Rotating Superconductors and the Frame-Independent London Equation

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A frame-independent, thermodynamically exact, London equation is presented, which is especially valid for rotating superconductors. A direct result is the unexpectedly high accuracy ($\sim 10^{-10}$) for the usual expression of the London moment. [S0031-9007(98)07338-4]

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The defining property of superconductors to expel any magnetic field from its bulk reverses when it is rotated, and a spontaneous field (or London moment)

$$B = -2\Omega/\gamma, \qquad \gamma \equiv e/mc, \tag{1}$$

appears, where Ω is the rotational velocity, while $m, e(<0)$ are, respectively, the bare mass and charge of the electron [1,2]. (In MKSA units, $\gamma = e/m$.) This is intriguing because some exact cancellations must have taken place before microscopic parameters such as e or m will appear on a macroscopic level, in a complicated many body system that any superconductor is. So in this sense, the London moment may perhaps be compared to the quantum Hall effect [3]. The bare charge e holds its place because the superfluid velocity,

$$\mathbf{v}_s = (\hbar/2m)\nabla\varphi - \gamma\mathbf{A}, \tag{2}$$

must be gauge invariant. ($\mathbf{A}$ is the vector potential, and φ is the macroscopic phase variable characterizing the spontaneously broken symmetry.) Although this type of arguments does not apply to m, a common prefactor in Eq. (2), many authors have convincingly argued why nevertheless, and to what extent, m remains unrenormalized in the London moment [2,4–6].

In this paper, a symmetry argument as simple as the gauge invariance is invoked to show why it is the bare mass that enters Eqs. (1) and (2). The basic point is the following: Only as defined, with the bare mass m, does v_s transform as a velocity.

There is a preferred coordinate system in superconductors, that of the lattice, and most useful properties are best understood in this system. Confining one's interest to these, the transformation property of v_s is irrelevant, and de Gennes' famously pointed statement, that the mass m is completely arbitrary, and might well be taken as that of the Sun [7], valid.

Sometimes, however, other coordinate systems are more advantageous, especially if the lattice itself (when rotating or subject to a sound field) is not an inertial system. A theory that consistently relates the results from different frames is then desirable. And how v_s transforms is an important input for such a frame-independent theory. As will be shown below, the London moment may be understood as a direct consequence of v_s being a velocity.

The superfluid velocity v_s as a thermodynamic variable is a concept perhaps more of superfluid helium, and less of superconductors. The reason frequently given is again the presence of the lattice, which breaks the translational invariance for the electrons. (Even without the lattice, a uniform background charge breaks the Galilean symmetry.) This is hardly convincing, as all the transformations may well be defined to include the ions. Then the usual invariance under translation, rotation, and Galilean boost is restored, and we retain local conservation of (total) momentum and angular momentum. So, v_s is a perfectly legitimate thermodynamic variable also in the context of superconductors, it characterizes the spontaneously broken phase symmetry, and (as we shall see) it gives rise to persistent mass and charge current simultaneously.

In contrast to the exact gauge symmetry, the Galilean transformation property of v_s, and hence Eq. (1), are approximative and subject to relativistic corrections. This has been carefully considered by various authors, who obtain values [2] between 10^{-4} and 10^{-5}. The relevant quantity here is the work function W of the electron, or the electrochemical potential, over mc^2. As pointed out by these authors, this value is subject to further scrutiny, as possibly important effects, such as the centrifugal force in the rotating frame [5] or the presence of the symmetry breaking lattice [6], have not been included.

Eliminating these sources of uncertainties, especially by including the ions explicitly, the accuracy is shown in this paper to be given by μ_{chem}/c^2, where μ_{chem} is the chemical potential of the superconductor, or the energy per unit mass of adding a (neutral) atom to the crystal. Because of the much greater mass of the atoms, μ_{chem}/c^2 is tiny, of order 10^{-10}. This makes the correction negligible for all practical purposes, and renders Eq. (1) about 3 orders of magnitude more precise than any present experimental technique to determine the electron mass directly [8].

To understand the connection between Eq. (1) and the Galilean transformation property of v_s, consider the London equation, which for more than half a century has been very useful to account for static properties of superconductors, especially the Meißner effect and the associated magnetic healing phenomena. It is obtained by taking $\mathbf{j}_s = \varrho_s\mathbf{v}_s$ in the static Maxwell equation $\nabla \times \mathbf{H} = \gamma\mathbf{j}_s$, where ϱ_s is the superfluid density, a stiffness constant. (The

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superconducting number density $n_s \equiv \varrho_s/m$ is probably more familiar.) Applying a further curl on both sides, we arrive at (what we for simplicity shall here call) the London equation, $\nabla \times \nabla \times \mathbf{H} = -\varrho_s \gamma^2 \mathbf{B}$ ($\mathbf{B} \equiv \nabla \times \mathbf{A}$), which yields the magnetic penetration depth $\lambda = \pm\gamma\sqrt{\varrho_s}$ (for $H = B$) [9]. Unfortunately, the same equation also insists on vanishing B fields in the bulk, irrespective of a possible rotation $\Omega \neq 0$.

Because of its curious ineptness to describe rotating superconductors, the London equation is not usually consulted in this context (a notable exception is [2]; see also [10]). Instead, Eq. (2) is considered alone, with a large dose of healthy intuition to compensate for its incompleteness: *If we rotate a bulk cylinder of superconductor, it is essential that over the sample as a whole* $v_s = v_{\text{lattice}}$, *because otherwise very large currents would flow*... [4]. This appears convincing, as taking the curl of Eq. (2) with $v_s = v_{\text{lattice}}$ yields Eq. (1) with $\mathbf{\Omega} = \frac{1}{2}\nabla \times v_{\text{lattice}}$, but it is not really consistent: Inserting $v_s = v_{\text{lattice}}$ also in $\mathbf{j}_s = \varrho_s \mathbf{v}_s$ leads exactly to the large electric current that was to be avoided in the first place. And setting $v_s = v_{\text{lattice}}$ while considering linear motions of the superconductor plays further havoc with physics.

This paper reports the thermodynamic derivation of a generalized London equation that is (i) valid in an arbitrary inertial frame and (ii) given in terms of the macroscopic, measured fields. It is

$$\nabla \times \nabla \times \mathbf{H}_0 = -\varrho_s \gamma(\gamma \mathbf{B}/\eta + 2\mathbf{\Omega}), \tag{3}$$

where $\mathbf{H}_0$ denotes the field in the local rest frame of the lattice, and

$$\eta \equiv 1 + \mu_{\text{chem}}/c^2 \tag{4}$$

is the relativistic correction. Clearly, this equation accounts for the bulk value of the field, $B = -2\eta\Omega/\gamma$, and the attendant magnetic healing, both with and without rotations.

Equation (3) is obtained by combining

$$\nabla \times \mathbf{H}_0 = \gamma \mathbf{j}_s, \tag{5}$$

$$\mathbf{j}_s = \varrho_s(\mathbf{v}_s/\eta - \mathbf{v}_n). \tag{6}$$

Both will be derived carefully below, but let us first establish a qualitative understanding: The static Maxwell equation (5) is an Euler-Lagrange equation, expressing the fact that the free energy is minimal with respect to variations in the vector potential δA; Eq. (6) is a constitutive relation, expressing the thermodynamic conjugate variable $\mathbf{j}_s \equiv \partial\varepsilon/\partial\mathbf{v}_s$ as a function of the two velocities, v_s and v_n, where v_n is the velocity of the lattice points. As the left side of Eq. (5) is invariant under a Galilean transformation, so must the right side $\sim j_s$ be, which may therefore depend only on the velocity difference. Equation (6) states this fact to linear order, including the relativistic correction. (This is where the information that v_s is a velocity enters the equations.)

To understand the transformation behavior of v_s, we start from the rest frame form of the Josephson equation,

$$(\hbar/2)\dot{\varphi} + \mu_- = 0. \tag{7}$$

Consider first the electrochemical potential μ_-: Take the thermodynamic energy density ε either as a function of the mass and charge density,

$$d\varepsilon = \mu d\varrho + \Phi d\rho_e, \tag{8}$$

or as a function of the numbers of ions and electrons,

$$d\varepsilon = \mu_+ dn_+ + \mu_- dn_-. \tag{9}$$

These two pairs of independent variables are related by

$$\varrho = Mn_+ + mn_-, \qquad \rho_e = |e|(n_+ - n_-), \tag{10}$$

with M, m as their mass and $|e|, e$ as their charge, respectively. As a result, the conjugate variables are related as

$$\mu_+ = M\mu - e\Phi, \qquad \mu_- = m\mu + e\Phi. \tag{11}$$

Note two points: (i) This is the only place where m enters our considerations independently from Eq. (2), yet since $\varrho, \rho_e, n_+, n_-$ are all strictly conserved, M, m, e are the bare parameters. (ii) Following the standard notation in relativistic physics, we take the energy density ε to include the rest energy, ϱc^2. Then all three chemical potentials contain a large constant term, especially

$$\mu = \mu_{\text{chem}} + c^2. \tag{12}$$

With the Josephson equation now given as

$$(\hbar/2m)\dot{\varphi} + \mu = -e\Phi/m, \qquad \dot{\mathbf{v}}_s + \nabla\mu = e\mathbf{E}/m, \tag{13}$$

we may define a 4-vector

$$u_\alpha \equiv (\hbar/2m)\partial_\alpha\varphi - \gamma A_\alpha = (-\mu/c, \mathbf{v}_s), \tag{14}$$

such that a transformed u'_α, in the system boosted by $\mathbf{v}$, is given as $(-\mu'/c, \mathbf{v}'_s) = (-[\mu + \mathbf{v} \cdot \mathbf{v}_s]/c, \mathbf{v}_s - \mu\mathbf{v}/c^2)$, especially

$$\mathbf{v}'_s = \mathbf{v}_s - \mu\mathbf{v}/c^2. \tag{15}$$

Taking $\mu/c^2 = 1 + \mu_{\text{chem}}/c^2 \equiv \eta$, we see that $\mathbf{v}_s/\eta$ transforms as a velocity, rendering the combination in Eq. (6) invariant.

As is clear from Eq. (8), μ_{chem} is the energy of adding an atom to the crystal at constant charge, divided by the mass of the atom. Estimating this energy as 10^4 K (boiling/melting temperature), or 10^{-19} J, and the atom as having 50 proton mass $\approx 10^5$ electron mass $\approx 10^{-25}$ kg, we have $|\mu_{\text{chem}}| \approx 10^6$ $(\text{m/s})^2$, or $\eta \approx 1\text{–}10^{-10}$.

The main correction being so small, any inaccuracy in Eq. (7) will be even less important. For instance, the velocity dependent terms in the Josephson equation (not considered above as the rest frame form was chosen) are smaller if the macroscopic velocity stays below $\sqrt{\mu_{\text{chem}}} \approx 10^3$ m/s. The same is true for the centrifugal force, which slightly compresses the outer rim, and decompresses the center, of the superconductor. As a result, the density ϱ, and, hence, $\mu_{\text{chem}}(\varrho)$ are inhomogeneous. However, since $\mu_{\text{chem}} - (\mathbf{\Omega} \times \mathbf{r})^2/2$ remains constant [11], this correction is again of order $(v/c)^2$.

The reasons previous authors arrived at much larger relativistic corrections vary: Some considered a different 4-vector $w_\alpha \equiv (\hbar/2m)\partial_\alpha\varphi$, which transforms as $\mathbf{w}_s' = \mathbf{w}_s - \mathbf{v}(\mu_-/mc^2)$. [See, for instance, Eq. (10) of [5].] Mainly because of the smallness of the electron mass m, the associated relativistic correction μ_-/mc^2 deviates more strongly from 1. Others did consider the 4-vector of Eq. (14), especially Cabrera [2], who, in fact, obtained the combination, $\mu_- - e\Phi$ as the relevant factor in the boost transformation. The absence of ions in his considerations, however, prevented an interpretation of this combination as the chemical potential of the solid. In addition, the term $e\Phi$ was taken to be given only in the corotating, and not in the laboratory frame, and was therefore neglected. Interestingly, the best experimental evidence to date [12] yields the value $\eta - 1 \approx 10^{-4}$, and thus contradicts (both in sign and magnitude) all the above theories, including the present one.

We now proceed to prove Eq. (5) by thermodynamic considerations, in a way that is similar to the minimization procedure of the Ginsburg-Landau functional [13], though the procedure is executed in a general inertial frame here [14], not necessarily that of the lattice. The basic Gibbs relation is

$$d\varepsilon = Tds + \mu d\varrho + \mathbf{v}_n \cdot d\mathbf{g}^{\rm tot} + \sigma_{ij} d\nabla_j u_i + \mathbf{E}_0 \cdot d\mathbf{D} + \mathbf{H}_0 \cdot d\mathbf{B} + \mathbf{j}_s \cdot d\mathbf{v}_s . \quad (16)$$

The first four terms are those of a normal solid [15], with $\varepsilon, s, \varrho, g^{\rm tot}$ denoting the densities of energy, entropy, mass, and momentum, respectively, while u_i is the displacement vector. The conjugate variables are defined by the respective derivatives, e.g., $T \equiv \partial\varepsilon/\partial s$, or $v_n \equiv \partial\varepsilon/\partial g^{\rm tot}$. The next two terms account for the presence of fields, where the subscript $_0$ denotes the respective field in the rest frame $\mathbf{v}_n = 0$. The last term appears in superconducting or superfluid phases, with the variable given by Eq. (2). Accepting that the energy is given as in Eq. (16), minimizing the free energy while keeping the conserved quantities constant, stationarity with respect to variations in the vector potential,

$$\delta \int (\varepsilon - Ts) = \int [\mathbf{H}_0 \delta(\nabla \times \mathbf{A}) - \mathbf{j}_s \delta(\gamma \mathbf{A})] + \dots ,$$

quickly leads to Eq. (5). The other equilibrium conditions are [11] $\partial_i v_j^n + \partial_j v_i^n = 0$, $\partial_t \mathbf{v}^n + \nabla\mu = 0$, $\nabla_j \sigma_{ij} = 0$, and $\mathbf{E}_0 = 0$.

Although the basic structure of Eq. (16) simply states the fact that the thermodynamic equilibrium of a superconducting crystal depends on the specified variables, there are perhaps three points that may seem puzzling at first: (i) Why is the term $\Phi d\rho_e$ of Eq. (8) substituted by its partially integrated form, $\mathbf{E}_0 \cdot d\mathbf{D}$ of Eq. (16)? (ii) Why do the two fields, $\mathbf{E}$ and $\mathbf{H}$, assume their local rest frame values? (iii) To what extent is $\mathbf{v}_n$ the velocity of the lattice? The answer to the first question is connected to locality: $\mathbf{E}_0$ is a local function of $\mathbf{D}$, while ρ_e from everywhere is needed to calculate Φ. Local thermodynamics and hydrodynamics can deal only with quantities that preserve locality. The second question is discussed in details in [16], and is a result of the fact that the conserved, total momentum density $g^{\rm tot}$ contains field contributions. The third question is discussed in [17], in which the dynamics of a hypothetical superfluid crystal is derived, where the equation of motion for the displacement vector u_i is shown to be $\dot{u}_i = v_i^n + Y^D$, with Y^D a dissipative term that accounts for diffusion of defects and interstitials [15]. So, if Y^D is zero (especially true in equilibrium), the lattice points move with v_n. This remains valid also for superconductors, the dynamics of which reduces to that of a superfluid crystal in the limit $e \to 0$ of Eq. (2).

We now revisit Eq. (6): When deriving the constitutive relation for j_s, the information that it must be an invariant quantity was a consequence of asserting the consistency of Eq. (5). More prudently, this should be obtained as a result. So, the form of j_s will be derived independently below, without the reference to Eq. (5), or even the input that v_s/η is a velocity—though the equivalent information of Eq. (13), the Josephson equation, is needed. As we shall see, this calculation will also provide insights in an analogy between superconductors and superfluid ^{3}He. We start the derivation by observing that the energy current $\mathbf{Q}$ contains the term $\mathbf{j}_s\mu$, then deduce the form of the total momentum density by the known symmetry of the energy stress 4-tensor, $\mathbf{g}^{\rm tot} = \mathbf{Q}/c^2$, and, finally, we obtain the explicit form of $\mathbf{j}_s$ via a Maxwell relation linking $\mathbf{g}^{\rm tot}$ to $\mathbf{j}_s$.

The energy current $\mathbf{Q}$ is obtained by evaluating $\dot{\varepsilon}$ via Eq. (16), and requiring that it is given as a total divergence, $\dot{\varepsilon} = -\nabla \cdot \mathbf{Q}$. In the rest frame $\mathbf{v}_n = 0$, and disregarding dissipative terms, we have $\dot{s}$, $\dot{u}_i = 0$; hence only four terms remain:

$$\dot{\varepsilon} = \mu\dot{\varrho} + \mathbf{E} \cdot \dot{\mathbf{D}} + \mathbf{H} \cdot \dot{\mathbf{B}} + \mathbf{j}_s \cdot \dot{\mathbf{v}}_s .$$

With $\dot{\mathbf{v}}_s$ given as in Eq. (13), the Maxwell equations and the continuity equation must contain the corresponding counter terms,

$$\dot{\mathbf{B}} = -c\nabla \times \mathbf{E} , \quad (17)$$

$$\dot{\mathbf{D}} = c\nabla \times \mathbf{H} - e\mathbf{j}_s/m , \quad (18)$$

$$\dot{\varrho} + \nabla \cdot \mathbf{j}_s = 0 , \quad (19)$$

such that the energy current is $\mathbf{Q} = \mathbf{j}_s\mu + c\mathbf{E} \times \mathbf{H}$. The same calculation in a general frame $\mathbf{v}_n \neq 0$ is tedious, but not more difficult. And of the new terms, all $\sim v_n$, the overwhelming one is that associated with the rest mass, $\varrho c^2 \mathbf{v}_n$. So the total momentum density is

$$\mathbf{g}^{\rm tot} = \varrho \mathbf{v}_n + \mathbf{j}_s\mu/c^2 + \mathbf{E} \times \mathbf{H}/c , \quad (20)$$

while the neglected terms are again relativistic corrections, ones that are completely irrelevant in the present context. Now consider the Maxwell relation,

$$\partial g_i^{\rm tot}/\partial j_j^s|_{v_n} = \partial v_j^s/\partial v_i^n|_{j_s} = (\mu/c^2)\delta_{ij} . \quad (21)$$

Evaluating the first expression via Eq. (20) we arrive at the third expression; and the second equals sign only allows a functional dependence as given in Eq. (6). This concludes the independent proof.

The total momentum density, Eq. (20), may be written as $\mathbf{g}^{\mathrm{tot}} = (\varrho - \varrho_s)\mathbf{v}_n + \varrho_s \mathbf{v}_s$ for $E = 0$, implying a literal *nonclassical rotational inertia* [18] for superconductors—though it is, of course, much smaller than in He II: Employing Eqs. (5) and (6), one finds in a superconducting cylinder of radius $R \gg \lambda$ that the superfluid velocity v_s deviates from the normal one, $\mathbf{v}_n = \mathbf{\Omega} \times \mathbf{R}$, by the amount $\lambda(2\Omega + \gamma B_{\mathrm{ext}}) \exp[(r - R)/\lambda]$ (r is the distance from the center, and B_{ext} the external field). Therefore, the fraction of ϱ_s/ϱ of the total mass does not quite participate in the rotation of the rest, $(\varrho - \varrho_s)/\varrho$, and reduces (or enhances) the moment of inertia accordingly—as a function of the external magnetic field.

Equations (18) and (19) show that $\mathbf{j}_s \equiv \partial\varepsilon/\partial\mathbf{v}_s$ transports both electric charge and mass in equilibrium, with a quotient given by the microscopic parameter e/m. As is clear from the derivation, this is intimately related to the form of the Josephson equation, and to the relationship between g^{tot} and j_s. If the Josephson equation, Eq. (13), were of the form $(\hbar/2m)\dot\varphi = -e\Phi/m$, the terms $\sim j_s$ in Eqs. (19) and (20) (i.e., in the momentum density and mass current) would vanish; if it were $(\hbar/2m)\dot\varphi + \mu = 0$, the persistent electric current, in Eq. (18), is zero. The first is a hypothetical case of pure superfluidity in the electric charge, and the second is a case of pure superfluidity in the mass, such as realized in He II. The specific sum in Eq. (13) characterizes a phase with condensed electrons, ones that carry mass and charge of the given ratio.

All this is reminiscent of the relative broken symmetries in the superfluid phases of ^{3}He [19]—especially ^{3}He-A_1, in which the condensation is in the up-spin population, of ^{3}He particles that carry both spin and mass, with the ratio $\hbar/2m$. And the corresponding broken symmetry is a linear combination of spin and phase symmetry—if one forgets the less obvious orbital one. The microscopic coefficient $\hbar/2m$ also characterizes the ratio between the equilibrium currents for mass and spin, and it enters the relevant Josephson equation [20] which—as in Eq. (13)—contains two thermodynamic conjugate variables, $\mu \equiv \partial\varepsilon/\partial\varrho$ and $\boldsymbol{\omega} \equiv \partial\varepsilon/\partial\mathbf{s}$. This gives rise to a number of useful analogies that should be explored.

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