Modulational Interaction of the Lower-Hybrid Waves with a Kinetic-Alfvén Mode

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A system of equations describing the coupling and modulational interaction of kinetic Alfvén mode with lower-hybrid waves is constructed and analyzed. A coupling mechanisms is created by the density modulation forced by Reynolds stresses of the lower-hybrid waves. The modulational instability resulting from the coupling of the lower-hybrid and kinetic-Alfvén waves has been investigated. The nonlinear evolution of the modulational instability leads to the formation of the magnetic-field-aligned envelope soliton of Alfvén waves with typical transverse dimension of the order of the electron skin depth, and with much shorter in wavelength lower-hybrid oscillations trapped inside the soliton. [S0031-9007(98)07303-7]

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A mechanism of transverse ion acceleration (TAI) is an important component of the magnetospheric physics. The mechanism provides injection in the magnetosphere of the energetic heavy ions of the ionospheric origin (e.g., energetic O^+ ions). Rocket observations performed by Kintner and his group revealed that the TAI phenomenon is localized in the narrow regions of the intensive lower-hybrid wave (LHW) activity located at the altitudes about 1000 km in the auroral ionosphere [1] with the frequencies close to the so-called lower-hybrid resonance [2]. TAI events usually occur during periods of field aligned electron bursts, which therefore can be identified as the driver for the LHW [3]. Two mechanisms are considered now as the most plausible explanation of the LHW excitation by the precipitated auroral electrons. They are two-stream instability driven by Cherenkov resonance between the lower-hybrid waves and the beam part of the precipitated electrons energy distribution [4], and the so-called "fan" instability driven by anomalous Doppler resonance with the energetic tail of the precipitated electrons [5]. Observations described in Ref. [1] have shown that regions of localization of the lower-hybrid waves and ion acceleration are thin field aligned filaments with the dimension across geomagnetic field on the order of the several hundred meters comparable for the ionospheric conditions with the hot ion gyroradius v_{Ti}/ω_{ci} or the electron skin depth c/ω_{pe}. These regions of localization are usually interpreted as the LH cavitons (see, e.g., [6]), and therefore the above-described rocket experiments are the first clear demonstration of the self-modulational instability and cavity formation for the LHW.

At first, a theoretical interpretation has been proposed that these observations result from the modulational interaction of the LHW with the large-scale quasineutral density perturbations similar to ion acoustic oscillations (see, e.g., [7,8]). Modulational interaction causes the short wavelength cascading of the LHW resulting in TAI due to ion resonant interaction with waves. However, the typical transverse caviton size comparable with the electron skin depth remained unexplained in this theory.

A new outlook on the picture of modulational interaction has been initiated by the wave data from the recently launched Freja satellite [9]. The data show a strong correlation in the auroral energization regions between enhancement in shorter wavelength and higher frequency electrostatic LHW turbulence and Alfvén wave activity. In some cases the observed caviton structures are identified as localizations of the kinetic Alfvén waves (KAW).

A kinetic Alfvén wave is created from oblique $(k_\parallel \ll k)$ to the ambient magnetic field shear Alfvén wave when the transverse frequency dispersion is important. In our case of an extremely low $\beta \ll m_e/m_p$ plasma (β is the ratio of the plasma kinetic pressure to the magnetic pressure), this dispersion is due to electron inertia, resulting in the following dispersion law of KAW [10]:

$$\omega = \frac{k_\parallel v_A}{\sqrt{1 + \frac{k_\perp^2 c^2}{\omega_{pe}^2}}}. \quad (1)$$

The goal of this Letter is to show that the observed solitary KAW structures can be produced by the Reynolds stresses exerted on a plasma by the lower hybrid or (close to them in nature) electrostatic whistler waves obeying the following dispersion law:

$$\omega = \omega_{\rm LH}\left(1 - \frac{\omega_{pe}^2}{k^2c^2} + k^2\rho^2 + \frac{m_p}{m_e}\frac{k_\parallel^2}{k^2}\right)^{1/2}. \quad (2)$$

In this expression the second term in brackets determines a small electromagnetic correction to the lower-hybrid frequency, the third term-short wavelength dispersion which is due to thermal motion (k is the wave number, ρ is an electron gyroradius), and the last term is the frequency dispersion due to deviation from the transverse direction of wave propagation $k_\parallel = 0$. All corrections are small in comparison with unity, so Eq. (2) describes waves at the frequencies close to the lower hybrid resonance $\omega_{\rm LH}$. Reynolds stresses created by the LHW result in the modulational interaction with KAW and formation of fairly deep plasma density modulation ($\sim$10%), in which a broadband electrostatic noise at the

LH frequency $\sim 10-15$ kHz is trapped. The threshold for modulational instability is definitely below the observed amplitudes of the LHW, and therefore, above described modulational interaction is an inalienable feature of the auroral ionosphere.

As usual (see, e.g., [11]), components of the electric and magnetic fields in KAW can be expressed through the scalar potential Ψ and the longitudinal component of the vector potential A:

$$B_z = B_0, \quad B_x = \frac{\partial A}{\partial y}, \quad B_y = -\frac{\partial A}{\partial x},$$
$$E_x = -\frac{\partial \Psi^*}{\partial x}, \quad E_y = -\frac{\partial \Psi}{\partial y}, \quad E_z = -\frac{\partial \Psi}{\partial z} - \frac{1}{c}\frac{\partial A}{\partial t}. \tag{3}$$

It follows from Maxwell equations for curl B that a deviation of Ψ^* from Ψ is a pure nonlinear effect due to the curl of the nonlinear diamagnetic current j_D:

$$\frac{\partial}{\partial t}\frac{\partial^2}{\partial x \partial y}(\Psi - \Psi^*) = \frac{eB_0^2}{n_0 m_i c^2}\left(\frac{\partial}{\partial y} j_{D_x} - \frac{\partial}{\partial x} j_{D_y}\right). \tag{4}$$

Here a diamagnetic current $\vec{j}_D = -e\langle \delta n \vec{v}_e \rangle$, δn is an envelope amplitude for the density variation in the LHW with the fast time dependence at the lower hybrid frequency singled out:

$$\delta n_{\mathrm{LH}} = \frac{1}{2}\,\delta n(t,\vec{r})\exp(-i\omega_{\mathrm{LH}} t) + \mathrm{c.c.}$$

It can be expressed through the corresponding electrostatic potential φ with the help of the following relationship:

$$\delta n = -\frac{n_0 m_e c^2}{eB_0^2}\nabla_\perp^2 \varphi,$$

$\vec{v}_e = \frac{c}{B_0^2}\vec{B}_0 \times \nabla\varphi$ is an envelope amplitude for the electron drift velocity, and brackets denote averaging over the fast time ($\sim \omega_{\mathrm{LH}}^{-1}$). Using relationships (3) it is possible to rewrite transverse components of the Maxwell equations for curl B in the form of the following equation coupling vector and scalar potential in the KAW:

$$\frac{\partial}{\partial y}\left(\frac{\partial \Psi}{\partial t} + \frac{B_0^2}{4\pi n_0 m_p c}\frac{\partial A}{\partial z}\right) = \frac{m_e}{4m_p}\frac{c}{B_0}\left\langle \frac{\partial \varphi}{\partial x}\nabla_\perp^2 \varphi^* + \mathrm{c.c.}\right\rangle. \tag{5}$$

In the KAW an electron field aligned motion is important; the velocity of this motion is determined by the following equation:

$$m_e \frac{\partial u_{ze}}{\partial t} = \frac{e}{c}\frac{\partial A}{\partial t} + e\frac{\partial \Psi}{\partial z} + \frac{ec}{iB_0\omega_{\mathrm{LH}}}\left[(\nabla\varphi^* \times \nabla)_z \frac{\partial \varphi}{\partial z} - \mathrm{c.c.}\right],$$

where we also included a nonlinear term describing Reynolds stress created by electron field aligned motion in the lower hybrid wave $(\vec{v}_e \cdot \nabla)v_{ez}$. Then the longitudinal component of the Maxwell equation gives the following relationship coupling Ψ and A in the kinetic Alfvén wave:

$$\frac{\partial}{\partial t}\left[1 - \frac{c^2}{\omega_{pe}^2}\nabla_\perp^2\right]A = -c\frac{\partial}{\partial z} \times \left[\Psi + \frac{c}{4iB_0\omega_{\mathrm{LH}}}(\nabla\varphi^* \times \nabla\varphi)_z\right]. \tag{6}$$

There are two nonlinear terms in Eqs. (5) and (6) driving modulational interaction of KAW with the lower hybrid oscillations. A nonlinearity in Eq. (5) is produced by an average component of the electron diamagnetic current; in Eq. (6) it is due to the Reynolds stresses in the electron field aligned motion. It is easy to see that input from the first nonlinearity is small as $\frac{m_e\omega_{\mathrm{LH}}}{m_p\omega_A}$ (ω_A is the Alfvén frequency), and we shall neglect it. Therefore, the final system of equations describing the nonlinear coupling of the fast lower hybrid oscillations with the kinetic Alfvén wave can be written as follows: An equation for the lower hybrid potential has the form (e.g., compare with [12]):

$$-\frac{2i}{\omega_{\mathrm{LH}}}\frac{\partial}{\partial t}\nabla_\perp^2\varphi + \frac{m_p}{m_e}\frac{\partial^2\varphi}{\partial z^2} + \frac{\omega_{pe}^2}{c^2}\varphi - \rho^2\nabla_\perp^4\varphi = i\frac{\omega_{pe}^2}{\omega_{ce}\omega_{\mathrm{LH}}}\left(\nabla_\perp\varphi \times \nabla_\perp\frac{\eta}{n_0}\right)_z. \tag{7}$$

Here $\frac{\eta}{n_0}$ is the relative plasma density variation in the KAW, which can be expressed through the vector potential of the wave with the help of the following relationship:

$$\frac{\partial}{\partial t}\frac{\eta}{n_0} = -\frac{c}{4\pi e n_0}\frac{\partial}{\partial z}\nabla_\perp^2 A. \tag{8}$$

Finally, an equation for Alfvén wave vector potential is of the form

$$\frac{\partial^2 A}{\partial t^2} - \frac{c^2}{\omega_{pe}^2}\nabla_\perp^2\frac{\partial^2 A}{\partial t^2} - v_A^2\frac{\partial^2 A}{\partial z^2} = i\frac{c^2}{4B_0\omega_{\mathrm{LH}}}(\nabla\varphi^* \times \nabla\varphi)_z, \tag{9}$$

where $v_A = B_0/\sqrt{4\pi n_0 m_p}$ is the Alfvén velocity.

Nonlinear coupling of the lower hybrid waves with the kinetic Alfvén mode results in the modulational instability. Small density depletions associated with the initial KAW serve as the potential wells for the LH oscillations. Trapping of these oscillations inside the potential wells results in modulation of their intensity. Because of modulation, Reynolds stresses exerted on plasma electrons by the LH mode are created. Under their action wells are deepening, leading to stronger modulation.

The initial stage of modulational instability can be described using the standard procedure of the linear stability analysis. A test Alfvén wave with the vector potential in a form of a plane wave at the frequency ω and with the wave vector $\vec{k}$ is coupled with the main (pump) lower hybrid wave at the frequency ω_0 and with the wave vector $\vec{k}_0$, and results in LH modulation by two satellites—red $(\omega_0 - \omega, \vec{k}_0 - \vec{k})$ and blue $(\omega_0 + \omega, \vec{k}_0 + \vec{k})$. The dispersion relation describing this process is of the following form:

$$\omega^2\left[1 + \frac{k_\perp^2 c^2}{\omega_{pe}^2}\right] = k_\parallel^2 v_A^2\left[1 - \frac{k_\perp^2 c^2}{4(\omega_{pe}^2 + \omega_{ce}^2)}\frac{E_0^2}{E_{\mathrm{TH}}^2}\frac{(\vec{k}\times\vec{k}_0)^2}{k^2 k_0^2} \times \left(\frac{k^2}{(\vec{k}_0 - \vec{k})^2}\frac{1}{\Delta_- + 2\omega/\omega_{\mathrm{LH}}} + \frac{k^2}{(\vec{k}_0 + \vec{k})^2}\frac{1}{\Delta_+ - 2\omega/\omega_{\mathrm{LH}}}\right)\right]. \qquad (10)$$

The following notations are used in this equation: $E_0 = k_0\varphi_0$ is an electric field of the pump wave, E_{TH} is a typical electric field strength characterizing efficiency of the modulational coupling:

$$E_{\mathrm{TH}}^2 = 4\pi n_0 T \frac{\omega_{ce}^2}{\omega_{pe}^2}\frac{m_e^2}{m_p^2\beta} = \frac{\omega_{ce}^2}{\omega_{pe}^2} B_0^2 \frac{m_e^2}{m_p^2}. \qquad (11)$$

Finally, Δ_+, Δ_- are dimensionless frequency mismatches, $\Delta_\pm = \frac{\omega_\pm - \omega_0}{\omega_{\mathrm{LH}}}$, between the frequencies of the LH satellites determined by Eq. (2) for the wave vectors $\vec{k}_0 \pm \vec{k}$ and that of the LH pump with a wave vector $\vec{k}_0$.

For small amplitudes of the pump wave dispersion equation (10) describes the parametric decay of a lower hybrid wave into another lower hybrid wave and KAW. Substituting into the dispersion equation $\omega = \omega_A + \varepsilon$, where ω_A is an eigenfrequency of the kinetic Alfvén mode and using a paremetric decay condition for frequencies, $\omega_\pm = \omega_0 \pm \omega_A$, it is easy to obtain the following relationship for $\varepsilon \ll \omega_{\mathrm{LH}}$:

$$\varepsilon_\pm^2 = \pm\frac{\omega_A\omega_{\mathrm{LH}}}{8}\frac{E_0^2}{E_{\mathrm{TH}}^2}\sin^2\alpha\frac{k^4c^2}{(\omega_{pe}^2 + \omega_{ce}^2)(\vec{k}\pm\vec{k}_0)^2}, \qquad (12)$$

where α is an angle between $\vec{k}$ and $\vec{k}_0$ vectors. It follows from this relationship that a parametric instability $(\varepsilon^2 < 0)$ is possible, as usual, only in the case of a red satellite excitation. Equation (12) is valid only for small amplitudes of the pump, when $|\varepsilon|/\omega_A \ll 1$, or

$$E_0 < E_{\mathrm{TH}}\sqrt{\omega_A/\omega_{\mathrm{LH}}} \ll E_{\mathrm{TH}}.$$

For pump wave amplitudes comparable with E_{TH} coupling is stronger resulting in the so-called "modified decay instability" with the growth rate exceeding an eigenfrequency of the Alfvén mode (e.g., see discussion of a similar situation for the decay of Langmuir waves in [12]). If we assume for simplicity that pump wave is sufficiently short wavelength $k_0 \gg k$ and propagates strictly perpendicular to the magnetic field $(k_0)_{\|=0}$, then $\Delta_+ = \Delta_- = \frac{k_\|^2}{k_0^2}$, and dispersion equation (10) can be written in a simpler form:

$$(\omega^2 - \omega_A^2)\left(\frac{4\omega^2}{\omega_{\mathrm{LH}}^2} - \Delta^2\right) = \omega_A^2\Delta\Gamma, \qquad (13)$$

where notation $\Gamma = \frac{k^2c^2}{2(\omega_{pe}^2+\omega_{ce}^2)}\frac{k^2}{k_0^2}\sin^2\alpha\frac{E_0^2}{E_{\mathrm{TH}}^2}$ is used.

It follows from a solution of this dispersion relation, that for sufficiently large amplitudes of a pump wave, when $\Gamma > \Delta$, strong aperiodic $(\omega^2 < 0)$ instability develops leading to the excitation of the kinetic Alfvén mode from the LH pump.

It is also noteworthy that since $\Delta \sim \omega_A/\omega_{\mathrm{LH}} \ll 1$, both parametric and strong aperiodic instabilities are possible for the values of the pump amplitude below E_{TH}. The latter value for the typical conditions of the auroral ionosphere ranges in the interval 300 mV/m $-$ 1 V/m. Estimation of the magnetic field in the KAW can be easily obtained from Eq. (9) for the vector potential. It follows from this equation that a pump LHW with an amplitude comparable with E_{TH} drives the magnetic field in the kinetic Alfvén mode of the order

$$B^* \sim \frac{k_\perp c}{\omega_{pe}}\frac{E_0^2}{B_0}\frac{\omega_{pe}^2}{\omega_{ce}^2} \sim \frac{m_e}{m_p}B_0 \sim (1\text{--}3)\,nT.$$

The relative modulation of the plasma density can be estimated from Eq. (8) as follows:

$$\frac{\eta}{n_0} \sim \frac{k_\perp^2c^2}{\omega_{pe}^2}\left(\frac{m_p}{m_e}\right)^{3/2}\frac{E_0^2}{B_0^2} \sim (3\text{--}10)\%.$$

As a result of the nonlinear evolution of the described modulational interaction, a two-dimensional soliton of the kinetic Alfvén wave can be formed. The soliton travels along the magnetic field with Alfvén speed, and across the field has the form of the localized dypole structure. For simplicity, we consider the case of the plane soliton geometry. In this case,

$$\varphi = \varphi(z - v_At, x)\exp[i(k_yy - \lambda t)], \quad A = A(z - v_At, x). \qquad (14)$$

Equation (9) for the vector potential in this case can be easily integrated to give the following relationship for the magnetic field inside the soliton:

$$\frac{dA}{dx} = -\frac{\omega_{pe}^2k_y}{4B_0v_A\omega_{\mathrm{LH}}}|\varphi|^2. \qquad (15)$$

We now consider a case where, in Eq. (7) for the LH potential, it is possible to neglect the left-hand side term with the longitudinal derivatives of the potential describing a deviation from strictly transverse polarization of the LH oscillations. Then, a form of traveling wave in a field aligned direction remains arbitrary determined by

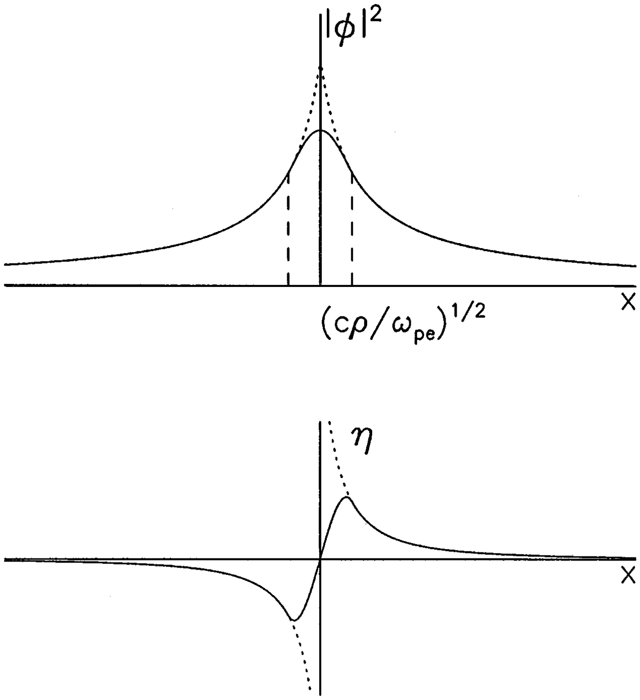

FIG. 1. Square of the amplitude of the lower hybrid potential (top) and the density variation (bottom) inside an Alfvén soliton.

the initial conditions, while the solitary structure across magnetic field is a solution of the following differential equation:

$$\delta\left(\frac{d^2}{d\xi^2} - \frac{c^2k_y^2}{\omega_{pe}^2}\right)^2\phi + \frac{2\lambda}{\omega_{\mathrm{LH}}}\frac{d^2\phi}{d\xi^2} - a\phi = b\phi\,\frac{d^2}{d\xi^2}\,\phi^2, \tag{16}$$

where the following dimensionless units and notations are used:

$$\phi = \frac{e\varphi}{m_e v_A^2}, \qquad \xi = \frac{x\omega_{pe}}{c}, \qquad \delta = \frac{\rho^2\omega_{pe}^2}{c^2},$$

$$a = \frac{\omega_{pe}^2}{\omega_{pe}^2 + \omega_{ce}^2} + \frac{2\lambda}{\omega_{\mathrm{LH}}}\frac{k_y^2c^2}{\omega_{pe}^2}, \quad b = \frac{k_y^2c^2}{\omega_{ce}^2}.$$

Assuming sufficiently smooth transverse structure with typical size on the order of the electron skin depth, it is possible to omit a term with the fourth derivative, proportional to δ. Then Eq. (16) has a first integral in the form

$$\left(-\frac{\lambda}{\omega_{\mathrm{LH}}} + b\phi^2\right)\left(\frac{d\phi}{d\xi}\right)^2 = -\,\frac{a}{2}\,\phi^2. \tag{17}$$

The structure of the soliton is sketched in a Fig. 1. It is a solitary structure with the spike at its center ($x = 0$). To avoid this singularity it is necessary to take into account a term with the fourth derivative and a small coefficient in front of it in Eq. (16). Thus, singularity in φ would be smoothed, forming a narrow region with the following typical size and the maximum value of the electric field in it:

$$\Delta x \approx \sqrt{\rho\,\frac{c}{\omega_{pe}}}, (E_x)_{\max} \approx E_{\mathrm{TH}}\sqrt{\frac{c}{\omega_{pe}\rho}}\,. \tag{18}$$

The density structure within the soliton can be determined with the help of Eqs. (8) and (15). It is of the dypole ($\eta > 0, \eta < 0$) type, and also shown in a figure.

In conclusion, we have demonstrated in the paper that the lower hybrid wave activity observed in the auroral ionosphere is subject to the modulational interaction with the kinetic Alfvén waves, as the result of which field aligned solitary structures of KAW are formed.

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