

# Search for Proton Decay via $p \rightarrow e^+ \pi^0$ in a Large Water Cherenkov Detector

M. Shiozawa,<sup>1</sup> B. Viren,<sup>15</sup> Y. Fukuda,<sup>1</sup> T. Hayakawa,<sup>1</sup> E. Ichihara,<sup>1</sup> K. Inoue,<sup>1</sup> K. Ishihara,<sup>1</sup> H. Ishino,<sup>1</sup> Y. Itow,<sup>1</sup> T. Kajita,<sup>1</sup> J. Kameda,<sup>1</sup> S. Kasuga,<sup>1</sup> K. Kobayashi,<sup>1</sup> Y. Kobayashi,<sup>1</sup> Y. Koshio,<sup>1</sup> M. Miura,<sup>1</sup> M. Nakahata,<sup>1</sup> S. Nakayama,<sup>1</sup> A. Okada,<sup>1</sup> M. Oketa,<sup>1</sup> K. Okumura,<sup>1</sup> M. Ota,<sup>1</sup> N. Sakurai,<sup>1</sup> Y. Suzuki,<sup>1</sup> Y. Takeuchi,<sup>1</sup> Y. Totsuka,<sup>1</sup> S. Yamada,<sup>1</sup> M. Earl,<sup>2</sup> A. Habig,<sup>2</sup> E. Kearns,<sup>2</sup> M. D. Messier,<sup>2</sup> K. Scholberg,<sup>2</sup> J. L. Stone,<sup>2</sup> L. R. Sulak,<sup>2</sup> C. W. Walter,<sup>2</sup> M. Goldhaber,<sup>3</sup> T. Barszczak,<sup>4</sup> W. Gajewski,<sup>4</sup> P. G. Halverson,<sup>4,\*</sup> J. Hsu,<sup>4</sup> W. R. Kropp,<sup>4</sup> L. R. Price,<sup>4</sup> F. Reines,<sup>4</sup> H. W. Sobel,<sup>4</sup> M. R. Vagins,<sup>4</sup> K. S. Ganezer,<sup>5</sup> W. E. Keig,<sup>5</sup> R. W. Ellsworth,<sup>6</sup> S. Tasaka,<sup>7</sup> J. W. Flanagan,<sup>8,†</sup> A. Kibayashi,<sup>8</sup> J. G. Learned,<sup>8</sup> S. Matsuno,<sup>8</sup> V. Stenger,<sup>8</sup> D. Takemori,<sup>8</sup> T. Ishii,<sup>9</sup> J. Kanzaki,<sup>9</sup> T. Kobayashi,<sup>9</sup> K. Nakamura,<sup>9</sup> K. Nishikawa,<sup>9</sup> Y. Oyama,<sup>9</sup> A. Sakai,<sup>9</sup> M. Sakuda,<sup>9</sup> O. Sasaki,<sup>9</sup> S. Echigo,<sup>10</sup> M. Kohama,<sup>10</sup> A. T. Suzuki,<sup>10</sup> T. J. Haines,<sup>11,4</sup> E. Blaufuss,<sup>12</sup> R. Sanford,<sup>12</sup> R. Svoboda,<sup>12</sup> M. L. Chen,<sup>13</sup> Z. Conner,<sup>13,‡</sup> J. A. Goodman,<sup>13</sup> G. W. Sullivan,<sup>13</sup> M. Mori,<sup>14,§</sup> J. Hill,<sup>15</sup> C. K. Jung,<sup>15</sup> K. Martens,<sup>15</sup> C. Mauger,<sup>15</sup> C. McGrew,<sup>15</sup> E. Sharkey,<sup>15</sup> C. Yanagisawa,<sup>15</sup> W. Doki,<sup>16</sup> T. Ishizuka,<sup>16,\*\*</sup> Y. Kitaguchi,<sup>16</sup> H. Koga,<sup>16</sup> K. Miyano,<sup>16</sup> H. Okazawa,<sup>16</sup> C. Saji,<sup>16</sup> M. Takahata,<sup>16</sup> A. Kusano,<sup>17</sup> Y. Nagashima,<sup>17</sup> M. Takita,<sup>17</sup> T. Yamaguchi,<sup>17</sup> M. Yoshida,<sup>17</sup> S. B. Kim,<sup>18</sup> M. Etoh,<sup>19</sup> K. Fujita,<sup>19</sup> A. Hasegawa,<sup>19</sup> T. Hasegawa,<sup>19</sup> S. Hatakeyama,<sup>19</sup> T. Iwamoto,<sup>19</sup> T. Kinebuchi,<sup>19</sup> M. Koga,<sup>19</sup> T. Maruyama,<sup>19</sup> H. Ogawa,<sup>19</sup> A. Suzuki,<sup>19</sup> F. Tsushima,<sup>19</sup> M. Koshiba,<sup>20</sup> M. Nemoto,<sup>21</sup> K. Nishijima,<sup>21</sup> T. Futagami,<sup>22</sup> Y. Hayato,<sup>22,††</sup> Y. Kanaya,<sup>22</sup> K. Kaneyuki,<sup>22</sup> Y. Watanabe,<sup>22</sup> D. Kielczewska,<sup>23,4</sup> R. Doyle,<sup>24</sup> J. George,<sup>24</sup> A. Stachyra,<sup>24</sup> L. Wai,<sup>24</sup> J. Wilkes,<sup>24</sup> and K. Young<sup>24</sup>

(Super-Kamiokande Collaboration)

<sup>1</sup>*Institute for Cosmic Ray Research, University of Tokyo, Tanashi, Tokyo 188-8502, Japan*

<sup>2</sup>*Department of Physics, Boston University, Boston, Massachusetts 02215*

<sup>3</sup>*Physics Department, Brookhaven National Laboratory, Upton, New York 11973*

<sup>4</sup>*Department of Physics and Astronomy, University of California—Irvine, Irvine, California 92697-4575*

<sup>5</sup>*Department of Physics, California State University, Dominguez Hills, Carson, California 90747*

<sup>6</sup>*Department of Physics, George Mason University, Fairfax, Virginia 22030*

<sup>7</sup>*Department of Physics, Gifu University, Gifu, Gifu 501-1193, Japan*

<sup>8</sup>*Department of Physics and Astronomy, University of Hawaii, Honolulu, Hawaii 96822*

<sup>9</sup>*Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan*

<sup>10</sup>*Department of Physics, Kobe University, Kobe, Hyogo 657-8501, Japan*

<sup>11</sup>*Physics Division, P-23, Los Alamos National Laboratory, Los Alamos, New Mexico 87544*

<sup>12</sup>*Physics Department, Louisiana State University, Baton Rouge, Louisiana 70803*

<sup>13</sup>*Department of Physics, University of Maryland, College Park, Maryland 20742*

<sup>14</sup>*Department of Physics, Miyagi University of Education, Sendai, Miyagi 980-0845, Japan*

<sup>15</sup>*Department of Physics and Astronomy, State University of New York, Stony Brook, New York 11794-3800*

<sup>16</sup>*Department of Physics, Niigata University, Niigata, Niigata 950-2181, Japan*

<sup>17</sup>*Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*

<sup>18</sup>*Department of Physics, Seoul National University, Seoul 151-742, Korea*

<sup>19</sup>*Department of Physics, Tohoku University, Sendai, Miyagi 980-8578, Japan*

<sup>20</sup>*The University of Tokyo, Tokyo 113-0033, Japan*

<sup>21</sup>*Department of Physics, Tokai University, Hiratsuka, Kanagawa 259-1292, Japan*

<sup>22</sup>*Department of Physics, Tokyo Institute for Technology, Meguro, Tokyo 152-8551, Japan*

<sup>23</sup>*Institute of Experimental Physics, Warsaw University, 00-681 Warsaw, Poland*

<sup>24</sup>*Department of Physics, University of Washington, Seattle, Washington 98195-1560*

(Received 19 May 1998)

We have searched for proton decay via  $p \rightarrow e^+ \pi^0$  using data from a 25.5 kton · yr exposure of the Super-Kamiokande detector. We find no candidate events with an expected background induced by atmospheric neutrinos of 0.1 events. From these data, we set a lower limit on the partial lifetime of the proton  $\tau/B_{p \rightarrow e^+ \pi^0}$  to be  $1.6 \times 10^{33}$  years at a 90% confidence level. [S0031-9007(98)07309-8]

PACS numbers: 13.30.Ce, 11.30.Fs, 14.20.Dh, 29.40.Ka

In the standard model, which is the modern paradigm of elementary particle physics, protons are assumed to be stable [1]. In grand unified theories (GUTs), however, the decay of the proton is one of the most dramatic predictions

of various models [2–5]. In the past two decades, several large mass underground detector experiments have looked for proton decay but no clear evidence has been reported [6–10]. In general, GUTs predict many modes of proton

decay. In many models, the  $p \rightarrow e^+ \pi^0$  mode is dominant and there are several GUTs which predict a decay rate within the observable range of Super-Kamiokande (see, for example, [11–13]). This decay mode has a characteristic event signature, in which the electromagnetic shower caused by the positron is balanced against the two showers caused by the gamma rays from the decay of the  $\pi^0$ . This signature enables us to discriminate the signal events clearly from atmospheric neutrino background. In this Letter, we report the results of our search for proton decay via  $p \rightarrow e^+ \pi^0$  in 414 live days of data corresponding to an exposure of  $25.5 \text{ kton} \cdot \text{yr}$  ( $8.52 \times 10^{33} \text{ proton} \cdot \text{yr}$ ) in Super-Kamiokande.

Super-Kamiokande is a large water Cherenkov detector located in a mine 2700 meters of water equivalent below the peak of Mt. Ikenoyama in Kamioka, Japan. The detector holds 50 ktons of ultrapure water contained in a cylindrical stainless steel tank and separated into two regions: a primary inner volume viewed by 11 146 50 cm photomultiplier tubes (PMTs) and a veto region, surrounding the inner detector, viewed by 1885 20 cm PMTs. More details can be found in Ref. [14].

The data sample we use for this analysis consists of events which are fully contained within the inner detector and is identical to that used for the atmospheric neutrino analysis [14]. The essential criteria of this selection are as follows: (1) No significant outer detector activity, (2) the total number of photoelectrons (p.e.'s) in the inner detector is greater than 200, (3) the ratio of the maximum number of p.e.'s in a single PMT to the total number of p.e.'s in the event is less than 0.5, and (4) the time interval from the preceding event is greater than  $100 \mu\text{sec}$ .

Essentially 100% of the cosmic ray muons are eliminated by criterion (1). Criterion (2) corresponds to a lower momentum cut of  $22 \text{ MeV}/c$  for electrons and  $190 \text{ MeV}/c$  for muons. Criterion (3) removes spurious electrical noise events. Criterion (4) removes electrons from the decay of stopping cosmic ray muons as well as the noise events caused by unwanted PMT signals following highly energetic events ("after pulsing"). By applying these criteria, the number of events is reduced from about  $400 \times 10^6$  to about 12 000. In addition, all events which follow a previous event within  $30 \mu\text{sec}$  are tagged as an electron from the decay of a muon.

After criteria (1)–(4) are applied, further reduction is done by scanners using an interactive graphic event display to eliminate most of the few remaining cosmic ray muons or electronic noise events. About 6000 events are classified as fully contained events. In this analysis, only events with a fitted vertex inside of the fiducial volume are used. This fiducial volume is defined as all points which are more than 2 m from the inner detector wall, giving a  $22.5 \text{ kton}$  fiducial mass. This volume cut removes any remaining entering cosmic ray muon events and assures us that the performance of the reconstruction algorithms is uniform throughout the fiducial volume. We observe 3468 fully contained events in the fiducial volume. The inefficiency

to recover  $p \rightarrow e^+ \pi^0$  candidates due to the criteria (1)–(4) and scanning is estimated to be less than 0.1%.

All measurement of physical quantities of an event, such as vertex position, the number of Cherenkov rings, momentum, particle type, and the number of decay electrons, is automatically performed [14] by reconstruction algorithms. The vertex position is estimated by finding the position at which the timing residual [(photon arrival time)–(time of flight)] distribution is most peaked. Using a Monte Carlo (MC) simulated data sample, the vertex resolution for  $p \rightarrow e^+ \pi^0$  events is estimated to be 18 cm. To find the rings in an event, the charge, viewed from the vertex as a function of  $\{\theta, \phi\}$ , is Hough transformed [15]. The resulting space is searched for peaks, giving the ring centers. With  $p \rightarrow e^+ \pi^0$  MC, 44% of the simulated events passing proton decay selection criteria (described below) are classified as 3-ring events and 56% are classified as 2-ring events. The 2-ring classification is primarily for events with one of the two  $\gamma$  rings taking only a small fraction of the  $\pi^0$ 's energy or overlapping too much with other rings.

The particle identification (PID) classifies a particle as a showering particle ( $e^\pm, \gamma$ ) or a nonshowering particle ( $\mu^\pm, \pi^\pm$ ), using the photon distribution of its Cherenkov ring. For single ring events, the particle misidentification probability is estimated to be less than 1.0% using the atmospheric neutrino MC. This is confirmed with stopping cosmic ray muons and their associated decay electrons. The PID performance was also checked using a 1 kton water Cherenkov detector with  $e$  and  $\mu$  beams from the 12 GeV proton synchrotron at KEK [16]. However, the misidentification probability differs between single ring and multiring events due to overlapping rings. Using a  $p \rightarrow e^+ \pi^0$  MC sample this misidentification is estimated to be 2%.

An appropriate momentum cut will reject atmospheric neutrino background but accept proton decay events. The momentum is estimated from the total sum of p.e.'s detected within a  $70^\circ$  half opening angle from the reconstructed ring direction. The number of p.e.'s collected in each PMT is corrected for light attenuation in water, PMT angular acceptance, and PMT coverage. In the momentum reconstruction, we assume that showering particles are electrons and nonshowering particles are muons. For single ring events, the reconstructed momentum resolution is estimated to be  $\pm[2.5/\sqrt{E(\text{GeV})} + 0.5]\%$  for electrons and  $\pm 3\%$  for muons. For multiring events, the fraction of p.e.'s in each PMT due to each ring is determined using the expected p.e. distribution. Then, the momentum for each ring is determined by the same method used for single ring events. The reconstructed momentum resolution is  $\pm 10\%$  for each ring in the  $p \rightarrow e^+ \pi^0$  events.

The energy scale stability is checked by the reconstructed mean energy of decay electrons from stopping cosmic ray muons. It varies within  $\pm 0.5\%$  over the exposure period. The absolute energy scale was checked with many calibration sources such as electrons from a linear accelerator [17], decay electrons from stopping cosmic ray

muons, stopping cosmic ray muons themselves, and the reconstructed mass of  $\pi^0$  events observed in atmospheric neutrino interactions. From comparisons of these sources and MC simulation, the absolute calibration error is estimated to be smaller than  $\pm 2.5\%$ .

Finally, the efficiency for detection of decay electrons is estimated to be 80% for  $\mu^+$  and 63% for  $\mu^-$  by a Monte Carlo study. The difference in these efficiencies is due to  $\mu^-$  capture on  $^{16}\text{O}$ . This efficiency was confirmed to an accuracy of 1.5% using stopping cosmic ray muons.

The main sources of background for this analysis are atmospheric neutrino interactions which could mimic a  $p \rightarrow e^+ \pi^0$  event. To estimate the number of background events, we have developed a detailed MC simulation of atmospheric neutrino interactions, meson propagation in the  $^{16}\text{O}$  nucleus, and propagation of secondary particles, as well as Cherenkov photons, in the detector water [14]. We use the atmospheric neutrino flux of Honda *et al.* [18]. For neutrino interactions in the detector, the following types of interaction are simulated: quasielastic scattering, single- $\pi$  production, multi- $\pi$  production, and coherent single- $\pi$  production for both charged current (CC) and neutral current (NC). For the  $p \rightarrow e^+ \pi^0$  mode, CC  $\pi$  production is the most important background because it could produce an  $e^\pm$  accompanied by a  $\pi^0$ . We use Rein-Sehgal's model to simulate single- $\pi$  production [19]. The pion cross sections in  $^{16}\text{O}$  are calculated with the model by Oset *et al.* [20]. Propagation of produced particles and Cherenkov light in water is simulated with a GEANT [21] based custom detector simulator. Propagation of charged pions in the detector water is simulated by a custom simulator [22] for less than 500 MeV/c pions and by the CALOR [23] simulator for more than 500 MeV/c pions. For the  $p \rightarrow e^+ \pi^0$  MC, the same simulator is used. In this, as well as the atmospheric neutrino MC, the Fermi motion of protons, the nucleon binding energy, and pion interactions in  $^{16}\text{O}$  are considered.

The observed atmospheric neutrino flavor ratio ( $\nu_\mu/\nu_e$ ) in Super-Kamiokande is significantly smaller than the expected value [14]. For comparison of data and atmospheric neutrino MC, the neutrino MC sample is normalized to the number of observed atmospheric neutrino events at Super-Kamiokande in the following manner. The number of  $\nu_e$  ( $\nu_\mu$ ) CC events is normalized by the ratio of the number of single ring events with a showering (nonshowering) PID in the data to the number of single ring events with a showering (nonshowering) PID in the atmospheric neutrino MC. For NC events, the same normalization factor as that of the  $\nu_e$  CC events is used.

To extract the  $p \rightarrow e^+ \pi^0$  signal from the event sample, these selection criteria are defined: (A) 6800 p.e. < total p.e. < 9500 p.e.; (B) the number of rings is 2 or 3; (C) all rings have a showering PID; (D)  $85 \text{ MeV}/c^2 < \pi^0$  invariant mass <  $185 \text{ MeV}/c^2$ ; (E) no decay electron; (F)  $800 \text{ MeV}/c^2 < \text{total invariant mass} < 1050 \text{ MeV}/c^2$  and total momentum < 250 MeV/c. Criterion (A) roughly corresponds to a total energy of 800 to 1100 MeV.

Criterion (C) selects  $e^\pm$  and  $\gamma$ . Criterion (D) applies only to 3-ring events. Here, at least one pair of rings must give a reconstructed invariant mass which is consistent with the estimated  $\pi^0$  mass resolution of  $135 \pm 35 \text{ MeV}/c^2$ . Criterion (E) is required since the desired  $e^+$  and  $\pi^0$  particles produce no decay electrons. In criterion (F), the total momentum is defined as  $P_{\text{tot}} = |\sum_i^{\text{all rings}} \vec{p}_i|$  where  $\vec{p}_i$  is a reconstructed momentum vector of  $i$ th ring. The total invariant mass is defined as  $M_{\text{tot}} = \sqrt{E_{\text{tot}}^2 - P_{\text{tot}}^2}$  where total energy  $E_{\text{tot}} = \sum_i^{\text{all rings}} |\vec{p}_i|$ . Criterion (F) checks that the total invariant mass and total momentum correspond to the mass and momentum of the source proton, respectively.

From Fig. 1(a), the detection efficiency of  $p \rightarrow e^+ \pi^0$  events is estimated to be 44%. The absorption, charge exchange, and scattering of  $\pi^0$ 's in the  $^{16}\text{O}$  nucleus are the dominant contribution to the detection inefficiency. To estimate the background from atmospheric neutrino interactions, we generate a MC sample of 900 kton · yr. By applying the proton decay selection criteria to this sample, we estimate the number of background events in the signal region to be 0.1 event in 25.5 kton · yr as seen in Fig. 1(b).

Finally, we apply the same criteria to the data to search for the  $p \rightarrow e^+ \pi^0$  signal. No events survive all criteria as shown in Fig. 1(c). Furthermore, the mass distribution of data in Fig. 2 is well reproduced by that of the neutrino MC and the event rates at different stages in the reduction for data and neutrino MC match, as shown in Fig. 3.

The primary result is based on cuts using kinematic reconstruction. Because a systematic error in this reconstruction could miss  $p \rightarrow e^+ \pi^0$  candidates a looser analysis is performed as a consistency check. The selection criteria are as follows: (1) Visible energy is within 200 MeV of the proton rest mass energy. (2) The light anisotropy is less than 30%. (3) No electrons from muon decays follow the primary products. The visible energy of an event is the total energy assuming all Cherenkov light is from the electromagnetic showers. The light anisotropy of an event is an estimate of the total momentum imbalance and is defined as the magnitude of the charge weighted vector sum of the directions from the vertex to each PMT. For a typical single ring event the light anisotropy is  $\sim 75\%$ . This simple analysis finds 4 events with a background of 3.5, which is consistent with the 0 candidates and 0.1 background of the primary analysis.

From these results, we conclude we do not find any evidence for proton decay via the mode  $p \rightarrow e^+ \pi^0$ . Therefore we set a lower limit on the partial decay lifetime. The method used to calculate this limit takes both the estimated background and the systematic uncertainties into account [24]. This calculation gives a limit on the partial lifetime for  $p \rightarrow e^+ \pi^0$  of  $\tau/B_{p \rightarrow e^+ \pi^0} > 1.6 \times 10^{33} \text{ yr}$  at a 90% C.L. If uncertainties are ignored the limit would increase by about 5%.

In calculating the limit, the parameter with the dominant uncertainty is the detection efficiency. This uncertainty

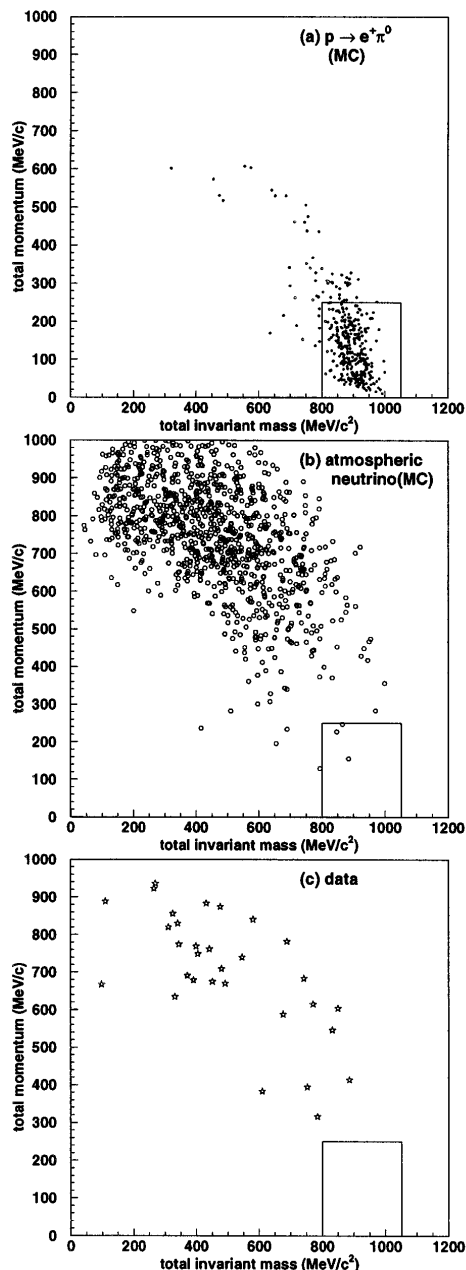


FIG. 1. The total invariant mass and total momentum distributions after criteria (A)–(E) (see text) for three samples: (a)  $p \rightarrow e^+ \pi^0$  Monte Carlo; (b) atmospheric neutrino Monte Carlo corresponding to 900 kton · yr; (c) data corresponding to 25.5 kton · yr. The boxed region in each figure shows the criterion (F) for the  $p \rightarrow e^+ \pi^0$  signal.

is primarily due to imperfectly known pion-nucleon cross sections in  $^{16}\text{O}$  nuclei and is estimated by comparing with another detailed model (based on [25]). This uncertainty is estimated to be 15%. In addition, systematic differences in the energy scale for data and MC contributes 1%, lack of uniformity in the detector gain contributes 2%, and fitting resolution contributes 5% to the uncertainty in the detection efficiency. The total uncertainty in the detection efficiency is then 16%. The statistical uncertainty in the background is 60% due to the small number of MC back-

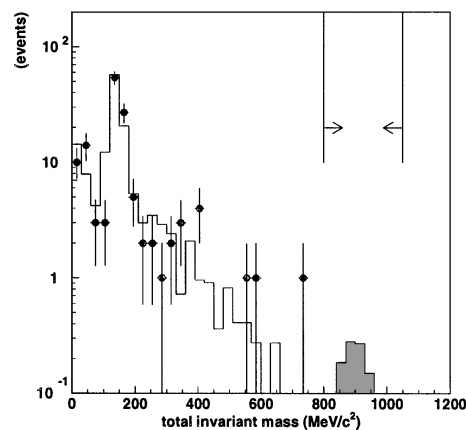


FIG. 2. The total invariant mass distributions of data (circles), normalized atmospheric neutrino Monte Carlo corresponding to 10 years (unshaded histogram), and  $p \rightarrow e^+ \pi^0$  Monte Carlo normalized to one event (shaded histogram) which satisfy the criteria (B)–(E) (see text) and have a total reconstructed momentum  $< 250$  MeV/c.

ground events passing the cuts. Finally, the uncertainty in the exposure is negligible.

In this Letter, we have reported the results of a proton decay search in Super-Kamiokande. We have no evidence of the proton decaying via the mode  $p \rightarrow e^+ \pi^0$  in the 25.5 kton · yr data. We set the most stringent limit on the partial lifetime of the proton to be  $1.6 \times 10^{33}$  yr at a 90% C.L., which should be compared with the previous experimental results,  $2.6 \times 10^{32}$  yr [7] and  $5.5 \times 10^{32}$  yr [8].

We gratefully acknowledge the cooperation of the Kamioka Mining and Smelting Company. The Super-Kamiokande experiment was built from, and has been operated with, funding by the Japanese Ministry of Education, Science, Sports and Culture, and the United States Department of Energy.

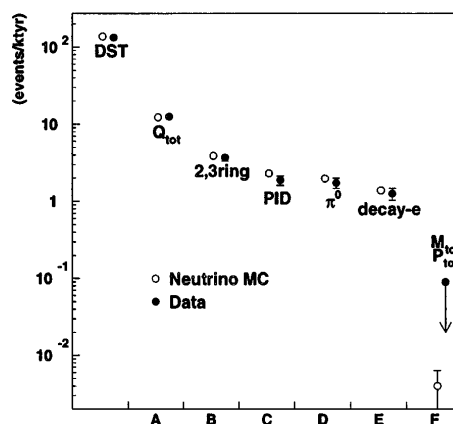


FIG. 3. The event rate after each proton decay selection criterion (see text) for data (filled circles) and atmospheric neutrino Monte Carlo (empty circles). There is no event in the data after criterion (F) and only the 90% C.L. upper limit is shown in the last bin.

- \*Present address: NASA, JPL, Pasadena, CA 91109.  
†Present address: Accelerator Laboratory, High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan.  
‡Present address: Enrico Fermi Institute, University of Chicago, Chicago, IL 60637.  
§Present address: Institute for Cosmic Ray Research, University of Tokyo, Tanashi, Tokyo 188-8502, Japan.  
\*\*Present address: Dept. of System Engineering, Shizuoka University, Hamakita, Shizuoka 432-8561, Japan.  
††Present address: Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan.
- [1] Because of the chiral anomaly in the standard model the proton could have a finite but unobservably long lifetime. G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976).  
[2] Joseph C. Pati and Abdus Salam, Phys. Rev. Lett. **31**, 661 (1973).  
[3] H. Georgi and S.L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).  
[4] P. Langacker, Phys. Rep. **72**, 185 (1981).  
[5] G.G. Ross, *Grand Unified Theories* (Addison-Wesley, Redwood City, CA, 1985).  
[6] T.J. Haines *et al.*, Phys. Rev. Lett. **57**, 1986 (1986).  
[7] K.S. Hirata *et al.*, Phys. Lett. B **220**, 308 (1989).  
[8] R. Becker-Szendy *et al.*, Phys. Rev. D **42**, 2974 (1990).  
[9] C. Berger *et al.*, Z. Phys. C **50**, 385 (1991).  
[10] C. Berger *et al.*, Nucl. Phys. **B313**, 509 (1989).  
[11] J. Ellis *et al.*, Phys. Lett. B **252**, 53 (1990); J. Ellis *et al.*, Phys. Lett. B **371**, 65 (1996).  
[12] N.T. Shaban and W.J. Stirling, Phys. Lett. B **291**, 281 (1992).  
[13] D. Lee, R.N. Mohapatra, M.K. Parida, and M. Rani, Phys. Rev. D **51**, 229 (1995).  
[14] Y. Fukuda *et al.*, Phys. Lett. B **433**, 9 (1998).  
[15] E.R. Davies, *Machine Vision: Theory, Algorithms, Practicalities* (Academic Press, San Diego, 1997).  
[16] S. Kasuga *et al.*, Phys. Lett. B **374**, 238 (1996).  
[17] Super-Kamiokande Collaboration, M. Nakahata *et al.*, "Calibration of Super-Kamiokande Using an Electron Linac" (to be published).  
[18] M. Honda *et al.*, Phys. Rev. D **52**, 4985 (1995); M. Honda *et al.*, Phys. Lett. B **248**, 193 (1990).  
[19] D. Rein and L.M. Sehgal, Ann. Phys. (N.Y.) **133**, 79 (1981); D. Rein, Z. Phys. C **35**, 43 (1987).  
[20] E. Oset *et al.*, Nucl. Phys. **A468**, 631 (1987).  
[21] CERN Program Library W5013 (1994).  
[22] M. Nakahata *et al.*, J. Phys. Soc. Jpn. **55**, 3786 (1986).  
[23] T.A. Gabriel *et al.*, IEEE Trans. Nucl. Sci. **36**, 14 (1989).  
[24] B. Viren, Super-Kamiokande Report No. 98-3, 1998 (unpublished but available from <http://www-sk.icrr.u-tokyo.ac.jp/doc/sk/pub>).  
[25] H.W. Bertini, Phys. Rev. C **6**, 631 (1972).