Increased Destabilization of the Parallel Velocity Shear Instability due to Reversed Magnetic Shear

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The nonlinear behavior of the parallel velocity shear instability in a shear magnetic field is studied. It is found that the nonlinear fluctuation levels and turbulent momentum transport depend strongly upon the direction of the magnetic shear. When the shear has the same sign as the second derivative of the parallel velocity with respect to the radial coordinate, the fluctuations grow to larger levels than if there were no magnetic shear. The physical mechanism controlling this effect is vortex merging between modes on either side of the velocity peak. It is likely that this behavior may be a general feature of all modes with structure parallel to the magnetic field. [S0031-9007(98)07377-3]

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One of the more remarkable experimental results over the past few years in fusion research was the discovery of improved confinement in Tokamak Fusion Test Reactor as well as DIII-D with the reversal of the magnetic shear [1,2]. This result has led to the idea that magnetic shear does not necessarily always provide a stabilizing influence, but can actually enhance growth of an instability. Previously, the conventional wisdom had been that magnetic shear, regardless of direction, always reduced the growth of an instability, a belief that was reinforced by the results of linear theory. This apparent contradiction between theory and experiment only further emphasizes what has been known for many years: linear theory is simply insufficient to describe most plasma behavior, and it is particularly inadequate when it comes to describing tokamaks.

An instability that has received renewed attention over the past five years is the electrostatic parallel velocity shear ($v_{\|}'$) instability. A primary reason for this interest is the evidence of large, nearly sonic parallel flows that have been measured in the tokamak edge [3], and recently, a study has shown that this instability is the likely source of edge turbulence in the CT-6B tokamak [4]. Neutral beam injection can also drive strong flows parallel to the magnetic field, thus also providing the driving mechanism for this instability. Such strong parallel flows, however, are not limited only to tokamak plasmas, but are ubiquitous in nature, occurring in the flanks of the earth's magnetosphere and along the auroral magnetic field lines [5].

The $v_{\|}'$ instability was first identified over 30 years ago by D'Angelo [6] and was studied shortly thereafter by Catto in a sheared magnetic slab [7]. The latter work, in which a kinetic nonlocal analysis was performed, gave the important result that the growth rate had the form $\gamma \propto \gamma_0 |L_s|/(2n + 1)$, where γ_0 is the growth rate from local theory, $|L_s|$ is the scale length of the magnetic shear, and n is the radial mode number. Therefore, linearly the mode should be weakened by strong magnetic shear (independent of direction) and should grow to longest radial wavelength. Although these studies did provide great insight into the behavior of a plasma with a strongly varying parallel velocity gradient, they were all based upon linear theory. Generally, linear theory, which assumes small fluctuation levels, is quite effective for understanding plasma behavior; however, this is simply not appropriate for the $v_{\|}'$ instability which is known both experimentally [8] and theoretically [9,10] to drive very large ($\sim$50%) fluctuation levels.

An interesting characteristic of the $v_{\|}'$ instability is that the mode is inherently three dimensional (3D) in nature; i.e., both parallel and perpendicular wave vectors are required for instability. Thus, the mode is "tilted" with respect to the magnetic field. The orientation of the tilting can be obtained from the local linear dispersion relation [9]

$$\gamma^2 = -k_z k_y \frac{cT}{eB} \frac{\partial v_z}{\partial x} - k_z^2 c_s^2 \,, \qquad (1)$$

where k_y and k_z are the perpendicular and parallel wave vectors, respectively, and $\partial v_z / \partial x$ is the radial derivative of the parallel velocity (standard notation is used for all other quantities). One can see from Eq. (1) that the unstable modes must obey the condition $k_z k_y (\partial v_z / \partial x) < 0$; i.e., for a positive (negative) parallel velocity gradient, the mode will be tilted downward (upward) with respect to the magnetic field. Thus, for a peaked parallel velocity profile, the tilting will have opposite orientations on either side of the peak. In other words, for a uniform magnetic field, the mode structure is effectively "sheared". Since a condition for instability is that $k_z < k_y v_z' (mc/eB)$, for a strongly magnetized plasma the mode is nearly aligned along $\mathbf{B}$; therefore, one would expect that this shearing could be removed with just a small rotation of $\mathbf{B}$.

In this Letter, we examine the physical impact of magnetic shear on the turbulent structure of this mode. We find that the orientation of the magnetic shear has a strong effect on the saturated mode structure, fluctuation levels, and momentum transport. When the mode saturates nonlinearly, the vortices on either side of the velocity peak merge with vortices on the other side with the same vorticity, thus producing increased fluctuations. When the magnetic shear further enhances the sheared structure of the mode the instability is reduced; however, when the magnetic shear is directed so as to align the modes on either side of the velocity profile peak, vortex merging between the two sides occurs over longer periods of time, thus increasing fluctuation levels and transport. Based upon numerical results and physical arguments, we provide quantitative estimates of the impact of magnetic shear.

The dimensionless nonlinear fluid equations used to describe this instability in a sheared slab are the ion continuity and parallel momentum equations:

$$\frac{\partial \Omega}{\partial t} + \frac{\partial \Omega v_x}{\partial x} + \frac{\partial \Omega v_y}{\partial y} + \frac{1}{\alpha}\frac{\partial n v_z}{\partial z} + \hat{s}x\frac{\partial n v_z}{\partial y} = 0, \tag{2}$$

$$\frac{\partial n v_z}{\partial t} + \frac{\partial n v_z v_x}{\partial x} + \frac{\partial n v_z v_y}{\partial y} + \frac{1}{\alpha}\frac{\partial n v_z^2}{\partial z} + \hat{s}x\frac{\partial n v_z^2}{\partial y} = -\frac{1}{\alpha}\frac{\partial n}{\partial z} - \hat{s}x\frac{\partial n}{\partial y}, \tag{3}$$

where $v_x = -\partial\phi/\partial y$, $v_y = \partial\phi/\partial x$, and we have defined the new variable $\Omega = n - \hat{\rho}^2\nabla_\perp^2 n$ (a detailed derivation of these equations for a shearless slab is given in Ref. [10]). In these equations, all convective terms are retained up to order $\rho_s^3 k_\perp^3$, T_i is assumed to be much smaller than T_e, and all averaged $\mathbf{E}\times\mathbf{B}$ flows are neglected. The x and y scales are normalized to $L_\perp$, z is normalized to $2\pi qR$, v_z to c_s, and $v_\perp$ to $\rho_s c_s/L_\perp$. The initial velocity profile is taken to be $v_z = \sin\pi x$, where x varies from 0 to 1, and is maintained by a source term that forces an average value of $2/\pi$. In this paper, we do not speculate as to what is the source of these flows, although there is evidence that such flows can be driven by asymmetric anomalous particle transport and are localized at the last closed flux surface [9]. There are three dimensionless parameters, $\alpha \equiv 2\pi qR\rho_s/L_V^2$, $\hat{\rho} \equiv \rho_s/L_V$, and $\hat{s}$. Physically, α is the ratio of the sound transit time to the growth time, and $\hat{\rho}$ is a simple ratio of the Larmor radius to the radial scale length. The strength of the magnetic shear is determined by $\hat{s}$ which is typically of order unity for the tokamak edge.

Before showing the results with finite magnetic shear, it is worthwhile to discuss the results with zero magnetic shear [10]. In the region $\alpha > 1$ (which is appropriate for a tokamak, which has a typical value of $\alpha \sim 10$, more examples of which can be found in Ref. [9]) and finite Larmor radius, the model evolved to a state that was isotropic in the $x - y$ plane with $m = 1$, where m is the mode number in the y direction. In the z direction, the mode evolves so that $n = \alpha/2$ where n is the mode number in the z direction. The fluctuation levels from all of these simulations were typically about 20–30% on the average, although in certain regions, the fluctuations extended to $\sim 50\%$ locally.

An essential point that must be understood about the behavior of these vortices is that on either side of the velocity peak, there is an effective drift (which is essentially the phase velocity if the mode). This drift results from the tilting of the mode in the y-z plane and its flow in the $+z$ direction. For example, if a mode is oriented so that $k_y k_z < 0$, it will appear to be directed down as one looks into the x-y plane; however, because of the flow in the z direction, the mode will appear to rise. Thus, for a slab with no magnetic shear, there will be an upward drift of vortices when $\partial v_z/\partial x > 0$ and a downward drift when $\partial v_z/\partial x < 0$. An estimate of this drift velocity can be obtained by dividing the frequency between modes $(k_z v_z)$ by the wave vector in the y direction $v_d \sim k_z v_z/k_z$. The reason that this drift is significant with regards to transport and fluctuation levels is that vortices of the same vorticity will tend to merge across the velocity peak. When such a merging occurs, the transport can rise (the radial scale length goes from L_V to $2L_V$) along with the fluctuation levels.

Figure 1 shows a drawing describing qualitatively the evolution of the vortices at a cut in the x-y plane for zero magnetic shear. Initially, there are two vortices on either side of the velocity peak each with opposing vorticity [10]. As the plasma flows, the vortices tend to drift upward on the right side and downward on the left side. When vortices of the same sign are aligned, they merge across the peak [Fig. 1(b)], forming a larger vortex with twice the radial scale length. This enlarged vortex [Fig. 1(c)], however, eventually breaks apart again

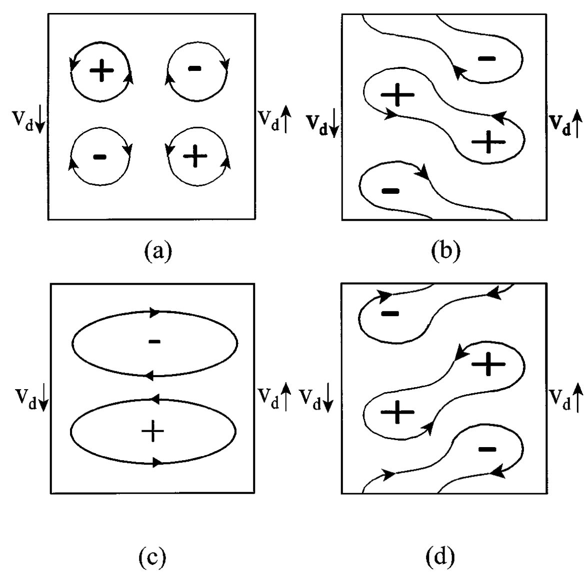

FIG. 1. A diagram of the vortex merging. (a) shows a typical saturated nonlinear state of the $v_\parallel'$ mode in the x-y plane, where the vortices on the left drift down, while the vortices on the right drift up. While drifting, a vortex will merge with another vortex with the same rotation as shown in (b), which will eventually form a large scale vortex [(c)]. The drift, however, continues, causing the large vortex to be torn apart [(d)] and return to the state depicted in (a).

into two smaller vortices as the upward/downward drift continues on the right/left side [Fig. 1(d)]. Thus, the transport for this mode should be characterized by a periodic sequence of higher and lower levels, depending upon whether or not the merging has occurred. When the magnetic shear is arranged so that it reduces the sheared structure of the mode, the vortices can remain merged for longer periods of time, and in the case of $\hat{s} = -\alpha/2$, the vortices will always be merged and will remain in a state similar to that shown in Fig. 1(c). This is the state of maximum transport and fluctuation levels, a fact supported by our simulations.

Equations (2) and (3) were solved numerically using a fully nonlinear 3D finite differencing code that evolved the equations using a second-order accurate explicit time stepping scheme [10]. As the purpose of this Letter is to understand the impact of magnetic shear on this mode, Fig. 2(a) shows the $\phi = 0.1$ isosurface of the electrostatic potential in the $\hat{s} = 0$ limit for $\alpha = 4$. One can see from this figure the tilting of the mode, the $n = 2$ structure, and more importantly, the apparent sheared structure of the vortices that occurs due to the peaked velocity profile. Figures 2(b) and 2(c) show the same isosurface for values of $\hat{s} = 1$ and $\hat{s} = -1$, respectively. For the case of positive shear [Fig. 2(a)], the field is oriented so as to increase the shearing between the opposite vortices, the end result being a reduction of the amount of vortex merging, therefore a reduction of the transport and the fluctuation levels. In the case of negative shear [Fig. 2(c)], the orientation of the magnetic field is such that it lessens the sheared structure of the mode, thus allowing the vortices on either side of the velocity peak to align with one another. The effect of this is that vortex merging will occur over the entire box, thus increasing the transport and fluctuation levels.

In order to show this effect on a quantitative level, Fig. 3 shows contours of the potential in the y-z plane for $\hat{s} = 0$, $\hat{s} = +1$, and $\hat{s} = -1$. Shown in the upper corners of each plot are the minimum and maximum values of the potential. From these plots, one can clearly see the changing mode structure with varying sign of magnetic shear, as well as the reduced fluctuation levels for positive shear and enhanced levels for negative shear.

The role of shear can be reasoned quantitatively by recognizing that the shear effectively alters the parallel wave vector as $k_{\parallel}' = k_{\parallel} + \hat{s}/L_s$, or in our dimensionless units, it affects the mode number as $n' = \alpha/2 + \hat{s}$. Thus when the shear is positive, the mode number increases, while when it is negative, the mode number decreases until $\hat{s} = -\alpha/2$. This is the situation that allows for the largest transport and fluctuation levels. An effective way to interpret this effect is to recognize that the velocity of the relative drift will now be given by $v_d = k_{\parallel}' v_z/k_y$; thus when $k_{\parallel}' \to 0$, the vortices will no longer drift, and can remain merged for long times. Shown in Table I are values of the mode number in the z direction, averaged fluctuation levels (defined as $|\tilde{\phi}| = \sqrt{\langle\tilde{\phi}^2\rangle_{yz}}$), and radial flux of the parallel momentum (defined as $n v_z v_x$) with varying magnetic shear for $\alpha = 4$. These results show an increase in both the fluctuations levels and the flux as the shear becomes more negative. Such a behavior was consistent for other values of α. An odd feature of this table is the

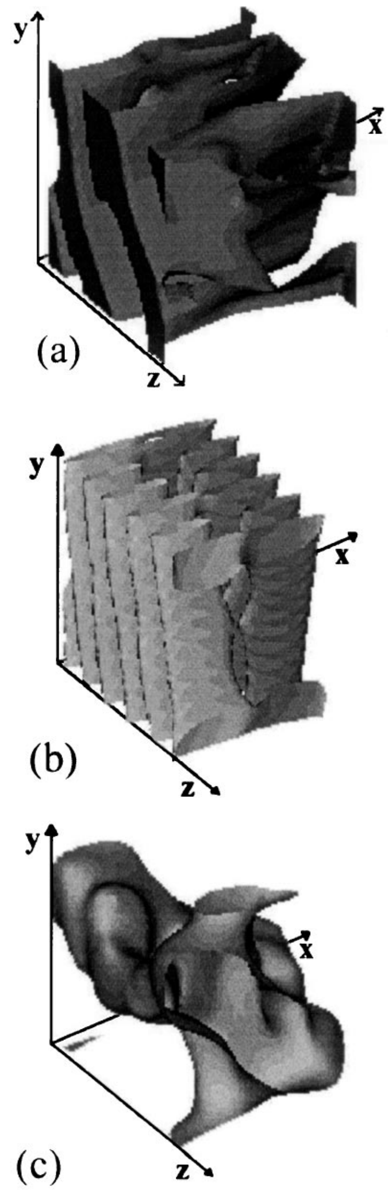

FIG. 2. Isosurfaces of ϕ for varying values of magnetic shear. (a) shows the $\phi = 0.1$ isosurface for the shearless slab. The $n_z = 2$ structure is apparent from this diagram, as is the tilting of the mode, which changes its orientation along x. The merging between the vortices can also be seen at certain points on this plot. (b) shows the same isosurface for the $\hat{s} = +1$ case. The increased tilting and structure in the z direction are apparent, as is the reduction in vortex merging. (c) shows the same isosurface for the $\hat{s} = -1$ case. There is no well defined structure here, and more pronounced vortex merging has occurred. The large scale structures in this plot, characteristic of vortex merging, occur throughout the entire box.

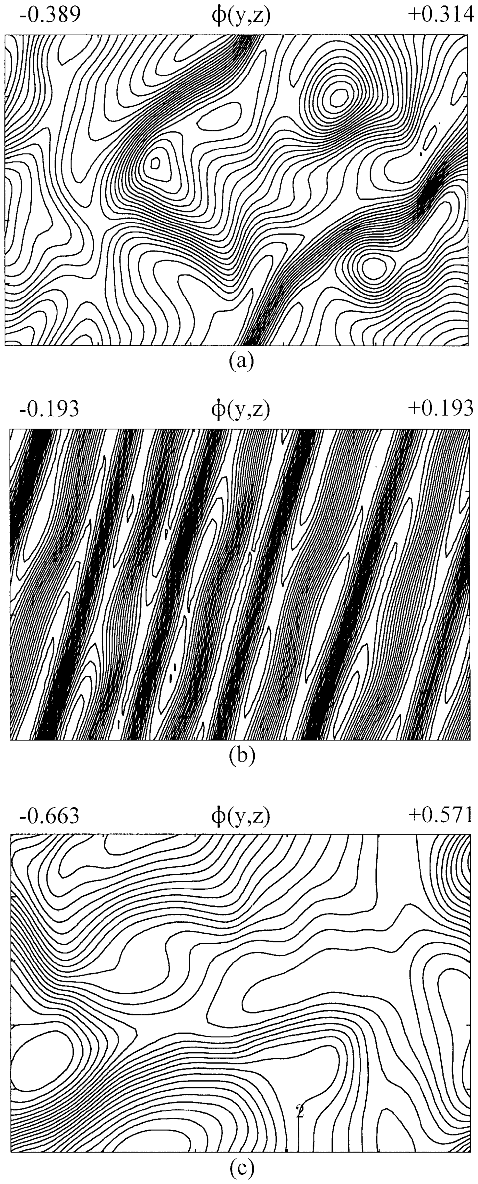

FIG. 3. Contour plots of ϕ in the y-z plane for varying values of magnetic shear. Shown in the upper left/right corners of each plot are the minimum/maximum values of ϕ. (a) shows the contours in the $\hat{s} = 0$ case, for which the fluctuation levels peak at about 35%. (b) shows contours for the $\hat{s} = +1$ case. The increased tilting can be clearly seen, as is the reduction in fluctuation levels. (c) shows contours of ϕ for the $\hat{s} = -1$ case. The structure in the z direction extends over much longer scales and the fluctuation levels now grow to 60%.

dramatic increase in $\tilde{\phi}$, but only the modest increase in flux. This apparent disparity can be explained when one realizes that the flux depends upon $v_x \sim k_y \tilde{\phi}$, and as can be seen from Fig. 3, the structure in the y direction is much broader in the case of negative shear than in the case of positive shear; therefore, k_y is smaller for $\hat{s} < 0$. An interesting feature is that when the shear becomes too negative, the fluctuation levels drop again, which is to be expected. If the vortices are tilted too much by the magnetic field,

TABLE I. Results for $\alpha = 4$.

$\hat{s}$	n_z	$\lvert\tilde{\phi}\rvert$	Flux
+2	5	0.10−0.11	0.072
+1	5	0.14−0.15	0.082
0	2	0.17−0.22	0.102
−1	1	0.24−0.31	0.106
−2	0.1	0.32−0.40	0.095
−3	1	0.33−0.38	0.085
−4	2	0.31−0.34	0.080

they become sheared again, and the relative drift resumes, just this time in the opposite direction.

In summary, we have identified a mechanism that explains why the saturated fluctuation levels of the $v'_{\parallel}$ instability will depend upon the orientation of the magnetic shear. A few final points must be emphasized. As this work is performed in a slab, not a torus, $\hat{s}$ is not defined as $\partial \ln q / \partial \ln r$; therefore, when we refer to "positive" and "negative" shear, it is with respect to our slab coordinate system with the parallel velocity flow in the $+z$ direction. The more accurate statement regarding the effect of the sign of magnetic shear is that when $\hat{s}$ has the same sign as $\partial^2 v_z / \partial x^2$, there will be an increase in the turbulent fluctuation levels. Another intriguing aspect of this work is that this mechanism does not depend upon the details of the $v'_{\parallel}$ instability, but just upon the fact that the saturated nonlinear state has a finite $k_{\parallel}$ that varies perpendicular to the field. Thus, for other modes that require a parallel wave vector, such as the η_i mode, this mechanism could indeed play a role in affecting the turbulence associated with these modes, which in turn could provide some insight into the previously mentioned experimental results [1,2].

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