## Comment on "Strongly Interacting Photons in a Nonlinear Cavity"

In a recent Letter [1] Imamoğlu *et al.* have proposed to use electromagnetically induced transparency (EIT), in order to create a giant Kerr nonlinearity while keeping absorption negligible. This could find application in the design of optical quantum logical gates [2], or in controlling the quantum noise of very low intensity light beams [3]. The scheme involves $N$ four-level atoms driven on one transition by a strong resonant laser field $\Omega_d$, while two other transitions are coupled to a resonator mode, driven by a weak coherent field $\Omega_s$ (see inset of Fig. 1). Adiabatic elimination of *all* atomic degrees of freedom yields an effective Hamiltonian $H_{\text{eff}}$ for the cavity mode alone, which describes physically cavity driving and damping, as well as a very large Kerr nonlinearity. The authors conclude that this system would implement a turnstile device for single photons, as the large dispersive nonlinearity would not allow more than one photon to enter the resonator at a time. These results hinge in a crucial way on the validity of the adiabatic elimination procedure. The purpose of this Comment is to point out that the adiabatic Hamiltonian $H_{\text{eff}}$ [Eq. (2) of [1]] does not give a complete account of the dynamics of the system.

Here we argue that adiabatic elimination is an approximation based on the existence of different time scales, and should be independent of the relative size of the fluctuations of the variables involved. We have thus chosen to examine the behavior of $H_{\text{eff}}$ in the case where the (weak) cavity field still contains enough photons so that its quantum fluctuations can be linearized. In that case, the preductions using $H_{\text{eff}}$ are in striking discrepancy with the ones obtained from a full four-level atomic model. Our model is a straightforward generalization of the one described in detail in Ref. [4], which was shown to be in very good agreement with recent experiments [3,5]. The relevant parameters of the model are the Rabi frequencies $\Omega_d$, $\Omega_s$, the one-photon coupling $g_1$, $g_2$, and the detuning $\Delta_{42}$. The decay rates are $\gamma_{\text{cav}}$ for the cavity, and $\gamma_{ij}$ for the transition from level $i$ to $j$. The cooperativity is defined as $C = N|g_1|^2/(\gamma_{\text{cav}}\gamma_{31})$. In Fig. 1 we have plotted a best squeezing spectrum, using either our linearized four-level model (full line) or the effective Kerr-effect Hamiltonian $H_{\text{eff}}$ (dashed line). A very dramatic feature emerges: though the noise reduction at zero frequency is roughly the same in both models, the *bandwidth* of the true atomic nonlinear response is extremely small. This feature has no critical dependence on parameter values.

An explanation for this is the following. The main point in the effective Hamiltonian derivation is that, due to EIT, the atomic polarization vanishes at first order if the cavity field is resonant with the 1-3 transition. However, slightly off-resonant frequency components see a very high refractive index, which switches them out of the cavity resonance [6]. The perturbative expansion underlying $H_{\text{eff}}$ is thus only valid very close to the center frequency component of the cavity field. On the other hand, $H_{\text{eff}}$ predicts a significant modification of the quantum fluctuations on the frequency scale of $\gamma_{\text{cav}}$. Such fast time response is not permitted by the atomic behavior displayed in Fig. 1, and therefore the conclusions of [1] are not warranted. Obviously, intriguing questions remain as to the behavior of the complete atomic model in the nonlinearized domain, and the work presented in [1] has significant merit to open them for investigation.

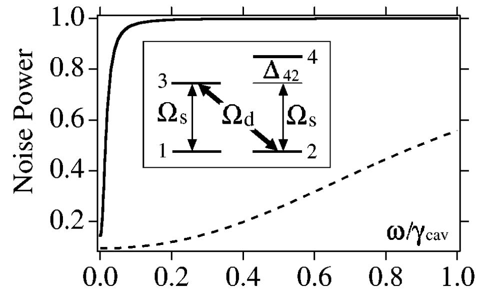


FIG. 1. Best squeezing spectrum for the system shown in the inset. The parameters $\Omega_d = \Delta_{42} = 30$, $\Omega_s = 0.1$, $C = 10^6$, $\gamma_{ij} = 1$, and $\gamma_{\text{cav}} = 0.1$ are deduced from Ref. [1]. The full line is from our four-level model, and the dashed line is from Ref. [1].

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