 (1995).
 - [8] A.E. Snyder and H.R. Quinn, Phys. Rev. D **48**, 2139 (1993).
 - [9] See, for example, ARGUS Collaboration, H. Albrecht *et al.*, Phys. Lett. B **308**, 435 (1993); E687 Collaboration, P.L. Frabetti *et al.*, Phys. Lett. B **331**, 217 (1994).
 - [10] E687 Collaboration, P.L. Frabetti *et al.*, Phys. Lett. B **407**, 79 (1997).
 - [11] J.D. Jackson, Nuovo Cimento **34**, 1644 (1964).
 - [12] I. Bediaga, C. Göbel, and R. Méndez-Galain, Phys. Rev. Lett. **78**, 22 (1997).
 - [13] J.D. Bjorken, Nucl. Phys. (Proc. Suppl.) **B11**, 325 (1989).
 - [14] J.F. Donoghue *et al.*, Phys. Rev. Lett. **77**, 2178 (1996).
 - [15] J.M. Gerard and J. Weyers, hep-ph/9711469.
 - [16] See, for example, A.S. Dighe, M. Gronau, and J.L. Rosner, hep-ph/9709223.
 - [17] F. James, *MINUIT, Function Minimization and Error Analysis*, CERN Program Library Long Writeup D506 (CERN, Geneva, 1994).
 - [18] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. **C3**, 1 (1998).
 - [19] N.G. Deshpande *et al.*, Phys. Rev. D **52**, 5354 (1995).
 - [20] G. Kramer and C.D. Lü, hep-ph/9707304.
 - [21] F. Parodi, P. Roudeau, and A. Stocchi, hep-ph/9802289.