Quantum Decay from Josephson π States

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Quantum decay from Josephson π states, which is inherent in the Josephson system with fixed particle numbers, is investigated. Since the π states have two different decay paths, they can interfere during the decay processes by quantum tunneling due to a topological phase originating from total particle-number restriction and result in parity effect for tunneling. [S0031-9007(98)07457-2]

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The Josephson effect is one manifestation of *primary* quantum aspects of macroscopic objects and was observed in superconducting materials. Recently it has been tested in various macroscopic systems to confirm their quantum-mechanical nature. In some of the systems, the total number of particles in the system is fixed. In such a restricted situation, it is somewhat difficult to observe the Josephson effect because the Josephson current cannot directly be detected as usual since measurement causes particle-number changes in the system. This is in contrast to the superconductors. Fortunately, one can observe the Josephson effect through the Josephson acceleration relation such as $d\Delta\varphi/dt = \Delta\mu$, where $\Delta\varphi = \varphi_R - \varphi_L$ is the phase difference between two condensates and $\Delta\mu$ is the chemical potential. We set $\hbar = 1$ hereafter. In fact, this kind of experiment has been done in the ^{3}He system [1,2].

In the past two decades, another conceptually distinct type of quantum effect for macroscopic objects has been studied in order to discuss the applicability of quantum mechanics on a macroscopic scale. These other effects are sometimes called *secondary* macroscopic quantum effects since they occur due to the Heisenberg's uncertainty principle between noncommuting macroscopic variables. One such secondary effect is macroscopic quantum tunneling (MQT), which has predominantly been studied in superconducting tunnel junctions with small capacitance [3–5]. In the Josephson systems, the phase and number differences between condensed states are noncommuting macroscopic variables. MQT studies require a metastable state. In superconducting materials, it was not difficult to realize this state by introducing an external bias current. However, there are both technical and conceptual difficulties for MQT studies in the above-mentioned restricted situation. In this paper, we show the existence of metastable states inherent in such restricted systems and discuss the *secondary* macroscopic quantum aspects of the system through studying quantum decay from the metastable states.

First, let us show the existence of metastable states inherent in the restricted system. Starting from the Feynman two-state model [6] taking into account the fixed number of particles in the system, the Josephson relations are modified as

$$\frac{d\Delta N}{dt} = -K\sqrt{N^2 - (\Delta N)^2}\,\sin\Delta\varphi\,, \qquad (1)$$

$$\frac{d\Delta\varphi}{dt} = (E_L - E_R) + \frac{K\Delta N}{\sqrt{N^2 - (\Delta N)^2}}\cos\Delta\varphi\,, \qquad (2)$$

where $N = N_L + N_R$ is the total number of particles in the weakly coupled macroscopic quantum system. For example, in superconductors, N is the total number of Cooper pairs. $\Delta N = N_L - N_R$ is the particle-number difference between two condensates. K is the coupling amplitude of the two-state system and is assumed to be real. E_L and E_R are the ground state energies of the two condensates. Since it is convenient to replace $E_L - E_R$ with $\Delta N/\chi$ in the usual manner where χ is the capacitance, these coupled equations can be derived from the following Hamiltonian:

$$H = \frac{N^2}{2\chi}z^2 - KN\sqrt{1 - z^2}\,\cos\Delta\varphi\,, \qquad (3)$$

where $z = \Delta N/N$. The first term represents the capacitive energy that enhances the particle nature of the phase particle, while the second term is the Josephson coupling energy which stems from the wave nature of the phase particle. Note that the Josephson coupling energy can be determined solely by its phase in the unrestricted case, while now in the restricted case, it depends on z in addition to $\Delta\varphi$. This difference produces the following new dynamical behavior of $\Delta\varphi$. The equation of motion for $\Delta\varphi$ is now given by

$$\frac{d^2\Delta\varphi}{dt^2} = -A\sin\Delta\varphi - B\sin 2\Delta\varphi + O(z^2)\,, \qquad (4)$$

where $A = KN^3/\chi$ and $B = (KN)^2$. The coefficient A is nothing but the square of the Josephson plasma frequency. The ratio $r \equiv A/B = N/K\chi$ characterizes the system either classically ($KN > N^2/\chi; r < 1$) or quantum mechanically ($KN < N^2/\chi; r > 1$) and can be changed by controlling the junction parameters such as χ and K within $0 < r < \infty$. The second term on the right-hand side of

Eq. (4) arises from the differential calculus of z in the Josephson coupling energy and exists even if a mean value of z is zero. The replacement, $E_L - E_R \rightarrow \Delta N/\chi$, that we made for our convenience before, is not necessary for the derivation of Eq. (4). This equation of motion implies that the potential term can effectively be rewritten as

$$U(\Delta\varphi) \simeq -\tilde{A}\cos\Delta\varphi - \frac{\tilde{B}}{2}\cos 2\Delta\varphi\,, \qquad (5)$$

where $\tilde{A} = KN$ and $\tilde{B} = \chi K^2/2$. Therefore, Eq. (4) describes a phase particle moving in the potential as shown in Fig. 1. This potential involves metastable states located at $(2m + 1)\pi$ when $r < 2$. Hereafter we call these "π states" [7]. To obtain the above results, we assume only particle-number restriction to the Feynman two-state model. Therefore, we stress here that *this is a general feature of the particle-number conserved system.* Corresponding to this, Smerzi *et al.* [8] discussed in detail similar states called "macroscopic quantum self-trapped (MQST) states," but only with regard to a pair of weakly coupled Bose-Einstein condensed (BEC) alkali atom gases. They argued MQST states originated from the nonlinear term in the Gross-Pitaevskii equation. MQST states are not inherent only in BEC systems but essentially come from the above-mentioned nature of restricted systems. In fact, they implicitly assume the restriction and are therefore equivalent to π states. Thus, it is possible to apply π states to other particle-number conserved systems. One candidate is the superfluid helium system. Very recently metastable π states in a superfluid ^{3}He weak link have been discovered by Backhaus *et al.* [9]. This must be explained by our model since their system is actually a closed system. Our model might be tested by comparing their measured frequencies at π and 2π with the theoretical values determined by the curvature at the bottom of the potential, as well as the oscillation amplitudes. Further discussion will appear elsewhere.

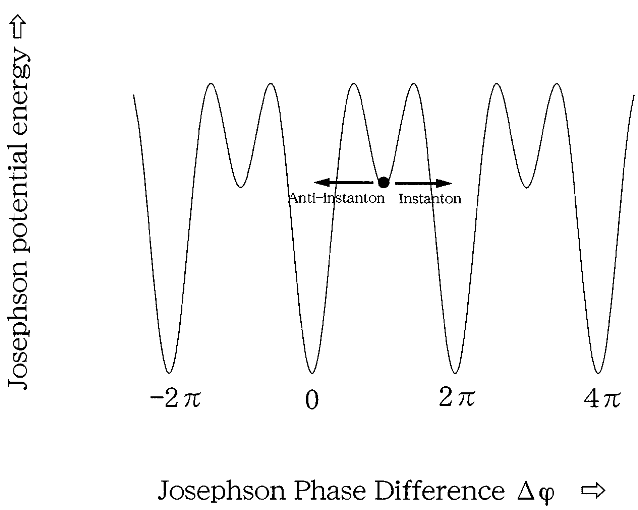

FIG. 1. A schematic diagram for Josephson π states. The arrows show two different decay paths: instanton and anti-instanton.

Next, let us consider quantum decay of phase particles from the metastable states. It is clear that there are two ways to decay from π states. This is apparently different from previous studies in Josephson systems. At first, it seems that, since the final states of the decay are essentially identical, one should expect interference between these two paths [10]. However, it is unlikely that tunneling paths interfere because the resulting Euclidean Lagrangian in an imaginary time path-integral scheme is real, not complex as usual. Next we see that the restricted case can change this situation.

Before we consider the origin of the tunneling path interference, we clarify the properties of the state vector in the restricted situation. The state vector for a whole system in unrestricted situations is given by a superposition of left and right vectors as follows:

$$|\psi\rangle = \sqrt{N_L}\, e^{i\varphi_L}|L\rangle + \sqrt{N_R}\, e^{i\varphi_R}|R\rangle\,. \qquad (6)$$

Under the restriction $N = N_L + N_R$, the state vector is modified as

$$|\psi\rangle = \sqrt{\frac{N}{2}}\, e^{i\alpha/2}(\sqrt{1+z}\, e^{-i\Delta\varphi/2}|L\rangle + \sqrt{1-z}\, e^{i\Delta\varphi/2}|R\rangle)\,, \qquad (7)$$

where $\alpha = \varphi_R + \varphi_L$, which is usually set to $\alpha = 0$ due to the uniqueness for wave functions. Note that this state vector is equivalent to that of a spin one-half system if $N = 1$ and $z = \cos\theta$, where θ is the colatitude in the spherical polar coordinate. In this case, the Hamiltonian (3) is expressed as

$$\hat{\mathcal{H}} = \frac{\sigma_z^2}{2\chi} - KN\sigma_x\,, \qquad (8)$$

where σ's are Pauli matrices. Thus, our system can be mapped onto the spin one-half system. This forces our state vector to move on 2-sphere S_2 and enables us to consider tunneling in spin space where tunnel paths interfere.

Now let us consider the origin of the tunneling path interference. Decay rate by tunneling can be estimated by the imaginary-time transition amplitude expressed as [11]

$$Z(T) \equiv \frac{\langle (2j+1)\pi{:}T|e^{-\hat{H}T}|(2i+1)\pi{:}0\rangle}{\langle (2j+1)\pi{:}T|e^{-\hat{H}_0T}|(2i+1)\pi{:}0\rangle} = \int_0^T \mathcal{D}\Delta\varphi\,\mathcal{D}z e^{-S_E}, \qquad (9)$$

where we have normalized the amplitude to that of the harmonic Hamiltonian $\hat{H}_0$. $S_E = \int d\tau\, \mathcal{L}_E$ is the Euclidean action, and i and j are integers. The Euclidean Lagrangian is

$$\mathcal{L}_E = -\langle\dot{\psi} \mid \psi\rangle + \langle\psi|\hat{\mathcal{H}}|\psi\rangle = -i\frac{N}{2}\Delta\dot{\varphi} + i\frac{N}{2}z\Delta\dot{\varphi} + \mathcal{H}(z,\Delta\varphi)\,, \qquad (10)$$

and

$$\mathcal{H} = \frac{N^2}{2\chi} z^2 + U(\Delta\varphi). \tag{11}$$

The dot on $\Delta\varphi$ means ∂_τ. The first two terms in Eq. (10) of the Euclidean action define the Wess-Zumino term and is complex even though we are now working in imaginary time. These terms have a topological meaning and are equal to the total area bounded by the trajectory on the 2-sphere S_2, forming the Berry phase. The second term, $iN\Delta\dot{\varphi}z/2$, shows the noncommuting relation between z and $\Delta\varphi$ and is incorporated into an ordinary Lagrangian after the integration for z, which can be exactly performed due to its bilinear form. The first term, $iN\Delta\dot{\varphi}/2$, is unique to our restricted Josephson system and is essential for interference of tunneling paths. In the usual unrestricted Josephson systems, the overlap of the states concerning different times does not produce this first kind of term. Since our system can be mapped onto a spin system, the origin of this term is exactly the same as in a spin system. Of course, this does not affect the classical equations of motion which result from δS_E. This Lagrangian is the same as those of previous work on spin tunneling [12,13] except for the potential forms.

Let us calculate the tunneling rate by the imaginary time path-integral method. In contrast to spin tunneling, we must take into account the energy difference between adjacent potential minima, the difference between the upper minimum $E_+ = \omega_+/2$ and the lower one $E_- = -\epsilon/2 + \omega_-/2$, because the coefficient A cannot be zero. We assume here that our potential in Eq. (5) can be approximated by an array of asymmetric double-well potentials. Then, we can employ the valley instanton technique developed by Aoyama *et al.* [14]. The action of the valley instanton is given by

$$S_E = Ci\frac{N}{2}\pi - \frac{\epsilon T}{2} + S_0^{cl}, \tag{12}$$

with

$$S_0^{cl} = S_0 + \frac{\epsilon}{2}, \tag{13}$$

where $C = \pm 1$ defines instantons ($C = 1$) and anti-instantons ($C = -1$). S_0 is the action for a symmetric double-well potential case ($\epsilon = 0$). The instanton can start one of the π states, $\Delta\varphi = m\pi$, and go to $\Delta\varphi = 2m\pi$. Likewise, the anti-instanton can go from $\Delta\varphi = m\pi$ to $\Delta\varphi = 2(m-1)\pi$. When doing the dilute-gas sum, there is no constraint that instantons and anti-instantons must alternate. The transition between the wells involves k instanton transitions in one direction and $2m - k$ anti-instanton transitions in the other, such that there are C_k^{2m} configurations. Summing up all instanton contributions we obtain

$$\begin{aligned} Z(T) &= \sum_{m=0}^{\infty}\sum_{k=0}^{2m} C_k^{2m}(De^{-S_0^{cl}}e^{iN\pi/2})^k(De^{-S_0^{cl}}e^{-iN\pi/2})^{2m-k}I_m(T) = \sum_{m=0}^{\infty}[De^{-S_0^{cl}}(e^{iN\pi/2} + e^{-iN\pi/2})]^{2m}I_m(T) \\ &= \sum_{m=0}^{\infty}\gamma^{2m}I_m(T), \end{aligned} \tag{14}$$

where D is the fluctuation determinant without zero mode. $I_m(T)$ describes the actual integrations over the positions of the valley instantons with zero-energy contributions of the determinant described by $\tilde{\epsilon} = \epsilon - (\omega_- - \omega_+)/2$:

$$I_m(T) \equiv \begin{cases} 1, & \text{for } m = 0, \\ \int_0^T d\tau_{2m}\int_0^{\tau_{2m}} d\tau_{2m-1}\cdots\int_0^{\tau_2} d\tau_1\, e^{\tilde{\epsilon}(\tau_{2m}-\tau_{2m-1}+\cdots+\tau_2-\tau_1)}, & \text{for } m \geq 0. \end{cases} \tag{15}$$

The finite series can be summed by the use of the generating function method [15] and yields

$$Z(T) = \frac{\lambda_+ e^{-\lambda_- T} - \lambda_- e^{-\lambda_+ T}}{\lambda_+ - \lambda_-}, \tag{16}$$

where $\lambda_\pm = -\tilde{\epsilon}/2 \pm \sqrt{\tilde{\epsilon}^2/4 + \gamma^2}$. Thus the energies of the two lowest states are given by

$$E_\pm = \frac{\omega_+}{2} + \lambda_\pm = \frac{\omega_+}{2} - \frac{\tilde{\epsilon}}{2} \pm \sqrt{\frac{\tilde{\epsilon}^2}{4} + \gamma^2}. \tag{17}$$

For $\epsilon = 0$, $I_m(T), Z(T)$, and $E_\pm$ are reduced to the zero bias case. Since these energy eigenvalues and the amplitudes are identical to those obtained by diagonalizing the following matrix,

$$H = \begin{pmatrix} \frac{\omega_+}{2} & \gamma \\ \gamma & -\epsilon + \frac{\omega_-}{2} \end{pmatrix}, \tag{18}$$

we can now read off the tunneling rate Γ:

$$\Gamma \equiv |\gamma| = 2D|\cos(N\pi/2)|e^{-S_0^{cl}}. \tag{19}$$

Evidently, the tunneling rate depends on the parity of the total number of particles in the system. If N is even, the total tunneling rate is of the same order as the single-instanton rate. But, if N is odd, the tunneling rate is zero. Since the $\cos(N\pi/2)$ factor arises directly from the topological phase, it represents an interference between the instanton and anti-instanton contributions to tunneling. Thus, the constructive interference between the instanton and anti-instanton occurs when N is even, while the destructive interference occurs when N is odd. This parity effect is quite different from that found in the supercondutor, which is based on the pair formation in the condensates [16,17].

In MQT, dissipation strongly affects the tunneling rate by reducing it. Since the parity effect stems from its topological origin, we expect that it will remain even in the presence of dissipation. It can be treated in the same manner in the biased double-well potential [18,19]. This will be discussed elsewhere.

In summary, we have showed the existence of the metastable states inherent in systems with fixed particle numbers, which we call π states. Two different decay paths from the π states interfere through the Wess-Zumino term resulting from restriction of the total number of particles to the wave function. Consequently, quantum decay rate depends on the parity of the total number of particles in the system.

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[1] S. V. Pereverzev, A. Loshak, S. Backhaus, J. C. Davis, and R. E. Packard, Nature (London) **388**, 449 (1997).

[2] S. Backhaus, S. V. Pereverzev, A. Loshak, J. C. Davis, and R. E. Packard, Science **278**, 1435 (1997).

[3] R. F. Voss and R. A. Webb, Phys. Rev. Lett. **47**, 265 (1981).

[4] L. D. Jackel, J. P. Gordon, E. L. Hu, R. E. Howard, L. A. Fetter, D. M. Tennant, R. W. Epworth, and J. Kurkijain, Phys. Rev. Lett. **47**, 697 (1981).

[5] A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. **46**, 211 (1981).

[6] R. Feynman, in *Quantum Mechanics*, The Feynman Lectures on Physics Vol. 3, (Addison-Wesley, Reading, MA, 1965).

[7] The π states have nothing to do with the "π junctions" discussed in connection with the d-wave symmetry of the order parameter in high-T_c superconductors.

[8] A. Smerzi, S. Fantoni, S. Giovanazzi, and S. R. Shenoy, Phys. Rev. Lett. **79**, 4950 (1997).

[9] S. Backhaus, S. Pereverzev, R. W. Simmonds, A. Loshak, J. C. Davis, and R. E. Packard, Nature (London) **392**, 687 (1998).

[10] A. J. Leggett, in *Quantum Tunneling of Magnetization—QTM'94*, edited by L. Gunther and B. Barbara (Kluwer Academic Publishers, Dordrecht, The Netherlands, 1995).

[11] S. Coleman, *Aspects of Symmetry* (Cambridge Universtiy Press, Cambridge, England, 1985).

[12] D. Loss, D. P. DiVincenzo, and G. Grinstein, Phys. Rev. Lett. **69**, 3232 (1992).

[13] J. von Delft and C. L. Henley, Phys. Rev. Lett. **69**, 3236 (1992).

[14] H. Aoyama, T. Harano, H. Kikuchi, M. Sato, and S. Wada, Phys. Rev. Lett. **79**, 4052 (1997).

[15] H. Aoyama and H. R. Quinn, Phys. Rev. D **31**, 885 (1985).

[16] K. A. Mateev, M. Gisselfalt, L. I. Glazman, M. Jonson, and R. I. Shekhter, Phys. Rev. Lett. **70**, 2940 (1993).

[17] P. Joyez, P. Lafarge, A. Pilipe, D. Esteve, and M. H. Devoret, Phys. Rev. Lett. **72**, 2458 (1994).

[18] H. Grabert and U. Weiss, Phys. Rev. Lett. **54**, 1605 (1985).

[19] M. P. A. Fisher and A. T. Dorsey, Phys. Rev. Lett. **54**, 1609 (1985).