Random Telegraph Noise in Microstructures

Sh. Kogan
Los Alamos National Laboratory, Los Alamos, New Mexico 87545
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The theory of random current switchings in conductors with S-type current-voltage characteristic is presented. In the range of bistability, the mean time spent by the system in the low-current state before a transition to the high-current state occurs, $\bar{\tau}_l$, decreases with voltage, and that for the high-current state, $\bar{\tau}_h$, grows with voltage; both variations are exponential-like. $\bar{\tau}_l = \bar{\tau}_h$ at a definite voltage in the bistability range. These results are in full accordance with experiments on microstructures. Because of the growth of both times with the size of the conductor, such noise is observable just in microstructures. [S0031-9007(98)07134-8]

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In many systems [reverse-biased p-n junctions, metal-oxide-semiconductor field-effect transistors (MOSFET's), metal-insulator-metal (MIM) tunnel junctions, small semiconductor resistors, or small metallic samples], the current randomly switches between two discrete values. The time intervals between switchings are random, but the two values of the fluctuating current are time-independent. This kind of noise is studied from the 1950s and is now called "random telegraph noise" (RTN) (for reviews see [1–3]). Recently, at least two groups observed RTN in microstructures in the range of voltages where the current-voltage characteristic (CVC) was S shaped [4–8]. The goal of this Letter is to present the theory of RTN in systems with S-type CVC which explains, in rather general terms, the dependence of RTN parameters on the voltage and on the size of the system.

An S-type dependence of the current density, j, on the electric field, F, is schematically shown in Fig. 1, curve 1 (for a review see [9]). In the range of the electric fields between the second and the first threshold fields F_{t2} and F_{t1}, the current is a three-valued function of the field. The states corresponding to the lower and upper branches of the CVC are locally stable, that is, stable against all small perturbations. The state corresponding to the negative differential resistance branch is unstable against some even small perturbations [9]. However, large fluctuations may cause transitions between the two locally stable states (LSS), producing switchings of the current, if the load impedance is sufficiently lower than the resistance of the sample, i.e., if the sample is under voltage-controlled regime. This system is an example of a bistable macroscopic system. Other examples include the tunnel diode (a system with N-type CVC), nonequilibrium chemical systems, and many others. The first theory of stochastic transitions between the LSS in a bistable electronic device was developed by Landauer [10] for the tunnel diode. The general theory is presented in [11,12].

Even though the specific mechanisms of the S-type CVC may be very different in different systems, some features of this phenomenon, not only the shape of the CVC, are general [9]. The rapid nonlinear growth of the current with voltage in semiconductors and semiconductor microstructures can be often attributed to the rapid growth of the number of free charge carriers due to some kind of breakdown, for instance, to the low-temperature impurity breakdown. Another possible mechanism is a rapid growth of the electron mobility in a semiconductor in which the times of momentum and/or energy relaxation grow with the electron temperature T.

We first study such S-type CVC which stem from the variation of the total number N of the free charge carriers in the sample. The master equation for the probability, $P(N)$, of the state with N free charge carriers can be

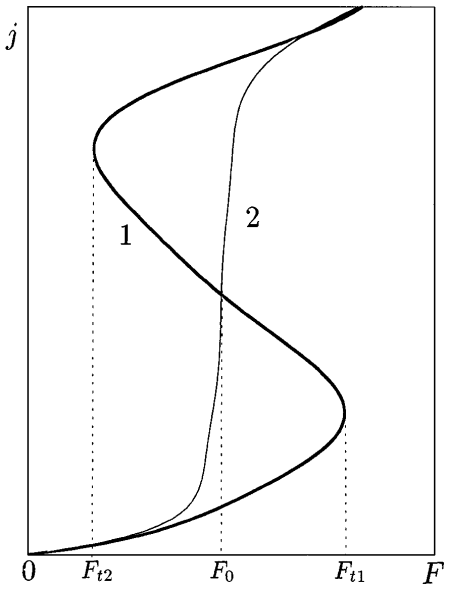

FIG. 1. The S-type current-voltage characteristics (schematically). j is the current density, and F is the electric field. Curve 1: The "intrinsic" dc S-type CVC. F_{t1}, F_{t2}, and F_0 are the first threshold field, the second threshold field, and the field at which $\bar{\tau}_l = \bar{\tau}_h$, respectively. Curve 2: The dc CVC of a microstructure measured under voltage-controlled regime and with the time of current averaging much greater than the mean times $\bar{\tau}_l$ and $\bar{\tau}_h$.

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written as follows:

$$\partial P(N)/\partial t = W^+(N-1)P(N-1) + W^-(N+1)P(N+1) - [W^+(N) + W^-(N)]P(N). \quad (1)$$

Here $W^+(N)$ and $W^-(N)$ are, respectively, the probability, per unit time, of generation (process $N \to N+1$) and trapping ($N \to N-1$) of one free charge carrier in the state with N such carriers. The theory of systems described by the master equation (1) (the random quantity is a whole number, its increment $\Delta N = \pm 1$ only) is well developed [10–14].

We consider conductors with $N \gg 1$. The rates $W^{\pm}(N)$ scale with the volume V of the system as $W^{\pm}(N) = V\gamma^{\pm}(n)$, where $\gamma^{\pm}(n)$ are the specific rates which depend on the density of the free charge carriers $n = N/V$ (if the system is quasi-2D the volume V must be replaced with the surface area). It means that the theory presented here is not concerned with the RTN produced by transitions (electron trapping and detrapping) involving one or few traps only.

In the systems under consideration, the probability density $p(n)$ of the charge carriers' density n is found by using the expansion in powers of V^{-1} [11,12]. For systems described by Eq. (1), the stationary probability distribution density equals [13]: $\bar{p}(n) = C\exp[-V\mu(n)]$, where

$$\mu(n) = -\int_{n_0}^{n} dn' \ln[\gamma^+(n')/\gamma^-(n')], \quad (2)$$

and in the exponent only the term $\propto V$ has been retained. The coefficient C and the density n_0 are determined by the normalization of $\bar{p}(n)$.

For each field F in the range of bistability, the specific rates in the steady states satisfy the equations $\gamma^+(n_i) = \gamma^-(n_i)$, where n_i is the density in any of these three states, i.e., low-current ($n_i = n_l$), high-current ($n_i = n_h$), and locally unstable state ($n_i = n_{lu}$). As follows from Eq. (2), these states correspond to the extrema of $\mu(n)$. The second derivative at an extremum equals:

$$\mu''(n_i) = \left\{\frac{d}{dn}[\gamma^-(n) - \gamma^+(n)]\right\}_{n_i} \Big/ \gamma^+(n_i). \quad (3)$$

Obviously, the state is locally stable if the difference $\gamma^-(n) - \gamma^+(n)$ grows with n, and is unstable in the opposite case. It follows then that $\mu(n)$ has minimums at $n = n_l$ and $n = n_h$, and a maximum at $n = n_{lu}$ (Fig. 2).

Another system with S-type CVC is an electron gas with frequent electron-electron scattering and with the time of electron momentum scattering by ionized impurities τ_p and/or the time of electron energy scattering by phonons τ_e growing with the electron temperature T. This "overheating" model was used for the analysis of systems with S-type CVC [9]. The random quantity is the electron energy E the increments of which in the random processes ϵ vary continuously. Actually, it is assumed in this model that $|\epsilon| \ll k_B T$ due to the inequality $\tau_p \ll \tau_e$ and to the smallness of the characteristic energy of emitted and absorbed acoustic phonons $\overline{\hbar\Omega} \ll k_B T$. The stochastic dynamics of the system is determined by the probability $W(E - \epsilon \to E)$ per unit time and per unit energy ϵ of a transition from a state with total energy $E - \epsilon$ to a state with energy E. The intensive random variable in this case is the specific energy $u = E/V$ (or the electron temperature $T \sim u/n$). The transition rates scale with the volume V as $W(E \to E') = V\gamma(u, E' - E)$.

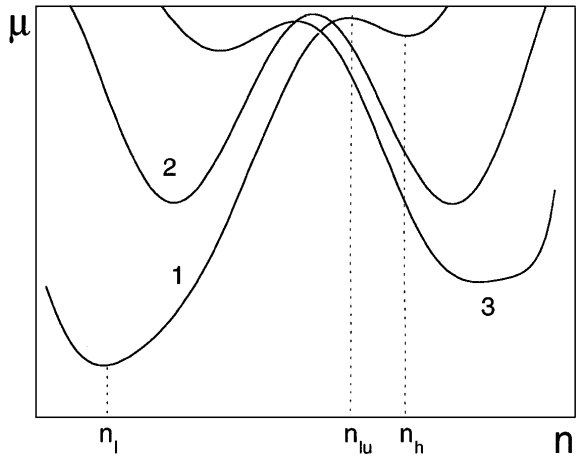

FIG. 2. The dependence of the "potential" μ on the electron density n (shown schematically) for three electric fields F: (1) $F_{t2} < F < F_0$, (2) $F = F_0$, and (3) $F_0 < F < F_{t1}$. The positions of $\mu(n)$ extrema, i.e., n_l, n_{lu}, and n_h, are shown for the curve 1. The dependence of μ on the specific electron energy u in the "overheating" model of the S-type CVC is qualitatively the same.

Because the change of the energy $|\epsilon|$ due to electron acceleration or deceleration by the applied electric field F and by emission and absorption of acoustic phonons is $\ll u/n \sim k_B T$, the master equation is reduced to the Fokker-Planck equation in its simplest form. In this approximation, $\overline{p}(u)$ is given by the same equation as $\overline{p}(n)$ with $\mu(n)$ replaced with $\mu(u) = -\int^u du' A_0(u')/A_1(u')$. Here $A_0(u) = \sigma(u)F^2 - Q(u)$ is the specific power, per unit volume, acquired by the electron gas from the electric field minus the power transferred to the phonon bath, the coefficient of energy diffusion $A_1(u) = A_1^{(\mathrm{imp})}(u) + A_1^{(\mathrm{phon})}(u)$, where $A_1^{(\mathrm{imp})}(u) = e^2 n F^2 D(u)$, and $A_1^{(\mathrm{phon})}(u) = n(\hbar\Omega)^2 \nu_{\mathrm{ph}}$. $D(u)$ is the ordinary diffusion coefficient, and ν_{ph} is the frequency of electron scattering by phonons. The steady states satisfy the condition $A_0(u_i) = 0$. The model assumes that there are three stationary values of u_i satisfying this equation. Obviously, a steady state is locally stable if $\sigma'(u)F^2 - Q'(u) < 0$, i.e., if $\mu''(u_i) > 0$, and unstable in the opposite case (the prime denotes differentiation with respect to u).

The experimentally measured are the mean times, $\bar{\tau}_l$ and $\bar{\tau}_h$, spent by the system in the low-current LSS (lower CVC branch) with $n = n_l(F)$ and in the high-current LSS (upper CVC branch) with $n = n_h(F)$, respectively, before a transition to the other LSS occurs. If the system happens to reach the locally unstable state it "falls" with equal probability (1/2) into any of the two LSS. Therefore, the

problem is to find the mean time $\bar{\tau}(n)$ necessary to reach for the first time the unstable state by starting from a state with a given density of free charge carriers n, where n is sufficiently lower than n_{lu} (for calculation of the time $\bar{\tau}_l$) or sufficiently higher than n_{lu} (for calculation of $\bar{\tau}_h$). For processes with increments $\Delta N = \pm 1$, the equation for the time $\bar{\tau}(n)$ was found in [13,14] (see also [11]). In the lowest approximation in $1/V$:

$$\bar{\tau}_l \equiv \bar{\tau}(n_l) = \frac{\pi \exp\{V[\mu(n_{lu}) - \mu(n_l)]\}}{\gamma^+(n_l)\sqrt{\mu''(n_l)\,|\mu''(n_{lu})|}}. \qquad (4)$$

The equation for $\bar{\tau}_h$ is found by replacing n_l by n_h. For the "overheating" model one can use the approach suggested by Weiss [15]. The result for $\bar{\tau}_l$ can be obtained from Eq. (4) by substituting u_i for n_i and $A_1(u_l)/2$ for $\gamma^+(n_l)$.

In the low-field part of the bistability range, i.e., at fields $F > F_{t2}$ but close to F_{t2}, the difference $\mu(n_{lu}) - \mu(n_h)$ is small and tends to zero as $F \to F_{t2}$ when n_h and n_{lu} coalesce. In this extreme case $\mu(n_l) < \mu(n_h)$ (Fig. 2), and the time $\bar{\tau}_l \gg \bar{\tau}_h$; i.e., the globally stable phase (GSP) corresponds to the lower branch of the CVC. The random transitions from the low-current phase to the high-current phase are comparatively rare (the time between consequent pulses is long) and the random current pulses are positive and short. In the opposite extreme case, when the field F is close to F_{t1}, $\mu(n_1) > \mu(n_h)$, and the time $\bar{\tau}_l \ll \tau_h$: the GSP is the high-current phase. Then the random current pulses are also short but negative.

A field, F_0, *must* exist, within the range of bistability, at which $\bar{\tau}_l = \bar{\tau}_h$. At this field (Fig. 2) the potential $\mu(n_l) = \mu(n_h)$ [the effect of the preexponential factors in Eq. (4) has been neglected]. It is known from the theory of conductors with S-type CVC (see [9]) that, under the current-controlled regime, a field exists within the same range of bistability ("sustaining" field) at which the two phases, corresponding to the lower and upper stable branches of the CVC, coexist and are separated by a boundary the thickness of which is some diffusion length l. Then the high-current phase has the form of a current filament with radius $R \gg l$, and the CVC is vertical. In the deterministic approximation (fluctuations neglected) this field is determined by the stability of the boundary which leads to the equation $H(n_l) = H(n_h)$, where $H(n) = \int^n dn'[\gamma^+(n') - \gamma^-(n')]D(n')$, and $D(n)$ is a phenomenological diffusion coefficient which determines the flow of charge carriers in the transition boundary region. The extrema of $H(n)$ and $\mu(n)$ coincide. However, the sustaining field and F_0 may differ.

The difference of the two potentials in the vicinity of the field F_0 varies linearly with F: $\mu(n_l) - \mu(n_h) = \alpha(F - F_0)$, where $\alpha = \{(d/dF)[\mu(n_l) - \mu(n_h)]\}_{F_0}$. It means that in the range of the field around F_0 the ratio of the two times,

$$\bar{\tau}_l/\bar{\tau}_h \approx \exp[-V\alpha(F - F_0)], \qquad (5)$$

varies exponentially with the field F. Of course, in the same vicinity of F_0, $\mu(n_{lu}) - \mu(n_l)$ and $\mu(n_{lu}) - \mu(n_h)$ also vary linearly with the field F. Therefore, each of the two times varies exponentially with F: $\bar{\tau}_l$ drops [because $\mu(n_{lu}) - \mu(n_l)$ drops] and $\bar{\tau}_h$ grows. Since the times $\bar{\tau}_l$ and $\bar{\tau}_h$ depend on the corresponding differences of potentials exponentially, and these differences depend on the electric field, one may expect a strong, exponential-like, dependence of the times on F in the entire range of bistability except the close vicinities of the threshold (critical) fields F_{t1} and F_{t2}. Just an exponential variation of the times in the low-current and high-current states with voltage is observed in all experiments [4,6–8].

As follows from Eq. (4), the times $\bar{\tau}_l$ and $\bar{\tau}_h$ grow exponentially with the volume V of the system which exhibits the S-type CVC. This dependence stems from the fact that the random transitions between the two locally stable macroscopic states are driven by large fluctuations in the system, and the greater the size of the system the rarer are such fluctuations which are necessary for the transition to happen. Therefore, the mean times spent in each of the LSS are many times greater than the characteristic microscopic times of electron transport in the system. This conclusion is in a good agreement with experiments: in Ref. [4] the times $\bar{\tau}_l$ and $\bar{\tau}_h$ vary from 10 ms up to 1 s, that is, in any case are by several orders higher than any microscopic transport time. The preexponential factor in Eq. (4) is independent of the system's size and, as a whole, can be considered as a microscopic time which is a combination of microscopic times related to the locally stable and locally unstable steady states.

In large systems, if the effect of nucleation is neglected (see below), the times $\bar{\tau}_l$ and $\bar{\tau}_h$ become so long that the current switching can be seen only at the threshold electric fields: at $F \approx F_{t1}$ and $F \approx F_{t2}$, where the differences $\mu(n_{lu}) - \mu(n_l)$ and $\mu(n_{lu}) - \mu(n_h)$, respectively, tend to zero (Figs. 1 and 2). When the field F is increased from small values and reaches F_{t1}, the current switches to the upper branch. When the field is then reduced it drops, at $F \approx F_{t2}$, to the lower branch, and a hysteresis loop is observed.

It was assumed above that the most probable of those states which separate the two domains of attraction [11], to the low-current and to the high-current steady state, respectively, is the spatially uniform state with $n = n_{lu}$ (or $u = u_{lu}$). A fluctuation with a volume smaller than V (but greater than some critical one) after being created may grow further and realize the transition. The role of such fluctuations is analogous to the role of nuclei of the new phase in the vicinity of a first-order equilibrium phase transition.

The fluctuations with at least one of the linear dimensions smaller than some diffusion length are suppressed by diffusion processes. It is reasonable therefore to analyze the evolution of strong fluctuations each of which constitutes a domain of one phase embedded in the second phase. Some conclusions can be made by comparing the evolution of two characteristic nuclei: one varying along

the current but uniform over the sample's cross section, the second varying in the direction perpendicular to the current but uniform in the direction of current.

If the nucleus is varying along the current only, the current density, j, not the field, is the same at both sides of the boundary between phases. Let the main phase be "cold" ($n = n_l$ or $u = u_l$) and the nucleus "hot" ($n = n_h$ or $u = u_h$) at time $t = 0$. The higher current density in the nucleus cannot be supported even under a stationary regime. For instance, in the "overheating" model, inside the nucleus the net specific power is, at $t = 0$, smaller than $A_0(u_h) = 0$ by $(1 - \sigma_l/\sigma_h)\sigma_l F^2$. Therefore, the value of u must decrease with time, and the nucleus must disappear. If, on the contrary, the main phase is "hot" and the nucleus is "cold" at $t = 0$, the net specific power inside the nucleus is greater than $A_0(u_l) = 0$ by $(\sigma_h/\sigma_l - 1)\sigma_h F^2 > 0$; i.e., u must grow with time until the cold nucleus disappears. Hence, the nuclei with boundaries normal to the current cannot grow.

If the nucleus has the form of a filament or a layer parallel to the current, the field is the same in the main phase and in the nucleus. The relative stability of the two LSS is determined by Eq. (4) for $\bar{\tau}_l$ and by the corresponding equation for $\bar{\tau}_h$: the GSP is the "cold" one at fields $F < F_0$ and the "hot" one at $F > F_0$. Because of the growth of GSP nuclei, the smaller of the two times, $\bar{\tau}_l$ or $\bar{\tau}_h$, stops to grow exponentially with V when the transverse size of the sample $L_\perp$ is greater than the critical transverse size l_c of the filamentary nucleus. However, RTN is produced by transitions in both directions and between definite two states only. Since the nuclei of the metastable phase (MP) in the GSP cannot grow freely, the waiting time for the transition of the entire system to the MP (only such transitions are observed as RTN) must grow exponentially with V. The random creation of finite ($<V$) nuclei of MP happens more frequently but it is observed only as comparatively small fluctuations of the current about its value in GSP, not as RTN. Hence, at $L_\perp \gg l_c$ RTN is not observable.

Because the mechanisms of the S-type CVC involve electron heating by the electric field and the energy diffusion lengths are usually no less than 1 μ, l_c may be greater than $L_\perp$ in many nanostructures. If, at $L_\perp \ll l_c$, the number of charge carriers in the sample is great (in this sense the system may be called mesoscopic), the equations for $\bar{\tau}_l$ and $\bar{\tau}_h$ remain valid. In the experiments [4,6–8] no change of the exponential dependence of $\bar{\tau}_l$ and $\bar{\tau}_h$ on voltage was observed at that voltage U_0 at which the two times become equal. One may conjecture that, in these experiments, the width $L_\perp$ of the active part of the conductor was smaller than l_c (in Ref. [4] $L_\perp \sim 100$–200 nm), and the RTN was caused by transitions of the sample as a whole.

In microstructures with a great number of transitions within the time of the experiment ("ergodic systems"), the measured dc current is its mean value averaged over these transitions:

$$\bar{I}(U) = [\bar{\tau}_l I_l(U) + \bar{\tau}_h I_h(U)]/(\bar{\tau}_l + \bar{\tau}_h). \qquad (6)$$

At the low-field side of the bistability range where $\bar{\tau}_l \gg \bar{\tau}_h$ the dc current is close to its lower branch value $I_l(U)$ (Fig. 1, curve 2). As the electric field approaches F_0, the current rapidly increases due to the exponential dependence of $\bar{\tau}_l$ and $\bar{\tau}_h$ on F [Eq. (5)] and to the fact that usually $I_h \gg I_l$. At $F = F_0$ it is equal to $I(F_0) = [I_l(F_0) + I_h(F_0)]/2$. At greater fields it is expected to follow the high-current branch of the intrinsic S-type CVC. Even though the intrinsic CVC is of S-type, the measured CVC under voltage-controlled regime is monotonous, shows no negative differential resistance, and has a point of inflection (it can be interpreted as "smeared phase transition"). Similar continuous dc CVC has been found in [7,8]. It is worth noticing that under the voltage-controlled regime the CVC may have either positive slope or current discontinuities.

In conclusion, the theory presented above explains all qualitative features of the experimental data: (1) The drop and growth of the mean times $\bar{\tau}_l$ and $\bar{\tau}_h$, respectively, with voltage. (2) The exponential dependence of these times on voltage. (3) The coincidence of these two times at a voltage (field F_0) in the bistability range. (4) The macroscopic values of the times $\bar{\tau}_l$ and $\bar{\tau}_h$. The dc CVC of microstructures under voltage-controlled regime measured with the time of current averaging $t_m \gg \bar{\tau}_l$, $\bar{\tau}_h$ has a positive slope and a point of inflection. RTN is observable in samples of small size (micro- and nanostructures).

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