Standing and Moving Gap Solitons in Resonantly Absorbing Gratings

Alexander E. Kozhekin and Gershon Kurizki
Chemical Physics Department, Weizmann Institute of Science, Rehovot 76100, Israel

Boris Malomed
Department of Interdisciplinary Studies, Faculty of Engineering, Tel Aviv University, Tel Aviv 69978, Israel
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We present hitherto unknown forms of soliton dynamics in the forbidden frequency gap of a Bragg reflector, modified by periodic layers of near-resonant two-level systems (TLS). Remarkably, even extremely low TLS densities create an allowed band within the forbidden gap. This spectrum gives rise, for *any* Bragg reflectivity, to a vast family of stable gap solitons, both standing and moving, having a unique analytic form, an *arbitrary* pulse area, and inelastic collision properties. These findings suggest new possibilities of transmission control, noise filtering, or "dynamical cavities" (self-traps) for both weak and strong signal pulses. [S0031-9007(98)07472-9]

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The study of light-matter interactions in periodic dielectric structures has developed into a vast research area. At the heart of this area is the interplay between the resonant reflections induced by the Bragg reflector, giving rise to photonic band gaps, and their dynamical modifications due to nonlinear light-matter interactions. The pulsed mode of propagation in such structures exhibits a variety of unique fundamentally and technologically interesting regimes: nonlinear filtering, switching, and distributed-feedback amplification [1]. Of particular interest are *gap solitons* (GS), i.e., moving or standing (quiescent) self-localized pulses, whose spectra are centered in a gap induced by the grating. GS in Kerr-nonlinear Bragg reflectors have been extensively analyzed [2] and experimentally observed [3]. Recently, GS have also been predicted in Bragg-reflecting second-harmonic generating media [4].

This Letter is dedicated to a different mechanism supporting GS in periodic media, which is based on *near-resonant* field-atom interactions. The first step in this direction has been made in Ref. [5], where an *exact* moving GS solution has been found in a periodic structure composed of thin layers of resonant two-level systems (TLS) separated by half-wavelength nonabsorbing dielectric layers, i.e., a *resonantly absorbing Bragg reflector* (RABR). In the soliton solution obtained in Ref. [5], the combined area of the forward- and backward-propagating pulses is 2π, characteristic of self-induced transparency (SIT) solitons in uniform media [6]. The existence of this soliton stems from the cooperative resonant atomic polarizability, which compensates for the periodic modulation of the linear polarizability in the Bragg reflector [5,7]. The analysis presented in Ref. [5] leaves several open questions of fundamental and applied importance: (i) Can one overcome the basic restriction implicit in this solution, namely, that the *cooperative length* over which a SIT pulse is formed, must be *shorter* than the Bragg reflection length? If this restriction is essential, then the soliton would only exist in weakly reflecting Bragg structures, which can hardly serve as efficient filters that block pulses other than GS. (ii) Are GS admitted in a RABR for weak pulses whose area is *less* than 2π? (iii) Is there a quiescent counterpart to the moving GS, which would imply *complete dynamic confinement* of light in the RABR? (iv) What is the result of collisions between moving GS in this system?

In this Letter, we present answers to the above questions: (a) The RABR supports a vast family of GS characterized by *two* parameters, the soliton amplitude and velocity (analogously to GS in Kerr-nonlinear gratings [2]), which exists for *any ratio* of reflection to cooperative lengths. (b) It includes a subfamily of quiescent or zero-velocity (ZV) solitons with an *arbitrary pulse area,* whose *exact* analytical form represents an essentially novel type of soliton solutions in nonlinear optics. (c) Moving GS are found analytically in the small-amplitude limit, where they reduce to the nonlinear-Schrödinger (NLS) solitons, and also numerically, as deviations from the exact subfamily found in [5] or from the exact ZV solitons. Our simulations indicate that both ZV and moving GS are *stable*. (d) We simulate collisions between moving GS, demonstrating that they may be either weakly or strongly inelastic. These findings reveal a basically new phenomenon: the multiple reflections in a grating can effectively change the pulse coupling to the TLS, so that soliton transmission is not restricted to pulses of 2π area, as in ordinary SIT, and occurs (at an appropriate frequency) for *any* ratio of reflection to cooperative lengths.

Our starting point is the equations for the sum Σ_+ of the amplitudes of the forward- and backward-propagating electromagnetic waves, polarization P, and population inversion w, derived in Ref. [5] from the coupled Maxwell-Bloch equations for the bidirectional propagation in RABR:

$$\left(\frac{\partial^2}{\partial\tau^2} - \frac{\partial^2}{\partial\zeta^2}\right)\Sigma_+ = 2\,\frac{\partial}{\partial\tau}P + 2i\eta P - \eta^2\Sigma_+\,, \quad (1)$$

$$\frac{\partial}{\partial \tau} P = w\Sigma_+ - i\delta P, \tag{2}$$

$$\frac{\partial}{\partial \tau} w = -\frac{1}{2}(P^*\Sigma_+ + P\Sigma_+^*), \tag{3}$$

where τ and ζ are the normalized time and propagation distance, and δ is the effective detuning of the field from the atomic resonance. The key parameter $\eta = l_c/l_r$ is the ratio of the Arrechi-Courtens cooperativity length ($l_c = 2c/\omega_p$, where ω_p is an effective plasma frequency [6]) to the reflection length ($l_r = 4d\epsilon_0/\pi\Delta\epsilon$, where d is the period and $\Delta\epsilon$ the variation of the dielectric index of the periodic structure with average dielectric index ϵ_0) [5]. The crucial assumption in the derivation of Eqs. (1)–(3) is that the *resonant absorbers are confined to thin layers*, periodically inserted between passive thick dielectric layers. The system (1)–(3) can be simplified: substituting into Eq. (3) the expression for Σ_+ following from (2), one obtains an equation that can be explicitly integrated to yield a simple algebraic relation

$$w = \pm\sqrt{1 - |P|^2}, \tag{4}$$

that should be further inserted into Eq. (1).

First, we linearize Eqs. (1)–(3) in order to obtain a crucially important characteristic of the model, its dispersion relation, by fixing $w = -1$ and substituting into the linearized equations $\Sigma_+, P \sim \exp(ik\zeta - i\omega\tau)$. The resulting dispersion relation,

$$(\omega - \delta)[\omega^2 - k^2 - (2 + \eta^2)] + 2(\eta - \delta) = 0, \tag{5}$$

is displayed in Fig. 1. The frequencies corresponding to $k = 0$ are $\omega = \eta$ and $\omega = -\frac{1}{2}(\eta - \delta) \pm \sqrt{2 + \frac{1}{4}(\eta + \delta)^2}$, while at $k^2 \to \infty$ the asymptotic expressions for different branches of the dispersion relation are $\omega = \pm k$ and $\omega = \delta + 2(\eta - \delta)k^{-2}$. Thus, the linearized spectrum always splits into *two* gaps, separated by an allowed band, except for the special case, $\eta = \eta_0 \equiv \frac{1}{2}\delta + \sqrt{1 + \frac{1}{4}\delta^2}$, when the upper gap closes down. The upper and lower band edges are those of the periodic structure, shifted by the induced TLS polarization in the limit of a strong reflection. They approach the SIT spectral gap for forward- and backward-propagating waves [7] in the limit of weak reflection. The allowed middle band corresponds to a collective atomic polarization excitation.

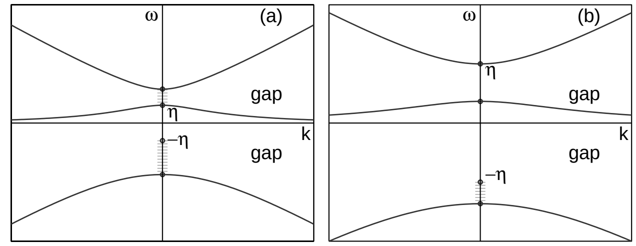

FIG. 1. The spectrum of the linearized RABR model (1)–(3). The parts of the upper and lower gaps filled with the solitons are shaded: (a) The weak-reflectivity case when the condition (10) holds ($\eta = 0.6$, $\delta = 0$); (b) the opposite case ($\eta = 2$, $\delta = 0$).

Inside these gaps, ZV solitons are sought in the form

$$\Sigma_+(\zeta,\tau) = e^{-i\chi\tau}\sigma(\zeta), \qquad P(\zeta,\tau) = ie^{-i\chi\tau}q(\zeta), \tag{6}$$

with real functions $\sigma(\zeta)$ and $q(\zeta)$, χ being the frequency detuning from the gap center. Using Eqs. (2) and (4), we can express the variables w and q in terms of σ:

$$w = \pm(\chi - \delta)[\sigma^2 + (\chi - \delta)^2]^{-1/2}, \quad q = \pm\sigma[\sigma^2 + (\chi - \delta)^2]^{-1/2}. \tag{7}$$

The remaining equation for $\sigma(\zeta)$ is

$$\frac{d^2\sigma}{d\varsigma^2} = \frac{dF}{d\sigma}, \qquad F(\sigma) \equiv -\left[\frac{1}{2}(\eta^2 - \chi^2)\sigma^2 \pm 2(\eta - \chi)\left(\sqrt{(\chi - \delta)^2 + \sigma^2} - |\chi - \delta|\right)\right], \tag{8}$$

where the $\pm$ signs correspond to those in Eqs. (7).

Equation (8) is the Newton equation of motion for a particle with the coordinate σ in the potential $F(\sigma)$. It gives rise to a solitonlike solution [8], provided that the upper sign is chosen in Eqs. (7), and the frequency χ satisfies the conditions $|\chi| > \eta$, $|(\chi + \eta)(\chi - \delta)| < 2$. These inequalities can be solved to yield, in an explicit form, *two* frequency intervals in which one has the ZV gap solitons: at $\chi > 0$, it is

$$\eta < \chi < \sqrt{\frac{1}{4}(\eta + \delta)^2 + 2} - \frac{1}{2}(\eta - \delta), \tag{9}$$

which exists only for small and moderate η (the weak Bragg-reflectivity limit)

$$\eta < \frac{1}{2}\delta + \sqrt{1 + \frac{1}{4}\delta^2}, \tag{10}$$

and another interval at $\chi < 0$, which exists for any reflectivity

$$\eta < -\chi < \sqrt{\frac{1}{4}(\eta + \delta)^2 + 2} + \frac{1}{2}(\eta - \delta). \tag{11}$$

On comparing these expressions with the spectrum shown in Fig. 1, we conclude that part of the lower gap is always empty from solitons, while the upper gap is completely filled with ZV solitons in the weak-reflectivity case (10) [Fig. 1(a)] and completely empty in the strong-reflectivity case (11) [Fig. 1(b)]. It is relevant to mention that a partly empty gap has recently been found in the model combining Bragg reflection and second harmonic generation [4].

Inside the frequency intervals (9) and (11), Eq. (8) can be integrated by means of the substitution

$$\sigma(\zeta) \equiv 2|\chi - \delta|\rho(\zeta)[1 - \rho^2(\zeta)]^{-1}. \tag{12}$$

This yields the soliton shape in an implicit form, i.e., ζ vs ρ:

$$|\zeta| = \sqrt{2\left|\frac{\chi - \delta}{\chi - \eta}\right|} \times \{(1 - \rho_0^2)^{-1/2} \tan^{-1}[\sqrt{(\rho_0^2 - \rho^2)/(1 - \rho_0^2)}] + (2\rho_0)^{-1} \ln[(\rho_0 + \sqrt{\rho_0^2 - \rho^2})/\rho]\}, \quad (13)$$

where $\rho_0^2 \equiv 1 - \frac{1}{2}|(\chi + \eta)(\chi - \delta)|$ [note that ρ_0^2 is positive under the above conditions (9)–(11)]. It can be checked that this ZV gap soliton is always *single* humped. Its amplitude can be found from Eq. (13),

$$\sigma_{\max} = 4\rho_0/\sqrt{|\chi + \eta|}. \quad (14)$$

The most drastic difference of these new solitons from the well-known SIT pulses [6] is that the area of the ZV soliton is not restricted to 2π, but, instead, may take an *arbitrary* value. As mentioned above, this basic new result shows that the Bragg reflector can enhance (by multiple reflections) the field coupling to the TLS, so as to make the pulse area *effectively* 2π. In the limit of the small-amplitude and small-area solitons, $\rho_0^2 \ll 1$, Eq. (13) can be easily inverted, the ZV soliton becoming a broad sechlike pulse:

$$\sigma \approx 2|\chi - \delta|\rho_0 \operatorname{sech}\left(\sqrt{2\left|\frac{\chi - \eta}{\chi - \delta}\right|}\rho_0\zeta\right). \quad (15)$$

In the opposite limit, $1 - \rho_0^2 \to 0$, i.e., for vanishingly small $|\chi + \eta|$, the soliton's amplitude (14) becomes very large, and further analysis reveals that, in this case, the soliton is characterized by a *broad central part* with a width $\sim(1 - \rho_0^2)^{-1/2}$ [Fig. 2(a)]. Another special limit is $\chi - \eta \to 0$. It can be checked that in this limit, the amplitude (14) remains finite, but the *soliton width diverges* as $|\chi - \eta|^{-1/2}$ [Fig. 2(b)]. Thus, although the ZV soliton has a single hump, its shape is, in general, strongly different from that of the traditional nonlinear-Schrödinger sech pulse.

The stability of the ZV gap solitons was tested numerically by means of direct simulations of the full system (1)–(3), the initial condition taken as the exact soliton with a small perturbation added to it. Running the simulations at randomly chosen values of the parameters, we have always found the ZV GS to be apparently *stable*. However, the possibility of their dynamical and structural instability needs to be further investigated as has been done in the case of GS in a Kerr-nonlinear fiber with a grating [9,10].

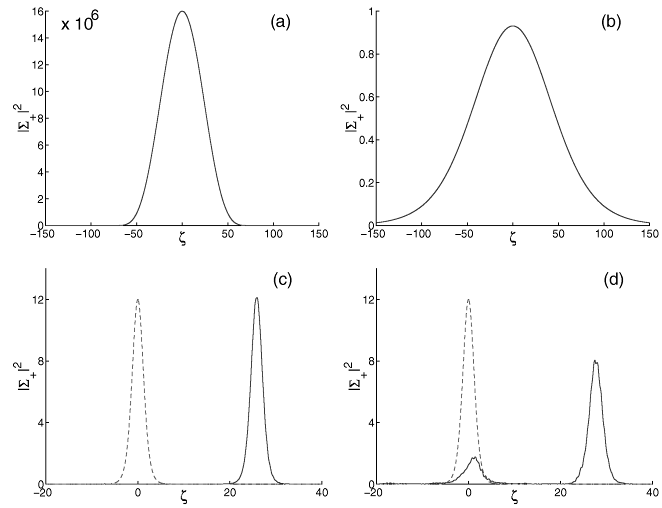

FIG. 2. Zero-velocity solitons $|\Sigma_+(\zeta)|^2$ (a) $\delta = 0$, $\eta = 0.9$, $\chi = -0.901$ (divergent width and amplitude); (b) the same as (a), but for $\chi = 0.901$ (divergent width and finite amplitude). (c) Pulse(s) obtained as a result of "pushing" of zero-velocity solitons (dashed lines) by the initial multiplier $\exp(-ip\zeta)$ after a sufficiently long evolution ($\tau = 400$) (solid lines). $\delta = 0$, $\eta = 4$, $\chi = -4.4$, and $p = 0.1$; (d) the same as (c), but for $p = 0.5$.

Although the system of Eqs. (1)–(3) is not explicitly Galilean or Lorentz invariant, translational invariance is expected on physical grounds. Hence, a full family of soliton solutions should have velocity as one of its parameters. This can be explicitly demonstrated in the limit of the small-amplitude large-width solitons [cf. Eq. (15)]. We search for the corresponding solutions in the form $\Sigma_+(\zeta,\tau) = \sigma(\zeta,\tau)\exp(-i\omega_0\tau)$, $P(\zeta,\tau) = iq(\zeta,\tau)\exp(-i\omega_0\tau)$ [cf. Eqs. (6)], where ω_0 is the frequency corresponding to $k = 0$ on any of the three branches of the dispersion relation (5) (see Fig. 1), and the functions $\sigma(\zeta,\tau)$ and $q(\zeta,\tau)$ are assumed to be slowly varying in comparison with $\exp(-i\omega_0\tau)$. Under these assumptions we arrive at the following asymptotic equation for $\sigma(\zeta,\tau)$:

$$\left[2i\,\frac{\omega_0(\omega_0 - \delta)^2 - \eta + \delta}{(\omega_0 - \delta)^2}\frac{\partial}{\partial\tau} + \frac{\partial^2}{\partial\zeta^2} + \frac{\omega_0 - \eta}{(\omega_0 - \delta)^3}|\sigma|^2\right]\sigma = \left(\eta^2 - \omega_0^2 + 2\,\frac{\omega_0 - \eta}{\omega_0 - \delta}\right)\sigma. \quad (16)$$

Since this equation is of the NLS form, it has the full two-parameter family of soliton solutions, including the moving ones [8].

In order to check the existence and stability of the *moving* solitons numerically, we simulated Eqs. (1)–(3) for an initial configuration in the form of the ZV soliton multiplied by $\exp(ip\zeta)$ with some wave number p, in order to "push" the soliton. The results demonstrate that, at sufficiently small p, the push indeed produces a moving stable soliton [Fig. 2(c)]. However, if p is large enough, the multiplication by $\exp(ip\zeta)$ turns out to be a more violent perturbation, splitting the initial pulse into two solitons, one quiescent and one moving [Fig. 2(d)].

A one-parameter subfamily of moving GS was found in an exact form in Ref. [5]:

$$\Sigma_{+} = A_0 \exp[i(\alpha\zeta - \Delta\tau)] \operatorname{sech}\left[\frac{1}{2} A_0 u(\zeta - u\tau)\right], \tag{17}$$

where Δ is the detuning from the gap center, and the squared amplitude $A_0^2 = (1 - u^2)^{-2}[8u^2(1 - u^2) - \eta^2(1 + u^2)^2]$ is expressed in terms of the velocity u (normalized to c). The values of u are restricted by the condition $A_0^2 > 0$, which, in particular, forbids $u = 0$. The present analysis strongly suggests, but does not rigorously prove, that the subfamily (17) belongs to a far more general two-parameter family, whose other particular representatives are the exact ZV solitons (13) and the approximate small-amplitude solitons determined by Eq. (16).

An issue of obvious interest is collisions between GS moving at different velocities. In the asymptotic small-amplitude limit reducing to the NLS equation (16), the collision must be elastic. To get a more general insight, we simulated collisions between two solitons given by (17). The conclusion is that the collision is *always inelastic,* directly attesting to the nonintegrability of the model. Typical results are displayed in Fig. 3, which demonstrates that the inelasticity may be strong, depending on the parameters.

The present findings can be demonstrated in periodically etched dielectric structures containing either strained quantum wells or gas as the active TLS. For example, we can use HF gas whose active dipole transition at the wavelength $\lambda = 84$ μm is resonant with a photonic band edge. A 4.5 mTorr gas pressure corresponds to cooperative length $l_c = 114$ cm and inhomogeneous dephasing length $cT_2^* \sim 10^4$ cm. In a structure with Bragg reflection length $l_r = 100\lambda \ll l_c$, we then have $\eta = -\delta = -135.7$, which allow for GS detuned by 2×10^3 cm^{-1} from both band edges with $\sim$2 ns width and 4×10^9 s^{-1} Rabi frequency. The suggested structures can be used to realize any kind of GS: standing (ZV) GS require initial *localized* excitation of the near-resonant medium by counterpropagating laser beams, whereas moving GS can be launched by a single near-resonant laser beam propagating along the structure.

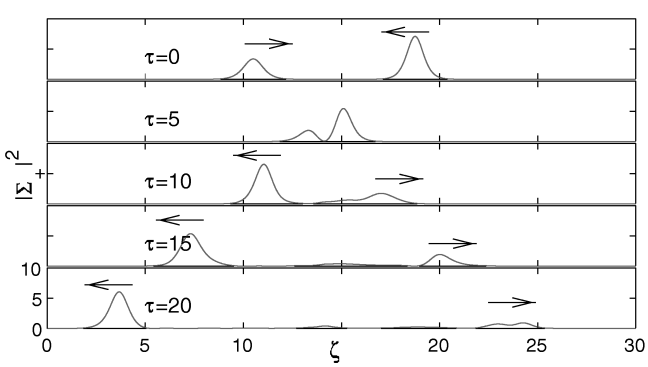

FIG. 3. Typical example of inelastic collisions between the solitons (17) at $\delta = 0$ and $\eta = 0.5$, with the velocities (normalized to c) $u_1 = 0.6$, $u_2 = -0.75$.

In conclusion, we have demonstrated that a periodic array of near-resonant two-level systems combined with a Bragg grating gives rise, for *any* Bragg reflectivity, to a vast family of stable gap solitons, both standing and moving, having a unique analytic form, an *arbitrary* pulse area (Fig. 2), and inelastic collision properties (Fig. 3). Remarkably, even extremely low TLS densities create an allowed band within the forbidden gap (Fig. 1). These findings reveal hitherto unknown forms of soliton dynamics in a nonintegrable, strongly nonlinear system. Their origin is the surprisingly rich interplay between multiple reflections and cooperative, near-resonant field-matter interaction, which removes the pulse-area restrictions of ordinary self-induced transparency. These findings can lead to the realization of novel, highly advantageous filters, which can stably transmit selected signal frequencies through their spectral gaps, while effectively blocking others, with no restriction on the signal pulse area. Alternatively, they can be used to spatially confine (self-trap) light in certain frequency bands, thus creating "dynamical cavities."

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