

## Dynamical Generation of Fermion Mass and Magnetic Field in Three-Dimensional QED with Chern-Simons Term

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(Received 13 February 1998)

We study dynamical symmetry breaking in three-dimensional QED with a Chern-Simons (CS) term, considering the screening effect of  $N$  flavor fermions. We find a new phase of the vacuum, in which both the fermion mass and a magnetic field are dynamically generated, when the coefficient of the CS term  $\kappa$  equals  $Ne^2/4\pi$ . The resultant vacuum becomes the finite-density state half filled by fermions. For  $\kappa = Ne^2/2\pi$ , we find the fermion remains massless and only the magnetic field is induced. For  $\kappa = 0$ , spontaneous magnetization does not occur and should be regarded as an external field. [S0031-9007(98)06439-4]

PACS numbers: 11.10.Kk, 11.30.Cp, 11.30.Qc

Field theoretical models in  $(2 + 1)$ -dimensional space-time have attracted much attention as effective theories at long distance in planar condensed matter physics. Especially, quantum electrodynamics in  $2 + 1$  dimensions (QED<sub>3</sub>) has been intensively studied in connection with the effective theories of high- $T_c$  superconductivity [1], as well as the probe for  $(3 + 1)$ -dimensional quantum chromodynamics. In  $2 + 1$  dimensions, there can be a topological gauge action known as Chern-Simons (CS) term. This term connects a magnetic field  $B$  with an electric charge density  $e\langle\psi^\dagger\psi\rangle$  for fermion field  $\psi$ . From this peculiar property, the CS term is used in the field theoretical understanding of the fractional quantum Hall effect [2].

As a natural extension of QED<sub>3</sub>, a theory in which gauge field action includes both the CS term and the Maxwell term was proposed by Ref. [3]. In this theory (CS-QED<sub>3</sub>), the coefficient  $\kappa$  for the CS term gives the photon a gauge invariant mass which explicitly violates the parity symmetry. Some years ago, Hosotani [4] showed that spontaneous magnetization occurs in CS-QED<sub>3</sub>, through breaking the most secret symmetry—Lorentz invariance. In this theory, the Gauss law  $\kappa B = -e\langle\psi^\dagger\psi\rangle$  follows from the equation of motion. Thus the magnetized vacuum corresponds to state with finite fermion density  $\langle\psi^\dagger\psi\rangle \neq 0$ . In Ref. [5] a chemical potential term  $\mu\psi^\dagger\psi$  was introduced as an explicit breaking term for Lorentz symmetry, and the condensation  $\langle\psi^\dagger\psi\rangle$  at  $\mu \rightarrow 0$  limit was studied. It was found that the above vacuum is stable if and only if the fermion bare mass is zero.

In this paper, we examine the possibility that Lorentz symmetry is broken in a theory in which fermion mass and magnetization are both spontaneously generated. We regard the fermion bare mass term  $-m\bar{\psi}\psi$  as an explicit breaking term for flavor  $U(2N)$  symmetry [6] as well as a chemical potential term  $\mu\psi^\dagger\psi$  for Lorentz symmetry. Solving the Schwinger-Dyson (SD) equation, we clarify whether or not two condensations,  $\langle\psi^\dagger\psi\rangle$  and  $\langle\bar{\psi}\psi\rangle$ , are dynamically realized at the symmetric limit

$\mu \rightarrow 0$ ,  $m \rightarrow 0$ . They signal fermion mass generation and magnetization, respectively. For  $\kappa = Ne^2/4\pi$ , we find a new phase of vacuum, in which both massive fermions and a magnetic field are dynamically generated so that both symmetries are simultaneously broken. The vacuum stability is also examined by calculating the Cornwall-Jackiw-Tomboulis (CJT) potential [7] and 2-loop effective potential.

For simplicity, we set the Dirac fermion in four components. The  $\gamma$ -matrices are given as follows:

$$\begin{aligned} \gamma^0 &= \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, & \gamma^1 &= \begin{pmatrix} i\sigma_1 & 0 \\ 0 & -i\sigma_1 \end{pmatrix}, \\ \gamma^2 &= \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}. \end{aligned} \quad (1)$$

We use the metric  $\text{diag}(g^{\mu\nu}) = (-1, 1, 1)$ , so that the  $\gamma$ -matrices satisfy the algebra  $\{\gamma^\mu, \gamma^\nu\} = -2g^{\mu\nu}$ . Our starting Lagrangian including explicit breaking terms is

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} - \frac{\kappa}{2} \epsilon_{\mu\nu\rho} \mathcal{A}^\mu \partial^\nu \mathcal{A}^\rho - \frac{1}{2\xi} (\partial \mathcal{A})^2 \\ &\quad + \bar{\psi} [\gamma^\mu (i\partial_\mu + e\mathcal{A}_\mu) - m + \mu\gamma^0] \psi, \end{aligned} \quad (2)$$

where  $\psi$  denotes the  $N$  flavor four-component fermion and we confine ourselves to the case that fermion mass is dynamically generated only as a parity conserving mass. From the above Lagrangian, we acquire the gauge field equation  $\partial_\nu \mathcal{F}^{\nu\mu} - (\kappa/2)\epsilon^{\mu\nu\rho} \mathcal{F}_{\nu\rho} = -e\bar{\psi}\gamma^\mu\psi$  whose vacuum expectation value gives the Gauss law constraint  $\kappa B = -e\langle\psi^\dagger\psi\rangle$  under the existence of constant magnetic field.

It is known that QED<sub>3</sub> is a super-renormalizable theory and its beta function for coupling constant has a nontrivial infrared fixed point [6]. Therefore we cannot neglect the screening effect of vacuum polarization at long distance. Following Ref. [6], we introduce dimensionful coupling  $\alpha = Ne^2/4\pi$  and keep  $\alpha$  finite when  $N$  is taken to infinity so that the vacuum polarization effect can be taken

into the effective action successively in  $1/N$  expansion. Since we also attempt to investigate the spontaneous magnetization, we separate the gauge field  $\mathcal{A}^\mu$  into background and propagating fields as  $\mathcal{A}_\mu = A_\mu^{\text{ext}} + A_\mu$ , with  $A_\mu^{\text{ext}}(x) = Bx_2\delta_{\mu 1}$ . The background field  $A_\mu^{\text{ext}}$  can be fully contained into the fermion propagator

$$S(x, y) = -\left\langle x \left| \frac{1}{\gamma^\mu(i\partial_\mu + eA_\mu^{\text{ext}}) - m + \mu\gamma^0} \right| y \right\rangle, \quad (3)$$

by using the proper time method [8]. It is determined as

$$S(x, y) = \exp\left(\frac{ie}{2}(x-y)^\mu A_\mu^{\text{ext}}(x+y)\right)\tilde{S}(x-y), \quad (4)$$

and the Fourier transform for  $\tilde{S}(x-y)$  is given as

$$\begin{aligned} \tilde{S}(k) &= i \int_0^\infty ds \exp\left[-is\left(m^2 - k_\epsilon^2 + \frac{\tan(eBs)}{eBs} \mathbf{k}^2\right)\right] \\ &\times \{[1 + \gamma_1\gamma_2 \tan(eBs)](m + \gamma^0 k_\epsilon) \\ &\quad - (\gamma^1 k_1 + \gamma^2 k_2) \sec^2(eBs)\}, \end{aligned} \quad (5)$$

where  $k_\epsilon := k^0 + \mu + i\epsilon \text{sgn}(k^0)$  which modifies the  $i\epsilon$  prescription to be consistent with the shift of Hamiltonian by  $\mu$ . The photon propagator can be read from  $\mathcal{L}$  and is

$$\begin{aligned} \Delta^{\mu\nu}(p) &= \frac{1}{p^2 + \kappa^2} \left[ g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} + i\kappa \frac{p_\rho \epsilon^{\mu\nu\rho}}{p^2} \right] \\ &\quad + \xi \frac{p^\mu p^\nu}{(p^2)^2}, \end{aligned} \quad (6)$$

where we see that the Chern-Simons coefficient  $\kappa$  plays a role of gauge invariant photon mass.

The effective gauge action improved by fermion 1-loop correction is constructed by integrating out the fermion field [9] and truncating up to the next to leading order in  $1/N$  expansion, that is,

$$\begin{aligned} \Gamma[A] &= -iN \text{Tr} \text{Ln} S^{-1} \\ &\quad + \int d^3x \left[ -\frac{1}{2} B^2 - \kappa B A^0(x) \right. \\ &\quad \left. + \frac{1}{2} A^\mu(x) D_{\mu\nu}^{-1}(-i\partial) A^\nu(x) \right], \end{aligned} \quad (7)$$

where  $D^{-1}$  denotes the inverse of the improved photon propagator  $D_{\mu\nu}(p) := [\Delta_{\mu\nu}^{-1}(p) - \Pi_{\mu\nu}(p)]^{-1}$ . The vacuum polarization  $\Pi_{\mu\nu}(p)$  can be regularized in a gauge invariant manner [10,11], as

$$\begin{aligned} \Pi_{\mu\nu}(p) &= -Ne^2 \int \frac{d^3k}{(2\pi)^3} \text{tr}[\gamma_\mu \tilde{S}(k) \gamma_\nu \tilde{S}(k-p)], \\ &= (p_\mu p_\nu - p^2 g_{\mu\nu}) \Pi_e(p) - ip^\rho \epsilon_{\mu\nu\rho} \Pi_o(p) \\ &\quad + (p_\mu^\perp p_\nu^\perp - p_\perp^2 g_{\mu\nu}^\perp) \Pi_\perp(p), \end{aligned} \quad (8)$$

where  $p_\perp^\mu = (0, p_1, p_2)$  and  $\text{diag}(g_{\mu\nu}^\perp) = (0, 1, 1)$ . We notice that the gauge invariant tensor is split into the parity conserving part and violating part. The latter has the same tensor structure as the CS term, or gauge invariant photon mass, in the effective action (7).

Now we will show that the possible values of  $\kappa$  under the existence of the constant magnetic field is restricted by the Gauss law  $\kappa B = -e\langle\psi^\dagger\psi\rangle$ . It is known that, unless  $\mu^2 = m^2 + 2n|eB|$ ,  $n = 1, 2, \dots$ , the charge condensation  $\langle\psi^\dagger\psi\rangle$  is related to the parity violating part of vacuum polarization  $\Pi_o$  with the relation:  $-e\langle\psi^\dagger\psi\rangle = B\Pi_o(0)$  [12]. The Gauss law becomes  $B[\kappa - \Pi_o(0)] = 0$  which means that the nonzero magnetic field can penetrate the system when the effective photon mass,  $\kappa_{\text{eff}} := \kappa - \Pi_o(0)$ , becomes zero. Otherwise, the system with  $\kappa \neq \Pi_o(0)$  excludes a magnetic field whether its origin is external or dynamical. In this case we should set the magnetic field to zero and investigate the dynamical generation of parity breaking fermion mass as well as  $U(2N)$  symmetry breaking [13]. We calculate  $\Pi_o(0)$ , when  $\mu^2 < m^2 + 2|eB|$ , as [11]

$$\Pi_o(0) = -2\alpha \text{sgn}(\mu eB) \theta(|\mu| - m), \quad (9)$$

where the step function  $\theta(x)$  has a value  $1/2$  at  $x = 0$  as a zero temperature limit of Fermi-Dirac statistics. We find that  $\Pi_o(0)$  has different values according to  $|\mu| < m$ ,  $|\mu| = m$ , and  $|\mu| > m$ . Therefore, the constraint  $\kappa = \Pi_o(0)$  forces us to take the different approach to a symmetric limit  $(\mu, m) \rightarrow (0, 0)$  for each value of  $\kappa$  so that the nonzero magnetic field can exist. The possible way of a symmetric limit for each value of  $\kappa$  can be read as

$$\kappa = \begin{cases} 0, & |\mu| < m \rightarrow 0 \\ -\alpha \text{sgn}(\mu eB), & |\mu| = m \rightarrow 0, \\ -2\alpha \text{sgn}(\mu eB), & m < |\mu| \rightarrow 0 \end{cases}, \quad (10)$$

which corresponds to empty, half-filled, and fully filled lowest Landau levels, respectively.

In the following, we investigate the dynamical generation of fermion mass and magnetic field for theories with  $|\kappa| = 0, \alpha, 2\alpha$ . We confirm that the fermion mass  $m_d$  and the chemical potential  $\mu_d$  are dynamically generated at the symmetric limit  $(m, \mu) \rightarrow (0, 0)$ . In order to be consistent with the nonzero magnetic field,  $m_d$  and  $\mu_d$  should satisfy the same relation that the explicit breaking parameters  $m$  and  $\mu$  satisfy in Eq. (10) for each  $\kappa$ .

It was shown in Ref. [14] that the strong magnetic field played a role of catalyst for  $U(2N)$  symmetry breaking in the  $(2+1)$ -dimensional Nambu-Jona-Lasinio model. Under the strong magnetic field, the wave function for a charged fermion is localized around the region with the size of magnetic length  $l := 1/\sqrt{|eB|}$ . So, the fermion behaves the same as in a  $0+1$  dimension and the condensation  $\langle\bar{\psi}\psi\rangle$  becomes easily formed such as in Bardeen-Cooper-Schrieffer theory [14]. Recently, Shpagin [15] showed that, by using the SD equation, the fermion mass term was dynamically generated for all of the number of flavors in QED<sub>3</sub> with an external magnetic field. In CS-QED<sub>3</sub>, when  $\kappa$  has the consistent values in Eq. (10), the photon becomes massless and our effective theory, described by Eq. (7), is identical to the one in QED<sub>3</sub> with an external magnetic field and the Gauss law

constraint. We, therefore, only have to extend Shpagin's analysis using the SD equation so as to include the  $\gamma^0$  component of fermion self-energy.

Now we construct the SD equation for fermion self-energy. According to Ref. [15], we assume the strong magnetic field  $m \ll 1/l$ , for which the higher Landau levels  $\sqrt{m^2 + 2n/l^2}$ ,  $n \geq 1$  decouple. Thus we only have to treat the lowest Landau level (LLL). We use the photon propagator improved up to the next to leading order in  $1/N$  expansion. The SD equation is [15]

$$G(x, y) = S(x, y) - ie^2 \int d^3z d^3t S(x, z) \times \gamma^\mu G(z, t) \gamma^\nu G(t, y) D_{\mu\nu}(t - z), \quad (11)$$

where we use the bare vertex approximation and  $G$  denotes the full fermion propagator which should be consistently determined through the SD equation. We assume that the full propagator  $G$  also has the same form as  $S$  in Eq. (4), where we only have to replace  $S$  with  $G$ .  $\tilde{S}(k)$  in Eq. (5) can be decomposed into the Landau level poles  $k_\epsilon^2 = m^2 + 2n/l^2$ ,  $n = 0, 1, \dots$ . We see that the higher Landau level poles decouple and only the LLL pole,  $k_\epsilon^2 = m^2$ , contributes to  $\tilde{S}(k)$  under the strong magnetic field:  $m \ll 1/l$ . Following Ref. [15], we approximate the Fourier transform of  $\tilde{S}$  and  $\tilde{G}$  with its LLL contributions. That is,

$$\tilde{S}(k) \simeq e^{-l^2 k_\perp^2} \frac{1}{m - \gamma^0 k_\epsilon} [1 - i\gamma^1 \gamma^2 \text{sgn}(eB)], \quad (12)$$

$$\tilde{G}(k) \simeq e^{-l^2 k_\perp^2} \tilde{g}(k^0) [1 - i\gamma^1 \gamma^2 \text{sgn}(eB)], \quad (13)$$

where  $1 - i\gamma^1 \gamma^2 \text{sgn}(eB)$  is a projection operator to spin state. We notice that the fermion on the lowest Landau level essentially behaves like  $(0 + 1)$ -dimensional objects.

Under the above approximation, the SD equation (11) is simplified as

$$\tilde{g}^{-1}(p^0) = m - \gamma^0(p^0 + \mu) - \frac{ie^2}{(2\pi)^3} \int_{-\infty}^{\infty} dk^0 \gamma^0 \tilde{g}(k^0) \gamma^0 \tilde{D}(p^0 - k^0), \quad (14)$$

where the function  $\tilde{D}$  is defined by

$$\tilde{D}(p^0) := - \int d^2 p_\perp e^{-l^2 p_\perp^2 / 2} D_{00}(p^0, p_\perp). \quad (15)$$

The function  $\tilde{g}$  is written in the form including the scalar component  $B$  and  $\gamma^0$  component  $\hat{B}$  such as

$$\tilde{g}^{-1}(p^0) = B(p^0) - A(p^0) \gamma^0 p^0 - \gamma^0 [\hat{B}(p^0) + i\epsilon \text{sgn}(p^0)], \quad (16)$$

without loss of generality. In Eq. (11), we use the bare vertex approximation, so we must set  $A(p^0) \equiv 1$  in order to maintain the consistency with the Ward-

Takahashi identity. We also assume  $B(p^0)$  is positive definite and  $B_\pm(p^0) := \hat{B}(p^0) \pm B(p^0)$  have a definite sign irrespective of its argument  $p^0$  so that we can carry out the Wick rotation  $p^0 = i\bar{p}$  uniquely in the SD equation.

Setting  $A(p^0) \equiv 1$  and substituting Eq. (16) into the SD equation (14), we acquire the two independent integral equations

$$B_\pm(\bar{p}) = \mu \pm m + \frac{e^2}{(2\pi)^3} \int_{-\infty}^{\infty} d\bar{k} \frac{B_\pm(\bar{k})}{\bar{k}^2 + B_\pm^2(\bar{k})} \times \tilde{D}(\bar{p} - \bar{k}). \quad (17)$$

At the limit  $\mu \pm m \rightarrow 0$ , the function  $\tilde{D}$  in Eq. (17) becomes the one calculated from the massless photon propagator and has logarithmic behavior in the infrared region of momentum. That is  $e^2 l \tilde{D}(\bar{p}) \simeq -8\pi^2 \alpha_0 \times \ln|l\bar{p}|$  with  $\alpha_0 := \alpha l / N(1 + c\alpha l)$ . The constant  $c$  denotes the parity conserving vacuum polarization effect  $\Pi_e(0) \equiv c\alpha l$  and is determined by the Riemann zeta function as  $c = -6\sqrt{2} \zeta(-1/2) \simeq 1.76397$ .

Since the integral in Eq. (17) is dominated at the infrared region, we put  $\bar{p} = 0$  and can replace  $lB_\pm(\bar{k})$  by its zero momentum values  $\omega_\pm := lB_\pm(0)$ . The SD equation at  $\mu \pm m \rightarrow 0$  finally becomes the gap equations:

$$\omega_\pm = -\frac{\alpha_0}{\pi} \int_{-\infty}^{\infty} ds \frac{\omega_\pm}{s^2 + \omega_\pm^2} \ln|-s|, \quad (18)$$

which have the nontrivial solution:  $|\omega_s| = -\alpha_0 \ln|\omega_s|$ , as well as the trivial one. We notice that  $\omega_s$  satisfies the condition  $|\omega_s| \ll 1$ , which is required to support the LLL approximation, since  $\alpha_0 < 1$  for any value of  $e^2$  and  $N$ . The dynamical variables  $m_d$  and  $\mu_d$  are given by  $m_d := (\omega_+ - \omega_-)/2l$ ,  $\mu_d := (\omega_+ + \omega_-)/2l$ . According to the values of  $\kappa$  in Eq. (10), the consistent solutions are determined as

$$(m_d, \mu_d) = \begin{cases} (|\omega_s|/l, 0) & \text{for } \kappa = 0 \\ (|\omega_s|/2l, \omega_s/2l) & \text{for } |\kappa| = \alpha \\ (0, \omega_s/l) & \text{for } |\kappa| = 2\alpha \end{cases}. \quad (19)$$

The solution for  $\kappa = 0$  coincides with that of Ref. [15] on the empty vacuum. The solution for  $|\kappa| = 2\alpha$  corresponds to the one in Ref. [4] on the fully filled vacuum. But, in our case, the massless fermion is shown to be dynamically generated. The solution for  $|\kappa| = \alpha$  is a new one which generates massive fermion as well as finite-density vacuum half-filled by fermions.

It was not clear whether or not the solutions of SD equation,  $m_d$  and  $\mu_d$ , are energetically favorable and the magnetization spontaneously occurs for each  $\kappa$ . So, we have to investigate the vacuum energy. We assume the strong coupling  $\alpha \gg l^{-1}$  and expand the vacuum energy  $V(B)$  with respect to  $1/\alpha l \sim \sqrt{B}/e^3$ .  $V(B)$  is constructed from four parts including the Maxwell energy, that is,  $V(B) = V_{\text{CJT}}(B) + V_F(B) + V_P(B) + B^2/2$ .  $V_{\text{CJT}}$  denotes the CJT potential [7], which gives the energy difference between the nontrivial vacuum and the trivial

one at the presence of the magnetic field. It is evaluated for each  $\kappa$ , in the LLL approximation, as [11]

$$V_{\text{CJT}}(B) = -\frac{N}{4\pi} \max\{m_d, |\mu_d|\} |eB|^{3/2} + \mathcal{O}(B^2), \quad (20)$$

which has a negative value and shows that the nontrivial solutions are energetically favorable irrespective of  $\kappa$ .

$V_F$  ( $V_P$ ) corresponds to the shift of zero-point energy for the fermion (photon) induced by the magnetic field at the symmetric limit.  $V_F$  is deduced from the 1-loop effective action (7) as  $iN \text{Tr} \ln S^{-1}$  in  $\mu \pm m \rightarrow 0$ , that is [11],

$$V_F(B) = -\frac{N}{4\pi} |eB|^{3/2} 4\sqrt{2} \zeta(-1/2) + \mathcal{O}(B^2). \quad (21)$$

$V_P$  is also calculated from the effective action (7) as a 2-loop contribution of  $-(i/2) \text{Tr} \ln D^{-1}$  in  $\mu \pm m \rightarrow 0$ , that is [11],

$$V_P(B) = -\frac{|\kappa|}{\pi^2} |eB| \arctan\left(\frac{2|\kappa|}{\pi\alpha}\right) + \mathcal{O}(B^{3/2}). \quad (22)$$

The linear term for  $B$  is acquired from  $\Pi_o$  in the photon propagator, since the energy shift of the photon's vacuum energy is an appearance of the difference of effective photon mass in the infrared momentum region [4].

In  $V(B)$ , the term proportional to  $B^{3/2}$  is dominated by that of  $V_F(B)$  and has a positive coefficient for each  $\kappa$  [11]. As to the linear term for  $B$ , it appears for only  $|\kappa| = \alpha, 2\alpha$ , which corresponds to the charge condensed vacua, and has a negative coefficient. Therefore  $V(B)$  has a stationary point at  $B \neq 0$  for  $|\kappa| = \alpha, 2\alpha$ , so the magnetic field is dynamically generated. For  $\kappa = 0$ , the spontaneous magnetization does not occur and we must regard a magnetic field as an external one.

In summary, we have investigated the dynamical symmetry breaking in Chern-Simons QED<sub>3</sub>. We have found that both the fermion mass and the magnetic field are dynamically generated when  $|\kappa| = \alpha$  and the corresponding vacuum is given as a half-filled lowest Landau level. This is a new phase of vacuum with nonvanishing fermion mass and broken Lorentz symmetry. For  $|\kappa| = 2\alpha$ , we have shown that the fermion remains massless and only the magnetic field is induced on the fully filled vacuum. This is the situation in Ref. [4]—vanishing fermion mass

and broken Lorentz symmetry. This configuration of vacuum has been certified to be realized through our argument in this paper. It should be noticed that the magnetic field does not necessarily enhance the mass generation if the vacuum is fully filled by fermions. This shows the sharp contrast to the results of Refs. [14,15], which are based on the empty vacuum and correspond to the case  $\kappa = 0$ .

We would like to thank K. Yamawaki and A. I. Sanda for their encouragement and enlightening discussions. We also thanks V. P. Gusynin, M. Hashimoto, K.-I. Kondo, V. A. Miransky, Y. Nagatani, T. Sato, M. Sugiura, and A. Takamura for valuable discussions. T. I. is grateful to K. Inoue for his encouragement.

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