High Beam Quality and Efficiency in Plasma-Based Accelerators

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The question of what beam quality and efficiency are ultimately achievable in plasma-based accelerators is addressed analytically and through self-consistent particle-in-cell simulations. A strategy for phasing and beam loading to minimize energy spread while at the same time achieving high energy extraction efficiency is proposed. Preservation of beam emittance is facilitated by the use of a hollow channel. [S0031-9007(98)07381-5]

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The past two years have seen great advances in laser-driven plasmas as media for particle acceleration [1,2]. Plasma waves (wakefields) of magnitude on the order of 100 GeV/m have been excited with relativistic phase velocities, electron beams accelerated to 100 MeV within a few millimeters, and peak currents of tens of kiloamperes generated [3]. Recent computational work [4] has added significant insight into the physical mechanisms underlying these experiments. The bulk of these near term experiments rely on an instability for plasma wave growth and typically result in 100% energy spread of accelerated particles. These works have emphasized high accelerating gradients and energy gains, and less attention has been paid to the qualities of the beams (energy spread, emittance, particle number). Moreover, the question of overall efficiency has not yet been seriously addressed.

In this Letter we address self-consistently the overall efficiency, beam emittance, and energy spread for a plasma accelerator. In order to overcome fundamental tradeoffs between these quantities we exploit adjustment of the insertion phase to equalize energy gain between head and tail and thereby minimize energy spread. Particle-in-cell (PIC) simulations are used to show that it is possible to accelerate beams in plasmas with high beam quality and high overall efficiency.

We consider one particular realization of a plasma accelerator, namely, a hollow channel plasma accelerator [5] with Gaussian beams. Wakefields in hollow channels can be excited by either a short laser pulse or a high current particle beam. In the simulations we consider the wake excited by an electron beam. However, most of the results apply to the laser-driven case as well. Channel formation is currently a topic of great experimental investigation. Leading approaches include laser-produced channels [6] and hollow capillary discharges [7]. The hollow channel case is significant because it offers not only laser guiding (for laser-driven wakefield accelerators) but also the ideal accelerating field structure (i.e., transversely uniform accelerating fields and minimum focusing/defocusing forces). Thus the results here address the question of what is the highest beam quality and efficiency ultimately possible in high gradient plasma-based accelerators.

We begin by considering an ideal hollow plasma (for simplicity, we analyze slab geometry; however, the conclusions apply to cylindrical geometry as well). The plasma is assumed to be homogeneous everywhere except between $y = -a$ to $y = +a$, between which it is empty. The typical wakefield excited by a laser pulse has been worked out previously [5] and is shown in Fig. 1. The wavelength of the longitudinal field can be approximated as $\lambda_{\rm ch} = \lambda_p\sqrt{1 + k_p a}$ where $k_p = 2\pi/\lambda_p = \omega_p/c$ and ω_p is the plasma frequency $= \sqrt{4\pi n_0 e^2/m}$. The transverse profile of the accelerating gradient is uniform across the channel and drops exponentially outside the channel. The focusing force is zero inside the channel for a very relativistic particle. The spikes at the channel walls are due to the plasma/vacuum boundaries. We comment that if particles hit the channel boundaries, the spikelike force can turn the particles around (like bouncing off of a wall). This may offer an interesting means of transporting very intense space charge dominated beams.

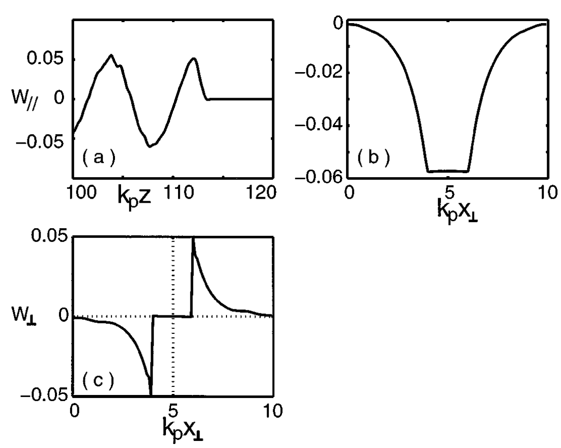

FIG. 1. Typical wakefield structure in a hollow plasma channel. (a) Accelerating gradient $W_\parallel$ vs $k_p z$; (b) accelerating gradient vs $k_p x_\perp$; and (c) focusing field $W_\perp$ vs $k_p x_\perp$.

The maximum number of particles that can be accelerated by the plasma wakefield can be found from superposition of the wakefield and the self-wake of the particles in the beam load. The wake excited by a particle beam in a hollow channel can be found in an analogous way to the calculation of the laser wake done in Ref. [5]. The result for the self-wakefield of an accelerated beam of radius b, density n_b, and length l is given by the following [8]:

$$\frac{eE_z(y,\xi)}{m\omega_p c} = \begin{cases} 0, & \xi > 0, \\ \frac{-1}{\sqrt{1+k_p a}}\left(\frac{n_b}{n_0}\right)(k_p b)\sin(k_{\mathrm{ch}}\xi), & -l < \xi < 0, \\ \frac{2}{\sqrt{1+k_p a}}\left(\frac{n_b}{n_0}\right)(k_p b)\sin\left(\frac{k_{\mathrm{ch}} l}{2}\right)\cos\left[k_{\mathrm{ch}}\left(\xi + \frac{l}{2}\right)\right], & \xi < -l. \end{cases} \tag{1}$$

This is the longitudinal wake inside the channel where n_0 is the background plasma density, $k_{\mathrm{ch}} = k_p/\sqrt{1 + k_p a}$, $\xi = z - ct$. The wake is zero in front of the beam by causality, rises as a *sine* function during the beam and oscillates sinusoidally after the beam.

The tradeoff between high beam loading efficiency and high beam quality is by now well known [9–11]. High efficiency requires heavy beam loading; however, the self-wake of these beams reduces the acceleration of the tail resulting in energy spread. For short bunch lengths (bunch length $\ll$ wake wavelength) the efficiency is approximately $2\eta - \eta^2$, when the energy spread induced in the beam by the beam's self-wake is η [11]. Thus, higher efficiency yields a larger energy spread. For example, a beam loading efficiency of 50% yields a 30% energy spread. In this Letter we include the effect of phase slippage and show that it can be used to reduce this energy spread from 30% to 3% without specialized shaping of the beam [9]. We also address for the first time overall efficiency, including self-consistently the drive beam to plasma coupling efficiency.

Because of the lack of transverse variations in E_z, the longitudinal dynamics of particle beams in a hollow channel are effectively simplified to one-dimensional phase slippage. We may take advantage of the slippage to minimize energy spread without shaping the beams [9]. By taking into account the combined effect of the accelerating wake, the beam's self-wakefield, and the phase slippage, we can find the injection position which will minimize energy spread (Fig. 2).

To quantify this idea, consider a slab beam of length l_t, width b, and density n_b riding on a wakefield $E_{zd}\sin(k_{\mathrm{ch}}\zeta)$ excited by a driving source. Assume the injection phase of the head is ζ_0 behind the accelerating peak, i.e., $\zeta_i = -\zeta_0 - \lambda_{\mathrm{ch}}/4$. To have maximum energy gain, the extraction position should be $\zeta_f = 0$. Thus the dephasing length can be approximated as $2\gamma_\phi^2(\lambda_{\mathrm{ch}}/4 + \zeta_0)$, where γ_ϕ is the Lorentz factor associated with the wake phase velocity. This dephasing length is obtained by calculating the distance that it takes for a particle moving at approximately c to outdistance the wake by $\lambda_{\mathrm{ch}}/4 + \zeta_0$. During this acceleration period, the particles at the tail of the beam will see a constant retarding force caused by the self-generated wake, whereas the self-generated wake is the energy loss of the tail, which can be estimated from Eq. (1) as

$$\Delta E_1 = 2\gamma_\phi^2\left(\frac{\pi}{2} + \phi_0\right)E_{zt}\cos\left(\frac{\phi_t}{2}\right)\sqrt{1 + k_p a}, \tag{2}$$

where $E_{zt} = (m\omega_p c/e)(2/\sqrt{1 + k_p a})(n_b/n_0)(k_p b) \times \sin(\phi_t/2)$ is the maximum amplitude of the wake excited by the trailing beam, $\phi_0 = k_{\mathrm{ch}}\zeta_0$, $\phi_t = k_{\mathrm{ch}} l_t$, and we have normalized ΔE_1 to mc^2.

This relative energy loss of the tail electrons with respect to the head electrons due to the beam self-wake can be compensated for modest beam loading by correctly choosing the insertion phase into the preexisting wake $E_{zd}\sin(k_{\mathrm{ch}}\zeta)$. The energy gains of the particles at different phases due to the preexisting wake can be found from the Hamiltonian formalism [12]: $\overline{H} = \sqrt{p^2 + 1} - E_0\cos(\phi) - v_\phi p$ where $\overline{H}$ is the normalized Hamiltonian of the particle-wave system, p is the normalized injection energy of the particle, and $E_0 = (eE_{zd}/m\omega_p c)\sqrt{1 + k_p a}$. For the particles at the head of the trailing beam, the injection position is $\zeta_i = -\zeta_0 - \lambda_{\mathrm{ch}}/4$ and the extraction position is $\zeta_f = 0$. Similarly, the particles at the tail of the beam have injection phase $\zeta_i = -\zeta_0 - \lambda_{\mathrm{ch}}/4 - l_t$ and extraction phase $\zeta_f = -l_t$. The energy spread of the trailing beam can be estimated by a simple two-particle model.

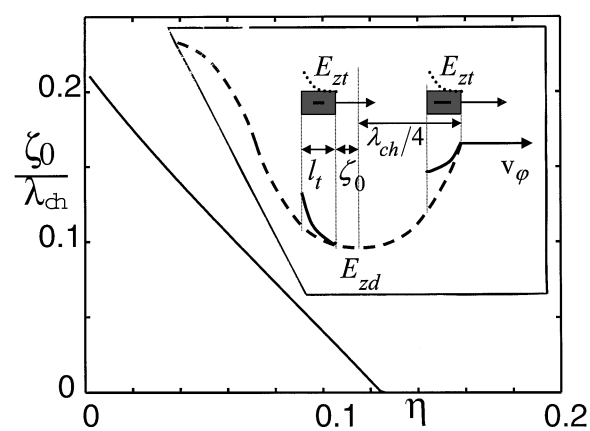

FIG. 2. The optimal injection position ($\zeta_0/\lambda_{\mathrm{ch}}$) vs the loading efficiency η for $l_t = 0.05\lambda_p$. The inset shows a sketch of the electric field seen by the beam at two different positions in the accelerating bucket. The dashed curve is the preexisting wakefield due to the driving source, the two dotted curves are the self-generated wakefields by the trailing beam, and the solid curve is the total wakefield.

By subtracting the energy gain of a particle at the beam head ($p_{hf} - p_{hi}$) from the energy gain of a particle at the beam tail ($p_{tf} - p_{ti}$, where the subscripts refer to the initial and final values for the particles at the tail and head), we obtain for the energy spread

$$\Delta E_2 = 2\gamma_\phi^2 E_0[\sin(\phi_0 + \phi_t) - \sin(\phi_0) + \cos(\phi_t) - 1], \quad (3)$$

where we have assumed all the variables $p \gg 1$. The condition that yields the same energy gain for the tail electrons as for the head can be found by equating Eq. (2) to Eq. (3). The result is

$$\eta\left(\frac{\pi}{2} + \phi_0\right)\cos\left(\frac{\phi_t}{2}\right) = \sin(\phi_0 + \phi_t) - \sin(\phi_0) + \cos(\phi_t) - 1, \quad (4)$$

where $\eta = E_{zt}/E_{zd}$ is the amplitude ratio of the wakefields excited by the trailing and the driving beams and is usually identified as the beam loading fraction (since $\eta = 1$ corresponds to 100% absorption of the wake energy). In terms of the number of accelerated particles (N), η is given by $N/N_{\max}$. For short ($k_p l \ll 1$) and narrow ($\ll c/\omega_p$) bunches, $N_{\max}$ follows from the last of Eqs. (1) to be $N_{\max} = (eE_{zd}/m\omega_p c)(3 \times 10^{17}/\sqrt{n_0})(1 + k_p a)$, where n_0 is in cm^{-3}. In Fig. 2 we show the optimal injection position ζ_0 versus the loading fraction η for a trailing beam of length $0.05\lambda_p$. For $\eta > 0.124$ (or 12.4% of the maximum possible beam load) the injection phase ζ_0 becomes negative, and the energy gain and gradient drop more severely.

In Figs. 3 we show simulation results of the energy gain and energy spread as functions of time. The beam lengths (full width at half maximum) are $1c/\omega_p$ for the driving beam and $0.3c/\omega_p$ for the trailing beam. Both beams are Gaussian in both y and z directions with beam radius $\sigma_y = 0.5c/\omega_p$. The emittance of the driving beam is much smaller than σ_y^2/L_d where L_d is the dephasing length, so that the beam divergence can be neglected. The peak density of the driving beam is the same as the plasma density n_0 and the peak density of the trailing beam is $0.33n_0$. The loading factor is $\eta \approx 10\%$. The energy spread $\delta\gamma$ is seen to increase then decrease as expected as the phase advances. At $\omega_p t = 600$, the energy gain $\Delta\gamma$ has reached 56.5, while the energy spread has been reduced to the initial root-mean-square value 0.5 ($\delta\gamma/\Delta\gamma = 0.76\%$). This is considerably smaller than what would be expected ($\delta\gamma/\Delta\gamma \approx \eta = 10\%$) if the usual injection phase choice ($\phi = 0$) were made and negligible phase slippage were allowed. The nearly perfect cancellation of the rms energy spread seen in the simulation appears to validate the simple two-particle model used in the analytical calculation.

In the following PIC simulation, we consider a heavy beam loading case. Parameters are similar to the previous example except that $\gamma_i \approx 50$ for both beams and we increase the peak density of the trailing beam to n_0. This corresponds to roughly 30% beam loading ($\eta = 0.3$). At the end of the simulation ($\omega_p t = 480$), the energy at the tail of the driving beam has decreased from $\gamma_i \approx 50$ to $\gamma_f \approx 5$. At this time, the energy gain of the trailing beam is $\Delta\gamma = 43.0$ and the energy spread is $\delta\gamma = 1.39$ or $\delta\gamma/\Delta\gamma = 3.2\%$. The normalized emittance has increased to 0.012 rad$/k_p$ which corresponds to $\epsilon_n \approx 2.5 \times 10^{-1}$ mm mrad for a channel of radius 20.5 μm and plasma density $n_0 = 6.7 \times 10^{16}$ cm^{-3}. Figure 4 shows the energy history of both the driving and trailing beams, normalized to the initial energy of the driving beam. This figure shows that more than 50% of the driving beam energy can be transferred to the plasma. Of this energy 50% is picked up by the trailing beam. Thus 25% overall efficiency from the driving beam to the trailing beam is achieved in this simulation case.

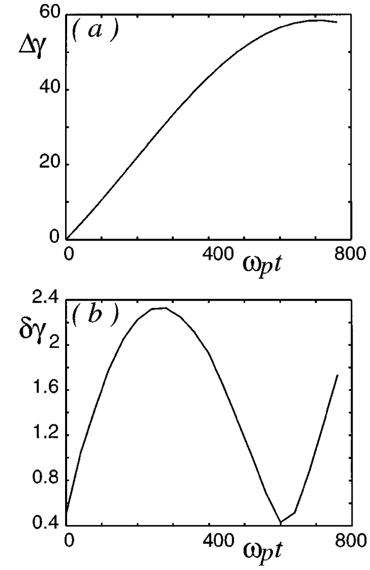

FIG. 3. PIC simulations of (a) energy gain and (b) energy spread vs time.

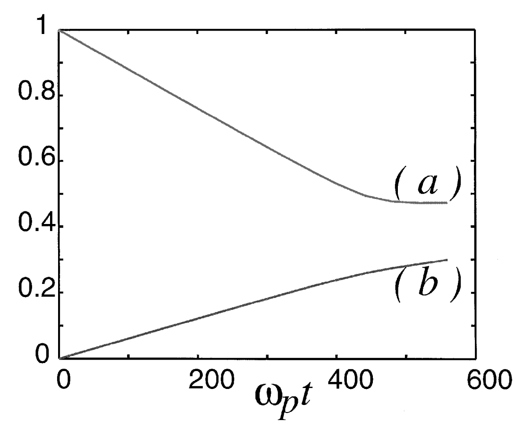

FIG. 4. Time history of beam energy. (*a*) The energy of the driving beam normalized to the initial total energy. (*b*) The energy gain of the trailing beam normalized to the initial driving beam energy.

In this Letter self-consistent PIC simulations were used to demonstrate that a heavy beam load ($\approx 10^8$ particles) can be accelerated with high efficiency (25% from the driving beam to the accelerated particles) while still preserving beam emittance ($\epsilon_n \approx 2.5 \times 10^{-1}$ mm mrad) and energy spread ($\delta\gamma/\Delta\gamma = 3.2\%$). With reasonable attention to the wall plug efficiency of the driving source (i.e., laser or particle beams), it appears that ultrahigh gradient plasma accelerators with high overall efficiency are possible.

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[1] T. Katsouleas, in *Advanced Accelerator Concepts*, edited by Swapan Chattopadhyay and Julle McCullough (AIP, New York, 1996), p. 175; A. Ting, C. I. Moore, K. Krushelnick, C. Manka, E. Esarey, P. Sprangle, R. Hubbard, H. R. Burris, R. Fischer, and M. Baine, Phys. Plasmas **4**, 1889 (1997); E. Esarey, P. Sprangle, J. Krall, and A. Ting, IEEE Trans. Plasma Sci. **PS-24**, 252 (1996).

[2] T. Tajima and J. M. Dawson, Phys. Rev. Lett. **43**, 267 (1979).

[3] J. R. Marques, J. P. Geindre, F. Amiranoff, P. Audebert, J. C. Gauthier, A. Antonetti, and G. Grillon, Phys. Rev. Lett. **76**, 3566 (1996); C. W. Siders, S. P. Le Blanc, D. Fisher, T. Tajima, and M. C. Downer, *ibid.* **76**, 3570 (1996); D. Umstadter *et al.*, Phys. Rev. Lett. **76**, 2073 (1996).

[4] K. C. Tzeng, W. B. More, and T. Katsouleas, Phys. Rev. Lett. **79**, 5258 (1997); E. Esarey *et al.*, Phys. Rev. Lett. **79**, 2682 (1997).

[5] T. C. Chiou *et al.*, Phys. Plasmas **2**, 310 (1995).

[6] H. Milchberg *et al.*, Phys. Plasmas **3**, 2149 (1996); W. Leemans (private communication); M. Downer (private communication); K. Krushelnick *et al.*, Phys. Rev. Lett. **78**, 4047 (1997).

[7] Y. Ehrlich *et al.*, Phys. Rev. Lett. **77**, 4186 (1996).

[8] T. C. Chiou, Ph.D. thesis, University of Southern California, 1998. The calculation for beams follows analogously for Ref. [5] for lasers.

[9] S. Van der Meer, CLIC Note No. 3 CERN/PS/85-86 (AA), 1985.

[10] R. Ruth, A. Chao, P. Morton, and P. Wilson, Part. Accel. **17**, 171 (1985).

[11] T. Katsouleas *et al.*, Part. Accel. **22**, 81 (1987); S. Wilks *et al.*, IEEE Trans. Plasma Sci. **PS-15**, 210 (1987).

[12] R. D. Ruth *et al.*, in *Laser Acceleration of Particles*, edited by P. J. Channell (AIP, New York, 1982).