Electroweak Radiative Corrections to $b \to s\gamma$

Andrzej Czarnecki and William J. Marciano
Physics Department, Brookhaven National Laboratory, Upton, New York 11973
(Received 19 March 1998)

Two loop electroweak corrections to $b \to s\gamma$ decays are computed. Fermion and photonic loop effects are found to reduce $R \equiv \mathcal{B}(b \to s\gamma)/\mathcal{B}(b \to ce\nu)$ by $\sim(8 \pm 2)\%$ and lead to the standard model prediction $\mathcal{B}(B \to X_s\gamma) = (3.28 \pm 0.30) \times 10^{-4}$ for inclusive B meson decays. Comparison of $R^{\text{theory}} = (3.04 \pm 0.25) \times 10^{-3}(1 + 0.10\rho)$, where ρ is a Wolfenstein Cabibbo-Kobayashi-Maskawa parameter, with the current experimental average $R^{\text{exp}} = (2.52 \pm 0.52) \times 10^{-3}$ gives $\rho = -1.7 \pm 1.9$ which is consistent with $-0.21 \le \rho \le 0.27$ obtained from other B and K physics constraints. [S0031-9007(98)06553-3]

PACS numbers: 13.40.Hq, 12.15.Hh, 12.15.Lk

The inclusive radiative decay $b \to s\gamma$ provides a quantum loop test of the standard model and sensitive probe of new physics [1]. That flavor changing neutral current reaction proceeds via virtual top, charm, and up quark penguin diagrams (see Fig. 1). As such, it depends on and can provide a determination of the quark mixing parameters $|V_{ts}V_{tb}|$ as well as a test of Cabibbo-Kobayashi-Maskawa (CKM) unitarity. Alternatively, a clear deviation from expectations could suggest evidence for additional "new physics" loops [2] from supersymmetry, charged Higgs scalars, anomalous W couplings, etc.

The CLEO Collaboration has published a branching ratio

$$\mathcal{B}(B \to X_s\gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4} \quad (\text{CLEO}), \tag{1}$$

and reported a preliminary update $2.50 \pm 0.47 \pm 0.39$ (CLEO II), for inclusive radiative B meson decays produced at the $\Upsilon(4S)$ resonance [generally assumed to be a 50-50 mixture of $B^{\pm}$ and B^0 ($\bar{B}^0$)] [3]. The ALEPH Collaboration has reported [4]

$$\mathcal{B}(h_b \to X_s\gamma) = (3.11 \pm 0.80 \pm 0.72) \times 10^{-4} \quad (\text{ALEPH}) \tag{2}$$

for inclusive $h_b = b$ hadrons (mesons and baryons) originating from Z decays. These two measurements are quite different and should not be expected to give identical results. A better ratio for comparing distinct experiments and theory is [5]

$$R \equiv \frac{\Gamma(b \to s\gamma)}{\Gamma(b \to ce\nu)} = \frac{\mathcal{B}(b \to s\gamma)}{\mathcal{B}(b \to ce\nu)} \simeq 0.96\,\frac{\mathcal{B}(B \to X_s\gamma)}{\mathcal{B}(B \to X_c e\nu)}, \tag{3}$$

where the 0.96 is a nonperturbative conversion factor specific to B meson decays which incorporates $1/m_b^2$ and $1/m_c^2$ corrections [6]. We do not know the analogous conversion factor for h_b decays at the Z pole, but assume it is not very different since h_b consists of about 75% B (B_u and B_d) mesons.

R should be less sensitive to the b parentage and other systematic uncertainties, as long as numerator and denominator have a common experimental origin. Also, from a theoretical perspective, R is less sensitive to b quark mass uncertainties which largely cancel, modulo QCD corrections, in the ratio [5].

Employing (after subtracting out a small $b \to ue\nu$ component)

$$\mathcal{B}(B \to X_c e\nu) = 10.34\% \pm 0.46\% \quad [\text{at } \Upsilon(4S)], \tag{4}$$

$$\mathcal{B}(h_b \to X_c e\nu) = 10.96\% \pm 0.20\% \quad (\text{at } Z \text{ pole}), \tag{5}$$

[Eq. (4) is from Ref. [7], and Eq. (5) is from Ref. [8]] leads to

$$\begin{aligned} R &= (2.42 \pm 0.59) \times 10^{-3} \quad (\text{CLEO II}), \\ R &= (2.84 \pm 1.06) \times 10^{-3} \quad (\text{ALEPH}), \end{aligned} \tag{6}$$

where all errors have been added in quadrature. [We cannot explain why the leptonic branching ratio in (5) appears to be somewhat larger than (4). It may be indicative of an underlying systematic normalization uncertainty [8].]

The results in (6) are consistent and can be combined to give

$$R^{\text{exp}} = (2.52 \pm 0.52) \times 10^{-3}. \tag{7}$$

That finding should be compared with the QCD refined standard model prediction

$$R^{\text{QCD}} = \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2}\,\frac{6\alpha_{\text{em}}F}{\pi g(m_c^2/m_b^2)}\,|D^{\text{eff}}|^2, \tag{8}$$

where we employ the notation and results of Chetyrkin,

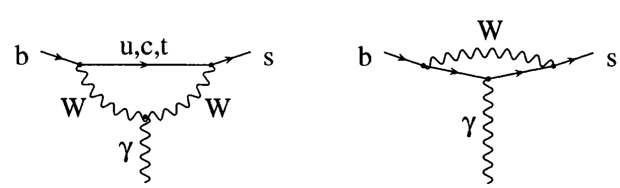

FIG. 1. One loop diagrams which give rise to the radiative decay $b \to s\gamma$.

Misiak, and Münz [5]. In that expression, g is a semileptonic decay phase space factor, F is a QCD correction, and $D^{\rm eff}$ is an effective $b \to s\gamma$ amplitude coupling (including gluon bremsstrahlung) corrected up to next-to-leading order in QCD [5,9–14] (our $|D^{\rm eff}|^2$ corresponds to $|D|^2 + A$ in Refs. [5,15,16]). We employ an update of $|D^{\rm eff}|^2$ consistent with the results in Refs. [15,16].

The electromagnetic coupling $\alpha_{\rm em} = e^2(\mu)/4\pi$ in (8) was renormalized at a short-distance scale μ_W, $m_b < \mu_W < m_W$ by the authors in Ref. [5]

$$\alpha_{\rm em}^{-1} = \alpha^{-1}(\mu_W) = 130.3 \pm 2.3 \tag{9}$$

and used in subsequent studies [15,16]. Since those analyses did not address QED corrections to R, the renormalization of $\alpha(\mu)$ was largely arbitrary. However, *a priori*, one should expect the fine structure constant $\alpha = 1/137.036$, renormalized at $q^2 = 0$, to be more appropriate for real photon emission. Our subsequent loop study confirms that expectation.

Employing [5,15,16]

$$\begin{aligned} \frac{|V_{ts}^* V_{tb}|}{|V_{cb}|} &= 0.976 \pm 0.010\,, \\ \frac{m_c(\text{pole})}{m_b(\text{pole})} &= 0.29 \to g(m_c^2/m_b^2) = 0.542\,, \\ F &= 0.926, \qquad |D^{\rm eff}| = 0.373\,, \end{aligned} \tag{10}$$

the value of $\alpha_{\rm em}$ in (9) and error analysis in Refs. [15,16] lead to

$$\begin{aligned} R^{\rm QCD} &= (3.32 \pm 0.27) \times 10^{-3}, \\ \mathcal{B}(B \to X_s\gamma) &= (3.58 \pm 0.33) \times 10^{-4}, \end{aligned} \tag{11}$$

which differ from the experimental results by about $1.4 \sim 1.6\sigma$.

Comparison of $R^{\rm exp}$ and $R^{\rm QCD}$ has been used to constrain or suggest the presence of new physics. It is a particularly sensitive probe of supersymmetry [17,18] and charged Higgs loops [2,16,19]. Future high statistics $\Upsilon(4S)$ studies are expected to further improve $R^{\rm exp}$ and lead to an interesting confrontation with supersymmetry expectations. Indeed, one can anticipate the error on $R^{\rm exp}$ to be reduced to about $\pm 10\%$ and, in the long term, perhaps $\pm 5\%$ is feasible. It is, therefore, important to fine-tune the theoretical prediction for R as much as possible. With that goal in mind, we examine here the $\mathcal{O}(\alpha)$ electroweak corrections to R.

A study of the $\mathcal{O}(\alpha)$ corrections to R must entail two loop contributions to $\Gamma(b \to s\gamma)$ as well as one loop corrections to $\Gamma(b \to ce\nu)$. A complete calculation of all contributions would be very difficult and would have to confront long distance hadronic uncertainties as well as experimental acceptance conditions. Such a thorough undertaking is not yet warranted by the data and may not even be required at the $\pm 5\%$ level. It is, however, important to consider potentially large effects and estimate their modification of R. Here, we examine two such classes of corrections: contributions from fermion loops in gauge boson propagators (γ and W) and short-distance photonic loop corrections. Examples of those effects for $b \to s\gamma$ are illustrated in Figs. 2 and 3. Analogous one loop corrections to $b \to ce\nu$ are pictured in Fig. 4.

We first present our result and then comment on the origin and magnitude of the various corrections. We find $R^{\rm theory} = R^{\rm QCD+EW}$ is given by

$$\begin{aligned} R^{\rm theory} = R^{\rm QCD}\left[\frac{\alpha}{\alpha_{\rm em}}\right]\Bigg\{1 - \frac{1}{|D^{\rm eff}|}\frac{\alpha_{\rm em}}{\pi}\left[\frac{1}{s_W^2} f\left(\frac{m_t^2}{m_W^2}\right) - \frac{8}{9} C_7^0\left(\frac{m_t^2}{m_W^2}\right)\ln\left(\frac{m_W}{m_b}\right) + \frac{104}{243}\ln\left(\frac{m_W}{m_b}\right)\right]\Bigg\} \\ \times \left[1 - \frac{2\alpha_{\rm em}}{\pi}\ln\left(\frac{m_Z}{m_b}\right)\right], \end{aligned} \tag{12}$$

where $\alpha_{\rm em}$ and $|D^{\rm eff}|$ can be found in (9) and (10) while

$$\begin{aligned} \alpha^{-1} &= 137.036\,, \qquad s_W^2 \equiv \sin^2\theta_W \simeq 0.23\,, \\ f(x) &= \frac{\pi^2(2-3x)(2-x)x}{48} + \frac{x(176 - 373x + 491x^2 - 468x^3 + 72x^4)}{192(1-x)^3} \\ &\quad + \frac{(2-x)x^2(5 - 14x + 11x^2 - 3x^3)}{8(1-x)^3}\,\mathrm{Li}_2\left(1 - \frac{1}{x}\right) - \frac{x(2+x)(7+16x-47x^2)}{96(1-x)^2}\ln(x-1) \\ &\quad + \frac{x(80 - 115x + 200x^2 - 425x^3 + 220x^4 - 83x^5)}{96(1-x)^4}\ln x + \frac{x^2(4-5x-3x^2)}{16(1-x)^3}\ln x \\ &\quad \times \left[\frac{5 - 3x + x^3}{(1-x)^2}\ln x - (2+x)\ln(x-1)\right] \simeq 0.77\,, \\ C_7^0(x) &= \frac{3x^3 - 2x^2}{4(x-1)^4}\ln x - \frac{8x^3 + 5x^2 - 7x}{24(x-1)^3} \simeq -0.19 \quad (\text{for } \overline{m}_t = 167 \text{ GeV})\,. \end{aligned} \tag{13}$$

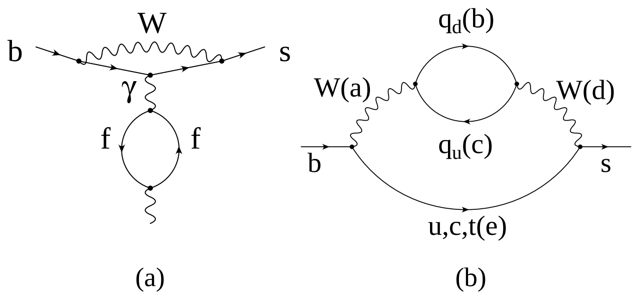

FIG. 2. (a) An example of vacuum polarization renormalization of α by the fermion loops. (b) Fermionic loop corrections to $b \to s\gamma$. Letters in parentheses label distinct lines from which the photon can be emitted. There is also a contribution from leptons in the W propagator loop.

The first correction factor in (12)

$$\frac{\alpha}{\alpha_{\rm em}} = \frac{130.3}{137.036} = 0.951 \tag{14}$$

is due to electric charge renormalization, illustrated in Fig. 2(a). Fermion loops decrease the value of $\alpha_{\rm em}(\mu)$ by 5% in going from the short-distance renormalization condition of Ref. [5] to the more appropriate $q^2 = 0$ physical condition. This effect is somewhat trivial in origin, but does represent an important 5% reduction neglected in recent analyses. Note that, unlike $\alpha_{\rm em}$, the physical fine structure constant has essentially no uncertainty.

The $f(m_t^2/m_W^2)$ term in (12) comes from fermion loop corrections in the W propagator and $WW\gamma$ vertex as illustrated in Figs. 2(b) and 4. It leads to a decrease in R by about 2.2%. Three loop QCD effects reduce that contribution by a factor $[\alpha_s(m_W)/\alpha_s(m_b)]^{16/23} \simeq 0.7$ [20].

The $\ln(m_W/m_b)$ terms in (12) originate from photonic corrections to $b \to s\gamma$ of the type illustrated in Fig. 3. In the effective theory language, the coefficients of those logs are given by the anomalous dimensions γ_{77} and γ_{27}, of which the latter is more difficult to compute, but can be obtained from analogous QCD calculations [12,14] using the translation $\alpha_s \to -\alpha/6$. Taken together, those terms reduce R by about 1%. Higher order leading QCD corrections decrease that result by a factor of $\sim$0.55 [21].

The final $\ln(m_Z/m_b)$ correction in (12) stems from short-distance photonic corrections to $b \to ce\nu$ of the type illustrated in Fig. 4. That type of correction is generic to all semileptonic charged current decays [22]. It is generally factored out of semileptonic B decays before the extraction of $|V_{cb}|$; hence, it must be explicitly included here [23]. That effect reduces R by about 1.4%.

All of the above corrections have the same sign and when taken together reduce R by about 8.3%. There are other electroweak (EW) radiative corrections of $\mathcal{O}(\alpha/\pi)$ and $\mathcal{O}(G_\mu m_t^2/8\sqrt{2}\,\pi^2)$ [24] as well as additional three loop QCD effects which we have not computed. We have examined some of those corrections and found typically $\lesssim \mathcal{O}(0.5\%)$ contributions of both signs. In addition, long-distance QED effects including two photon radiation could be several percent. For example, in Ref. [23] it was shown that final state Coulomb interactions are likely to increase $\Gamma(B^0 \to X_c e\nu)$ by about 2% relative to $\Gamma(B^+ \to X_c e\nu)$. However, currently such corrections may be effectively absorbed in $|V_{cb}|$ and its uncertainty, since experimental studies do not address their presence. Hence, we do not include them here. We see no reason for the sum of neglected effects to be particularly large but, nevertheless, assign a conservative $\pm 2\%$ uncertainty to them. In that way, we find

$$R^{\rm theory} = R^{\rm QCD+EW} \simeq (0.917 \pm 0.020) R^{\rm QCD}. \tag{15}$$

Since $R^{\rm QCD}$ previously contained a $\pm 1.8\%$ uncertainty due to $\alpha_{\rm em}$ [5] which our analysis does not have, the overall theoretical uncertainty in R remains essentially unchanged. From (15) and (11), we find the reduced predictions

$$\begin{aligned} R^{\rm theory} &= (3.04 \pm 0.25) \times 10^{-3}, \\ \mathcal{B}(B \to X_s\gamma)^{\rm theory} &= (3.28 \pm 0.30) \times 10^{-4}, \end{aligned} \tag{16}$$

which are in better agreement with experiment.

Rather than using the value of $|V_{ts}^* V_{tb}|/|V_{cb}|$ in (10), we can employ three generation unitarity which implies

$$\frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} = |V_{tb}|^2 \left(1 - \frac{|V_{td}|^2 - |V_{ub}|^2}{|V_{cb}|^2} \right) \tag{17}$$

or using $|V_{cb}| = 0.039 \pm 0.003$, and the Wolfenstein parametrization [25,26] of the CKM matrix

$$\frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} = 0.950(1 + 0.10\rho). \tag{18}$$

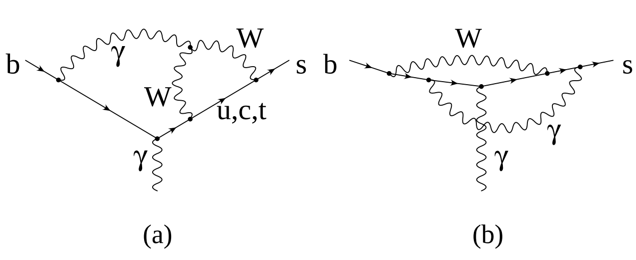

FIG. 3. Examples of the two-loop diagrams where a virtual photon exchange gives a short-distance logarithmic contribution.

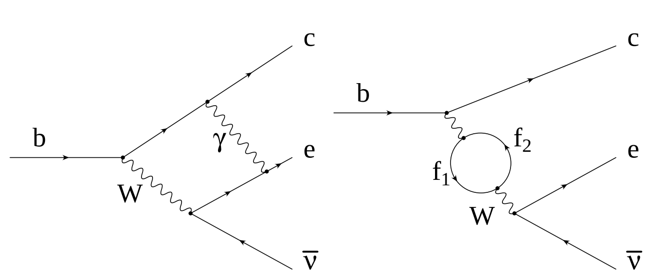

FIG. 4. Examples of electroweak corrections to the decay $b \to ce\nu$.

Constraints from $b \to ue\nu$, $B^0 - \bar{B}^0$ oscillations, and $K^0 - \bar{K}^0$ mixing currently limit ρ to [26]

$$\rho \simeq 0.03 \pm 0.24, \qquad (19)$$

and the corresponding range for $|V_{ts}^* V_{tb}|/|V_{cb}|$ in (10). However, keeping ρ arbitrary, we find

$$R^{\text{theory}} = [(3.03 \pm 0.25) \times 10^{-3}](1 + 0.10\rho), \qquad (20)$$

which on comparison with (7) implies

$$\rho = -1.7 \pm 1.9. \qquad (21)$$

That result is consistent with (19). Future $\pm 5\%$ measurements of R^{exp} would determine ρ to ± 0.5. Although not competitive with other methods for pinpointing ρ, such studies would be very valuable for constraining or providing evidence for new physics.

In summary, we have found a $(8 \pm 2)\%$ reduction in the standard model prediction for R^{theory} due to EW radiative corrections. That result improves agreement with experiment,

$$R^{\text{exp}}/R^{\text{theory}} = 0.83 \pm 0.18. \qquad (22)$$

It can be used to constrain new physics effects such as supersymmetry; however, those studies are beyond the scope of this paper. Further reduction in the $\pm 8\%$ uncertainty of R^{theory} will require a better determination of $m_c(\text{pole})/m_b(\text{pole})$ (currently the main theoretical uncertainty), perhaps by studies of semileptonic B decay spectra. This uncertainty could also be reduced by re-analyzing QCD corrections to $b \to s\gamma$ using low-scale running quark masses [27].

We look forward to future improved measurements of $\mathcal{B}(B \to X_s\gamma)$ and $\mathcal{B}(B \to X_c e\nu)$ and their confrontation with theory.

A C. thanks K. Melnikov for collaboration at an early stage of this project and M. Misiak for helpful remarks. This work was supported by the DOE Contract No. DE-AC02-98CH10886.

[1] B. A. Campbell and P. J. O'Donnell, Phys. Rev. D **25**, 1989 (1982).

[2] J. Hewett, in *Proceedings of the SLAC Summer Institute: Spin Structure in High Energy Processes* (SLAC, Stanford, 1993), p. 463.

[3] M. S. Alam *et al.,* Phys. Rev. Lett. **74**, 2885 (1995); R. Briere (private communication).

[4] ALEPH Collaboration, R. Barate *et al.,* Report No. CERN-EP/98-044 (unpublished).

[5] K. Chetyrkin, M. Misiak, and M. Münz, Phys. Lett. B **400**, 206 (1997).

[6] M. B. Voloshin, Phys. Lett. B **397**, 275 (1997); Z. Ligeti, L. Randall, and M. B. Wise, Phys. Lett. B **402**, 178 (1997); A. K. Grant, A. G. Morgan, S. Nussinov, and R. D. Peccei, Phys. Rev. D **56**, 3151 (1997); G. Buchalla, G. Isidori, and S. J. Rey, Nucl. Phys. **B511**, 594 (1998).

[7] B. Barish *et al.,* Phys. Rev. Lett. **76**, 1570 (1996).

[8] M. Feindt, contribution to the International Europhysics Conference on High-Energy Physics, Jerusalem, Israel, 1997 (unpublished).

[9] A. Ali and C. Greub, Z. Phys. C **49**, 431 (1991); Phys. Lett. B **259**, 182 (1991); **361**, 146 (1995).

[10] K. Adel and Y.-P. Yao, Phys. Rev. D **49**, 4945 (1994).

[11] A. J. Buras, M. Misiak, M. Münz, and S. Pokorski, Nucl. Phys. **B424**, 374 (1994).

[12] B. Grinstein, R. Springer, and M. B. Wise, Nucl. Phys. **B339**, 269 (1990).

[13] C. Greub, T. Hurth, and D. Wyler, Phys. Rev. D **54**, 3350 (1996).

[14] M. Ciuchini, E. Franco, L. Reina, and L. Silvestrini, Nucl. Phys. **B421**, 41 (1994).

[15] A. J. Buras, A. Kwiatkowski, and N. Pott, Phys. Lett. B **414**, 157 (1997); update hep-ph/9707482 v3.

[16] M. Ciuchini, G. Degrassi, P. Gambino, and G. F. Giudice, hep-ph/9710335 (unpublished).

[17] J. L. Hewett and J. D. Wells, Phys. Rev. D **55**, 5549 (1997).

[18] R. Garisto and J. N. Ng, Phys. Lett. B **315**, 372 (1993).

[19] T. M. Aliev and E. O. Iltan, hep-ph/9803272 (unpublished).

[20] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Phys. Rev. D **18**, 2583 (1978).

[21] After the first version of this paper was posted, A. L. Kagan and M. Neubert, hep-ph/9805303, found that leading QCD corrections reduce the QED short-distance logs by a factor of 0.55.

[22] A. Sirlin, Nucl. Phys. **B196**, 83 (1982).

[23] D. Atwood and W. J. Marciano, Phys. Rev. D **41**, R1736 (1990).

[24] After the completion of this work, an e-print by A. Strumia, hep-ph/9804274, examined the large m_t limit of radiative corrections to R^{QCD}. For fermion loops due to the top quark, where comparison is possible, the result is in accord with the leading large x behavior of our $f(x)$ in Eq. (13). Other nonfermionic loop $\mathcal{O}(G_\mu m_t^2)$ corrections to R given in that paper range from $\sim 0.6\%$ to 1.5% depending on the Higgs scalar mass. They are not included in our analysis but covered by the $\pm 2\%$ uncertainty.

[25] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[26] J. L. Rosner, contribution to the 5th International Workshop on Physics in Hadron Machines (Beauty 97), Santa Monica, CA, 1997; hep-ph/9801201 (unpublished).

[27] N. G. Uraltsev, Int. J. Mod. Phys. A **11**, 515 (1996).