# Liquid $^3$He in Aerogel: Crossover from Drude's to Hagen-Poiseuille's Law

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Charge flow in metals and mass flow in a neutral Fermi liquid show distinct profiles and are governed by different conservation properties. The connections between these two aspects of flow become apparent when we study the viscous flow of normal liquid $^3$He in the presence of aerogel. While the thermal flow is trivially affected, we predict that the resistance to mass flow shows a crossover from Drude's to Hagen-Poiseuille's law with varying temperature and low aerogel concentration. This crossover should be observable in dc-flow experiments. [S0031-9007(98)07528-0]

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One of the remarkable aspects of the Fermi liquid behavior of liquid $^3$He is the fact that the mean free path of its thermal excitations, the Landau quasiparticles [1], increases $\propto T^{-2}$ without bound when the temperature is lowered because it is entirely due to *momentum conserving* inelastic two-particle scattering processes. For $T \to 0$ it is eventually limited by the measuring cell's dimensions, and one enters the Knudsen flow regime [2].

In charged Fermi liquids scattering processes caused by impurities or lattice phonons lead to the relaxation of momentum in the bulk, whereas in the case of neutral Fermi liquids momentum relaxation is confined to the boundaries of the flow channel. As a result, Drude's law for electronic transport in metals (finite current decay rate, flat velocity profile) and Hagen-Poiseuille's (HP's) law for the flow of neutral Fermi liquid quasiparticles through a flow channel (vanishing current decay rate in the bulk, parabolic velocity profile) are traditionally viewed as entirely distinct descriptions of flow dissipation.

The connection between the two kinds of flow behavior was not clear until the discovery that low density correlated silica (aerogel) [3], immersed in liquid $^3$He serves as a random distribution of short ranged impurity potentials [4,5]. The aerogel provides a new (approximately elastic) scattering channel for the quasiparticles, and leads to a *new type of Fermi liquid* in which the transport mean free paths acquire an upper bound given by the aerogel-limited mean free path in the low temperature limit [6]. In contrast to other currents, the mass current (momentum density) loses its conservation property in the presence of aerogel. As a consequence, the mass flow and viscous transport in a system as simple as normal liquid $^3$He in aerogel is complex. The destruction of the momentum conservation property of the system leads to a strong modification of the (effective) shear viscosity, which displays a non-Fermi-liquid (i.e., nonmonotonic) temperature variation.

In this Letter, normal liquid $^3$He in a channel filled with aerogel is investigated as a model system for the continuous breakdown of momentum conservation and the consequent observability of both Drude and HP flow phenomena. The theoretical description deals with both electronic transport in metals and the mass transport of liquid $^3$He in aerogel. It is treated on the simplest possible level compatible with the hydrodynamic conservation laws and uses the linearized Landau-Boltzmann transport equation. We first investigate the transport parameters of charged and neutral Fermi liquids with two (elastic and inelastic) scattering channels. Then we specialize to the case of transverse stationary flow and calculate the effective viscosity of $^3$He in aerogel. We select experimental parameters such as temperature, pressure, and aerogel concentration which would allow the observation of different regimes of flow behavior.

We start with the usual description of normal Fermi liquids, characterized by charge $e$, Fermion mass $m$, particle density $n = N/V$, quasiparticle excitation spectrum $\epsilon_k = \mu + v_F(p - p_F) = \xi_k + \mu$ with $\mu$ the chemical potential, $\mu(0) = E_F$, Fermi velocity $v_F = p_F/m^*$ with $m^*$ the effective mass and $p_F$ the Fermi momentum, and group velocity $\mathbf{v}_k = \hbar\mathbf{k}/m^*$. In a state of global thermodynamic equilibrium the Fermi liquid is described by the Fermi distribution $n^0_{\mathbf{p}} = n^0(\xi_{\mathbf{p}}) = [\exp(\xi_{\mathbf{p}}/k_BT) + 1]^{-1}$ and its derivative $\varphi_{\mathbf{p}} = \{4k_BT\cosh^2(\xi_{\mathbf{p}}/2k_BT)\}^{-1}$. In what follows we shall abbreviate momentum sums as $\langle\cdots\rangle \equiv (1/V)\sum_{\mathbf{p},\sigma}\ldots$. A relevant quantity is $\langle\varphi\rangle = N_F = m^*p_F/\pi^2\hbar^3$, the density of states at the Fermi surface. Let us now introduce elastic

$$\Gamma^e = 1/\tau^e \tag{1}$$

and inelastic (only Fermi liquid two-particle scattering is discussed here)

$$\Gamma^i_{\mathbf{p}}(T) = \frac{1}{\tau_N(\xi_{\mathbf{p}}, T)} = \frac{\langle W\rangle}{32\hbar}\,\frac{\xi^2_{\mathbf{p}} + (\pi k_BT)^2}{\mu} \tag{2}$$

collision rates, which describe the relaxation of the quasiparticles to global equilibrium. Here $\langle W\rangle$ denotes the angular average of the effective two-particle scattering cross section [7]. It is sufficient to consider the energy average of the inelastic scattering rate

$$\bar\Gamma^i = \frac{1}{\bar\tau^i} = \frac{\langle\varphi\Gamma^i\rangle}{\langle\varphi\rangle} = \frac{4}{3}\,\frac{1}{\tau_N(0,T)}\,. \tag{3}$$

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Next we study deviations from global equilibrium $\delta n_{\mathbf{p}} = n_{\mathbf{p}} - n_{\mathbf{p}}^0$ induced by external perturbation potentials that induce the response $\delta n_a = \langle a \delta n \rangle$ of the macroscopic densities of charge ($a_{\mathbf{p}} = e$), mass ($a_{\mathbf{p}} = m$), entropy ($a_{\mathbf{p}} = \xi_{\mathbf{p}}/T$), and momentum ($a_{\mathbf{p}} = \mathbf{p}$). For this general level of description we specify external potentials to be of the form

$$\delta \xi_{\mathbf{p}}(\mathbf{q}, \omega) = \sum_a a_{\mathbf{p}} [\Phi_a^{\rm ext}(\mathbf{q}, \omega) + \mathbf{v}_{\mathbf{p}} \cdot \mathbf{A}_a^{\rm ext}(\mathbf{q}, \omega)]. \quad (4)$$

These potentials give rise to generalized forces characterizing the electromagnetic response and the mass flow, respectively,

$$\mathbf{F}_a^{\rm ext} = -i\mathbf{q}\Phi_a^{\rm ext} - i\omega \mathbf{A}_a^{\rm ext} = \begin{cases} -i\mathbf{q}\Phi^{\rm ext} + \frac{i\omega}{c}\mathbf{A}^{\rm ext}, \\ -i\omega m \mathbf{u}^{\rm ext} \simeq m \frac{\nabla \delta P}{\rho}. \end{cases} \quad (5)$$

Here $\Phi^{\rm ext}$ and $\mathbf{A}^{\rm ext}$ are the electromagnetic scalar and vector potentials, and $\mathbf{u}^{\rm ext}$ and $\delta P$ are the sample cell velocity and the applied pressure along the flow channel.

The linear response of the Fermi liquid to the perturbation potentials (4) can be described by the quasiclassical limit of the Landau-Boltzmann equation [1]

$$\omega \delta n_{\rm p} - \mathbf{q} \cdot \mathbf{v}_{\mathbf{p}} h_{\mathbf{p}} = i \delta I_{\mathbf{p}}; \qquad h_{\mathbf{p}} \equiv \delta n_{\mathbf{p}} + \varphi_{\mathbf{p}} \delta \xi_{\mathbf{p}}. \quad (6)$$

The energy change $\delta \xi_{\mathbf{p}}$ contains the contribution from both the external potentials (4) and the molecular potentials induced by the long range Coulomb and short range Fermi liquid forces for charged and neutral Fermi systems, respectively. These effects cause dielectric screening (Coulomb interaction) or become relevant only at higher frequencies (Fermi liquid interaction) and will be considered in a separate publication [8].

The collision integral $\delta I_{\mathbf{p}} = \sum_\nu \delta I_{\mathbf{p}}^\nu$ is assumed to consist of two separate contributions due to elastic ($\nu = e$) and inelastic ($\nu = i$) scattering processes and is treated within the conserving relaxation time approximation [9]:

$$\delta I_{\mathbf{p}}^\nu = -\frac{h_{\mathbf{p}}}{\bar\tau^\nu} + \frac{\varphi_{\mathbf{p}}}{\bar\tau^\nu}\left( \lambda_{a0}^\nu a_{\mathbf{p}} \frac{\langle a h \rangle}{\langle \varphi a^2 \rangle} + \lambda_{a1}^\nu a_{\mathbf{p}} \mathbf{v}_{\mathbf{p}} \frac{\langle a \mathbf{v} h \rangle}{\langle \varphi a^2 \mathbf{v} : \mathbf{v} \rangle} + \ldots \right). \quad (7)$$

Here the quantities $\lambda_{a\ell}^{e,i}$ are scattering parameters, which describe conservation ($\lambda_{a\ell}^{e,i} = 1$) and relaxation ($\lambda_{a\ell}^{e,i} \neq 1$) properties of the observables associated with the vertex $a_{\mathbf{p}}$ and the index $\ell = 0$ (densities), $\ell = 1$ (currents) (see below). In Eq. (7), $\mathbf{v} : \mathbf{v}$ denotes the dyadic product of the vectors $\mathbf{v}$, i.e., $\{\mathbf{v} : \mathbf{v}\}_{ij} = v_i v_j$. As an example one finds the set of continuity equations, which may be derived from (6) and (7) using scattering parameters appropriate for density conservation $\lambda_{a0}^\nu \equiv 1$, $\nu = e, i$.

$$\omega \delta n_a = \mathbf{q} \cdot \mathbf{j}_a; \qquad \mathbf{j}_a = \langle a \mathbf{v} h \rangle. \quad (8)$$

The superposition of the two different collision processes leads to (i) effective quasiparticle scattering rates

$$\Gamma^* = 1/\tau^* = \Gamma^e + \bar\Gamma^i(T) \quad (9)$$

and (ii) effective transport rates, which describe the decay or the conservation properties of the observables associated with the vertex $a_{\mathbf{p}}$

$$\Gamma_{a\ell}^* = \frac{1}{\tau_{a\ell}^*} = \frac{1 - \lambda_{a\ell}^e}{\tau^e} + \frac{1 - \lambda_{a\ell}^i}{\bar\tau^i(T)}. \quad (10)$$

This superposition leads to transport mean free paths

$$\lambda_{a\ell}^*(T) = v_{\rm F} \tau_{a\ell}^* = \frac{v_{\rm F}}{\Gamma_{a\ell}^e + \Gamma_{a\ell}^i(T)}, \quad (11)$$

which, contrary to the behavior in ordinary Fermi liquids, are bounded from above. This effect is shown for the quantity $\lambda_{m2}^*$ (viscous mean free path) in the inset of Fig. 1. The kinetic equation (6) together with the model collision integral (7) leads to the following generalized constitutive relation between the current $\mathbf{j}_a$ and the fields $\mathbf{F}_a^{\rm ext}$ as well as gradients in the densities

$$\mathbf{j}_a = \overleftrightarrow{T}_{aa} \cdot \left\{ \mathbf{F}_a^{\rm ext} - \frac{\Gamma^* - \Gamma_{a0}^*}{-i\omega + \Gamma^*} i\mathbf{q} \frac{\delta n_a}{\langle \varphi a^2 \rangle} \right\}. \quad (12)$$

Here $\overleftrightarrow{T}_{aa}$ is a generalized transport tensor

$$\overleftrightarrow{T}_{aa} = \left\langle \frac{\varphi a^2 \mathbf{v} : \mathbf{v}}{-i\omega + \Gamma_{a1}^* + O(\mathbf{q}^2)} \right\rangle \quad (13)$$

for which we list examples for $\omega \to 0$ in Table I.

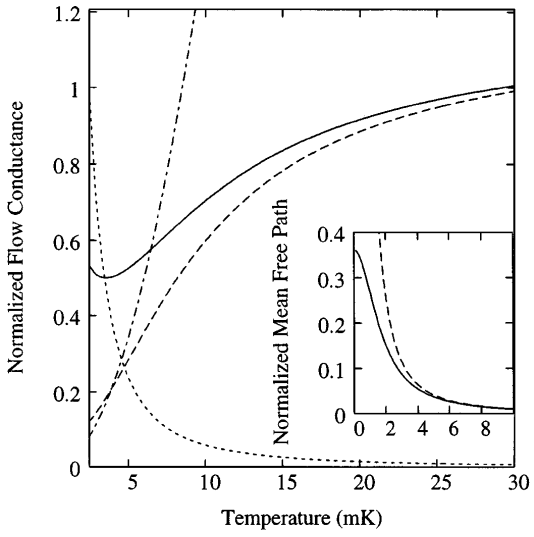


FIG. 1. The normalized conductance $\tau^{\rm eff}(T)/\bar\tau^i(1\ {\rm mK})$ vs temperature for 2 $\mu$m spaced parallel plates containing 99.9% open aerogel including slip (full line), without slip (dashed line), and without aerogel (dash-dotted line). Also shown is the normalized slip length $\zeta_0/d$ (dotted line). Inset: The normalized viscous mean free path $[\lambda_{m2}^*(T)/\lambda_{m2}^i(1\ {\rm mK})]$ with (full line) and without (dashed line) aerogel.

TABLE I. Fermi liquid transport parameters for a given vertex $a_{\mathbf{p}}$.

| | $a_{\mathbf{p}}$ | $\overleftrightarrow{T}_{aa}$ |
|---|---|---|
| Electronic conductivity | $e$ | $\sigma_e^* = N_{\mathrm{F}} e^2 v_{\mathrm{F}}^2 \tau_{e1}^*/3$ |
| Mass conductivity | $m$ | $\sigma_m^* = N_{\mathrm{F}} m^2 v_{\mathrm{F}}^2 \tau_{m1}^*/3$ |
| Thermal conductivity | $\xi_{\mathbf{p}}/T$ | $\kappa^* = C_V v_{\mathrm{F}}^2 \tau_\kappa^*/3$ |
| Shear viscosity | $\mathbf{p}$ | $\eta^* = \rho v_{\mathrm{F}}^2 \tau_\eta^*/5$ |

Here $\rho = mn$ is the mass density and $C_V$ is the specific heat, and $\tau_\kappa^* \equiv \tau_{\xi 1}^*$ and $\tau_\eta^* \equiv \tau_{m2}^*$. Table I emphasizes the analogy between electronic transport in metals and transport in the $^3$He-aerogel system. The electronic conductivity $\sigma_e^*$ shows the well-known crossover from the impurity-limited low temperature offset to the temperature dependence dictated by the relevant inelastic scattering processes. The neutral Fermi liquid analog $\sigma_m^*$ represents a *new* transport parameter of $^3$He which is finite only in the presence of aerogel. In contrast, the thermal conductivity of $^3$He in aerogel crosses from a high temperature (Fermi liquid, $\propto T^{-1}$) to a low temperature (aerogel-limited, $\propto T$) behavior, because the thermal current is not conserved even in the absence of the elastic scattering channel.

The situation is different in the case of the shear viscosity. In the presence of aerogel, the momentum density relaxes ($\Gamma_{m1}^* > 0$), similar to the current in a metal with imperfections. An effective shear viscosity of liquid $^3$He in aerogel can be defined but differs from the ordinary viscosity in a nontrivial way. To explore this further we consider the situation of transverse mass flow ($a_{\mathbf{p}} = m$) in the $x$ direction with spatial variation in the $z$ direction. An inspection of (13) shows that in the limiting case where $\Gamma_{m1}^* = 0$ (momentum conservation in the absence of aerogel), it is important to evaluate the $O(\mathbf{q}^2)$ term explicitly. The result for the mass current $g_x = \langle m v_x h \rangle$ reads

$$g_x = \frac{n F_{mx}}{-i\omega + \Gamma_{m1}^* + [\eta_{zz}^{\mathrm{eff}}(T)/\rho] q_z^2 + O(q_z^4)} \qquad (14)$$

and can be obtained by solving the coupled equations (8) and (12) for ($a_{\mathbf{p}} = m$). In Eq. (14) the denominator describes relaxation due to both a Drude ($\Gamma_{m1}^*$) and a viscous contribution to the flow resistance. The latter is characterized by an effective viscosity of $^3$He in aerogel

$$\eta_{zz}^{\mathrm{eff}}(T) = \frac{\Gamma^*(T) - \Gamma_{m1}^*}{\Gamma^*(T) - i\omega} \underbrace{\frac{1}{5} n p_{\mathrm{F}} v_{\mathrm{F}} \tau_\eta^*(T)}_{\eta_{zz}^*(T)} . \qquad (15)$$

In the absence of elastic scattering processes $\Gamma_{m1}^* = 0$ the mass current response function (14) is dominated by the usual viscous diffusion pole which is characterized by the normal Fermi liquid viscosity and reflects the momentum conservation law. $\eta_{zz}^*(T)$ displays a simple temperature crossover behavior analogous to that of the thermal conductivity which saturates at low temperature. However, its prefactor in (15) leads to a nonmonotonic temperature dependence of $\eta_{zz}^{\mathrm{eff}}(T)$ because of the nonvanishing contribution of the new transport rate $\Gamma_{m1}^*$ introduced by the aerogel. In the simplest case when $\lambda_{m1}^{\mathrm{e}} = 0$, the prefactor $\to 1$ for high temperatures and varies $\propto T^2$ for $T \to 0$. This behavior is masked as $T \to 0$ by both the superfluid and the Knudsen transition (Fig. 1).

To apply this result to the experiment we consider the case of parallel plates at $z = \pm d/2$ and solve the associated differential equation in real space. We obtain the cross sectional average of the mass current

$$\langle g_x \rangle = \frac{1}{d} \int_{-d/2}^{d/2} dz\, g_x(z) = n \tau^{\mathrm{eff}} F_{mx} \qquad (16)$$

with an effective flow relaxation time:

$$\tau^{\mathrm{eff}} = \frac{1}{-i\omega + \Gamma_{m1}^*} \left\{ 1 - \frac{\frac{2\ell^*}{d} \tanh(\frac{d}{2\ell^*})}{1 + \frac{\zeta_0}{\ell^*} \tanh(\frac{d}{2\ell^*})} \right\}$$
$$\overset{\omega \to 0}{=} \begin{cases} \tau_{m1}^*; & \ell^*/d \to 0 \text{ (Drude)}, \\ \frac{\rho d^2}{12 \eta_{zz}^{\mathrm{eff}}} (1 + 6\frac{\zeta_0}{d}); & \ell^*/d \to \infty \text{ (HP)}. \end{cases} \qquad (17)$$

Here $\ell^*$ is a new length which is defined by

$$\ell^*(T) \equiv \sqrt{\frac{\eta_{zz}^{\mathrm{eff}}(T)}{\rho(-i\omega + \Gamma_{m1}^*)}} . \qquad (18)$$

In the derivation of (17) we used a slip boundary condition $g_x(\pm d/2) = \mp \zeta_0 g_x'(\pm d/2)$ with $\zeta_0$ the Fermi liquid slip length [9]. The flow of liquid $^3$He in aerogel is determined by the ratio of $\ell^*(T)$ and the plate separation $d$. For $\ell^*(T) < d$ flow is dominated by aerogel scattering and follows Drude's law. Only when $\ell^*(T) > d$ will the contributions from wall scattering events become significant [10], and the flow resistance will show a transition from Drude's to HP's law as modified by the slip effect.

Although experiments at Cornell [4,11,12], Northwestern [5,13], and Manchester [14] have explored the properties of superfluid $^3$He in aerogel, there have been no corresponding investigations of normal $^3$He. Theoretical work has also concentrated on the superfluid phases [6,15–17]. To estimate the mean free path, we have to rely on models of aerogels [18], which show that for the most dilute aerogel (99.9% open) the mean free path $\lambda_{m1}^* \equiv \lambda_A = 7.2\ \mu$m. Such an aerogel could be grown in a 2 $\mu$m tall, 1 cm wide, 0.5 mm long micromachined flow channel. A measurable [19] flow of about $10^{-12}$ kg/s would result for a pressure differential of $10^3$ dyn/cm$^2$ and $\eta_{zz}^{\mathrm{eff}}$ of 0.02 poise.

At a $^3$He pressure of 30 bar, $\bar{\tau}^i(1\text{ mK}) = 180$ ns, $v_{\mathrm{F}} = 33.6$ m/s [20], $d = 2\ \mu$m, $\lambda_A = 7.2\ \mu$m, and $\zeta_0 = 0.579\cdots\lambda_{m2}^i$. For these experimental parameters, we plot in Fig. 1 the normalized flow conductance as a function of temperature, together with the conductance without slip in both the presence and the absence of

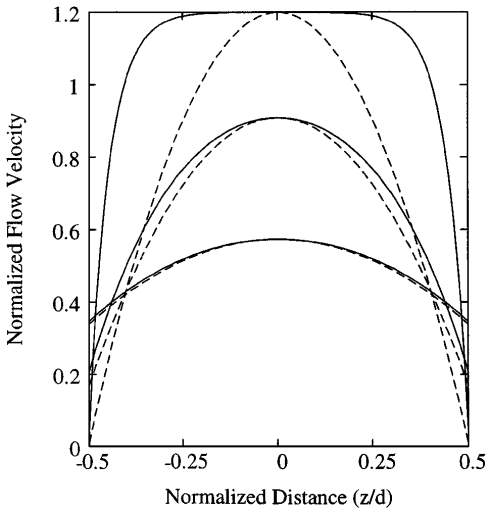


FIG. 2. The normalized profiles $g_x(z)/nF_{mx}\bar{\tau}^i(1\text{ mK})$ for $^3$He at 30 bars with 99.9% open aerogel at 3 mK (lower), 10 mK (middle), and 50 mK (upper curves). Full lines represent the general result including slip, and dashed lines are the scaled profiles $[g_x^0(z)/g_x^0(0)]g_x(0)$ in the absence of aerogel, with $g_x^0(z) = \lim_{\ell^*\to\infty} g_x(z) = nF_{mx}\tau_{m1}^*(d^2/8\ell^{*2})[1 + 4(\zeta_0/d - z^2/d^2)]$. The dashed lines are normalized to the calculated flow in the center of the channel and illustrate the difference between Poiseuille flow and the flow with aerogel present.

aerogel. We also show the ratio $\zeta_0/d$. At high temperature, the conductance is temperature independent (corresponding to the Drude regime). As the temperature is lowered, the conductance decreases and passes through a minimum. Hagen-Poiseuille flow $\tau^{\text{eff}} \propto T^2$ is never observed for any finite aerogel density, because the transition to Drude behavior will always occur for sufficiently small $\lambda_{m2}^i \propto T^{-2}$.

The flow profiles $g_x(z)$ in the channel and their normalized form $[g_x^0(z)/g_x^0(0)]g_x(0)$ in the Poiseuille flow limit $\ell^* \to \infty$, with $g_x^0(z) = \lim_{\ell^*\to\infty} g_x(z) = nF_{mx}\tau_{m1}^*(d^2/8\ell^{*2})[1 + 4(\zeta_0/d - z^2/d^2)]$ are shown in Fig. 2. At low temperature (<4 mK in Fig. 2) one has clearly reached the limit of HP flow as modified by slip effects. With increasing temperature the profile deviates from the form expected from Hagen-Poiseuille's law and approaches the flat Drude-like result in the presence of aerogel (50 mK in Fig. 2).

The behavior may be understood qualitatively by recognizing that the walls affect flow over a distance comparable to the length $\lambda_{m1}^*(T)$. At high temperatures, this is short so that the elastic scattering length dominates the flow which approaches the Drude profile. At low temperatures, $\lambda_{m1}^*(T)$ is long and the wall's presence is discernible in the interior providing the elastic scattering length is sufficiently large and the flow profile reverts to the Hagen-Poiseuille behavior as corrected for slip. Knudsen flow would dominate if the inelastic scattering length were much longer than the flow channel dimension. In pure $^3$He (in the absence of slip and superfluidity), the mean free path and viscosity increase without bound. In contrast, the mean free path (and viscosity) are restricted resulting in the larger low temperature relative conductance (Fig. 1) of the channel containing aerogel.

In summary, we have demonstrated that the flow properties of normal liquid $^3$He in aerogel can display a rich complexity of phenomena including Drude's and Hagen-Poiseuille's law *manifested in the same system.* We have shown that charge transport in metals and mass flow of pure $^3$He through flow channels are closely related. With aerogel present the mass flow of Landau quasiparticles occurs in an intermediate regime between Drude's and Hagen-Poiseuille's law. We predict that the transition from Drude to Poiseuille flow occurs for aerogel concentrations where $\lambda_{m1}^*/d > 1$. Such a transition could be seen in dc flow experiments at not too low a pressure.

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