Quantum Shot Noise at Local Tunneling Contacts on Mesoscopic Multiprobe Conductors

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New experiments which measure the low-frequency shot-noise spectrum at local tunneling contacts on mesoscopic structures are proposed. The current fluctuation spectrum at a *single* tunneling tip is determined by local partial densities of states. The current-correlation spectrum between *two* tunneling tips is sensitive to the *nondiagonal* density of states elements which are expressed in terms of products of scattering states of the conductor. Such an experiment permits the investigation of correlations of electronic wave functions. We present specific results for a clean wire with a single barrier and for metallic diffusive conductors. [S0031-9007(98)07203-2]

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Since the original implementation of scanning tunneling microscopy (STM) [1], a multitude of related scanning probe techniques [2,3] have permitted one to obtain an unprecedented wealth of information on the nanoscopic scale. It is the purpose of this paper to present theoretical predictions of the shot noise measured at a point tunneling contact. Shot noise arises due to the quantization of the charge in the presence of transport [4]. Measurements of the shot noise with a weak tunneling contact (such as the tip of a STM) are interesting not only because they would permit one to create a map of the spatial distribution of the shot noise but also, as we will show, because they permit a measurement of the correlation of wave functions. This is in contrast to conductance or tunneling measurements which are related to density of states and thus to absolute squares of wave functions. Below we show that an investigation of the current-current correlation at two tunneling contacts permits us to also extract information on the phase of an electronic wave function relative to that of another wave function.

The typical arrangement in which scanning tunneling microscopy is used to investigate surface effects corresponds to a two terminal setup: The sample provides one terminal and the tip provides the other terminal. In this case, the tunneling current is proportional to the local density of states $\nu(x)$ at the location of the tip. In this paper we consider a mesoscopic structure that supports a transport current. Thus the sample must already have at least two contacts which provide a source and sink for the carrier current (see Fig. 1). In this case we have to treat a three-terminal structure, and it depends in general on whether one is concerned with the tunneling conductance from the tip to the right or left contact. Instead of the total density of states, the tunneling conductance is related to a local partial density of states (LPDOS) $\nu(x,\alpha)$ defined in [5], where $\alpha = 1,2$ labels the contacts of the conductor. The DOS of the mth tranverse channel of reservoir α is $1/hv_{\alpha m}$ with the velocity $v_{\alpha m} = \sqrt{2(E_F - E^0_{\alpha m})/m^\star}$, where $m^\star$ is the effective electron mass, E_F is the Fermi energy, and $E^0_{\alpha m}$ is the threshold energy of the channel. With the help of the scattering states $\psi_{\alpha m}(x)$ incident from contact α, the LPDOS can be expressed as

$$\nu(x,\alpha) = \sum_{m\in\alpha} \frac{1}{hv_{\alpha m}} |\psi_{\alpha m}(x)|^2. \tag{1}$$

For a derivation of this result closely related to the discussion given below, we refer to Ref. [6]. The LPDOS determines the charge injected from contact α into a region at position x in response to an increase of the Fermi energy of contact α. We can thus refer to a LPDOS also as the injectivity of contact α. The local density of states (LDOS) is the sum of the injectivities of all contacts, $\nu(x) = \sum_\alpha \nu(x,\alpha)$. Below, as a first step, we show that the injectivities also determine the shot noise measured at a single tunneling contact [see Fig. 1 (only tip 1 is present)].

In a second step, we consider two tunneling contacts, a four-terminal geometry, and evaluate the correlation of the shot noise measured at these contacts. The correlation cannot be expressed with the help of the injectivities (which depend only on the absolute square of wave functions) but are determined by nondiagonal (nonlocal) elements of a density of states operator defined as

$$\nu(x,x',\alpha) = \sum_{m\in\alpha} \frac{1}{hv_{\alpha m}} \psi_{\alpha m}(x)\psi^\star_{\alpha m}(x'). \tag{2}$$

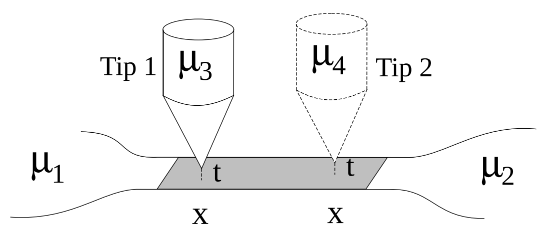

FIG. 1. Mesoscopic conductor with contacts at potentials μ_1 and μ_2 and a tunneling contact at potential μ_3, tip 1. A second tunneling contact tip 2 (dashed lines) at potential μ_4 is present only for the measurement of the current-correlation spectrum. The tunneling tips couple locally with strength t at the points x, respectively, x' to the wire.

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We note that these elements are not real but depend on the phase difference which the wave function accumulates between the location of the two tips at x and x'. The measurement of such a correlation thus permits the determination not only of the absolute square of the wave function but also of the phase of the wave function. In a recent paper, Byers and Flatté suggested a conductance experiment with two tunneling probes on a surface [7]. They found that to second order in the tunneling strength the current is determined by nondiagonal terms of the Greens functions, i.e., spatial correlations of the wave functions. In a conductance measurement spatial correlations represent a small correction to a dominant first order term. In contrast, in the shot noise experiment proposed here, the wave function correlations provide the leading term. We illustrate our results for two particular geometries: a ballistic wire which contains a single barrier and a metallic diffusive wire.

There has been continued strong interest in the shot noise of mesoscopic samples [4]. Since the initial experiments [8], the development of highly sensitive and accurate measurement techniques [9] has permitted a close comparison between experimental techniques and theoretical predictions [4,10,11]. It is thus justified to assume that similar techniques can be applied to the shot-noise measurement at tunneling contacts.

Our theoretical starting point is a general formula which expresses the shot noise in mesoscopic multiprobe conductors in terms of quadrupoles of scattering matrices [11]. The spectrum of the current correlations in two contacts α and β of a mesoscopic multiprobe conductor is defined as the Fourier transform of the current-current correlator, $S_{\alpha\beta}(\omega) = \int dt\, e^{i\omega t}\langle \Delta I_\alpha(t + t_0)\Delta I_\beta(t_0)\rangle$, where $\Delta I_\alpha(t) = I_\alpha(t) - \langle I_\alpha(t)\rangle$ is the fluctuation of the current in contact α away from its time average. In the low-frequency limit the correlation spectrum can be expressed in terms of the current matrix $A_{\delta\gamma}(\alpha) = \mathbf{1}_\alpha \delta_{\alpha\delta}\delta_{\alpha\gamma} - \mathbf{s}^\dagger_{\alpha\delta}(E)\mathbf{s}_{\alpha\gamma}(E)$ and the Fermi functions $f_\delta(E)$ of the electron reservoirs [11],

$$S_{\alpha\beta} = \frac{2e^2}{h} \sum_{\delta\gamma} \int dE\, \mathrm{Tr}[A_{\delta\gamma}(\alpha) A_{\gamma\delta}(\beta)] f_\delta(1 - f_\gamma)\,. \tag{3}$$

Here, $\mathbf{s}_{\alpha\beta}$ is the submatrix of the scattering matrix of the sample which describes scattering from all channels of contact β into the channels of contact α. We use Eq. (3) to find the fluctuation spectrum of the current at the tunneling tip, S_{33}, as shown in Fig. 1 (only tip 1 is present). The tip couples locally at a point x to the wire with a coupling strength t. We use the Hamiltonian formulation of the scattering matrix [12] to expand the scattering matrix of the full system (wire and tip) to the lowest order in the coupling strength t. The current fluctuations in the tip can then be expressed with the help of the scattering matrices of the two isolated systems and the coupling constant t. We assume an applied voltage $eV = \mu_1 - \mu_2$ at the two contacts of the wire and set the electrochemical potential at the tip $\mu_3 = [\nu(x,1)\mu_1 + \nu(x,2)\mu_2]/\nu(x)$ such that the average current into the tip vanishes [6]. With the two-terminal tip-to-sample conductance ($\mu_1 = \mu_2$) $G(x) = (e^2/h)4\pi^2\nu_{\mathrm{tip}}|t|^2\nu(x)$, where ν_{tip} is the LDOS of the isolated tip, we find, at zero temperature and in linear response to the applied potentials, the shot-noise spectrum

$$S_{33} = 2eG(x)V2\,\frac{\nu(x,1)}{\nu(x)}\left(1 - \frac{\nu(x,1)}{\nu(x)}\right). \tag{4}$$

Thus the noise is determined by $\nu(x,1)$, the injectivity of contact 1 at the coupling point x in the wire [Eq. (1)]. For small potential differences all densities have to be taken at the Fermi energy. Equation (4) suggests that the ratio $\nu(x,1)/\nu(x)$ plays the role of an effective local distribution function. It is an exact quantum mechanical quantity which contains information on the carrier propagation from contact 1 all the way to the point of observation. This is in contrast to the distribution functions used in the semiclassical Boltzmann equation approach [4,13,14] which contain no phase information. We now illustrate Eq. (4) for the case of a ballistic one channel conductor with a δ barrier at $x = 0$ leading to the transmission probability $T = 0.7$ and reflection probability $R = 0.3$. The injectivity of the left contact is

$$\nu(x,1) = \begin{cases} \frac{1}{hv}[1 + R + 2\sqrt{R}\cos(2k_F x + \phi)] & x < 0 \\ \frac{1}{hv} T & x > 0 \end{cases}, \tag{5}$$

where ϕ is the phase acquired by reflected particles. This injectivity together with the LDOS and the current fluctuations, is shown in Fig. 2.

As a function of the tip position x, the fluctuation spectrum

$$S_{33} \propto T\left(1 - \frac{T}{2}\,\frac{1}{1 + \sqrt{1 - T}\cos(2k_F x + \phi)}\right) \tag{6}$$

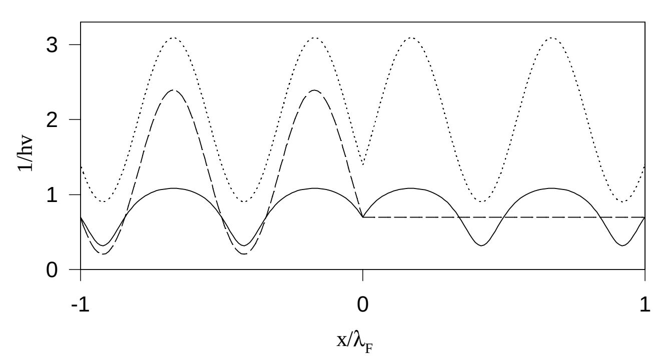

FIG. 2. Spatial variation of the current fluctuations and L(P)DOS of a ballistic wire with a δ barrier with transmission probability $T = 0.7$. The distance x is measured relative to the barrier at $x = 0$. The injectivity of the left contact $\nu(x,1)$ (dashed line) and the LDOS $\nu(x)$ (dotted line) are measured in units of $1/hv$. The solid line is the current fluctuation spectrum S_{33} [Eq. (6)].

shows an oscillating behavior with the period of half a Fermi wavelength $\lambda_F = 2\pi/k_F$. If we average this spectrum over one oscillation period, we find $\langle S_{33}\rangle_{\text{ave}} \propto T(1 - \sqrt{T}/2)$. Note that this differs from the fluctuation spectrum that would be measured at a massive contact, $S_{11} \propto T(1 - T)$. The dependence on $\sqrt{T}$ instead of T has its origin in the interference of incident and reflected waves. It is tempting to say that the fluctuations in the tip reflect directly the intrinsic fluctuations in the wire. Note, however, that even though a perfect ballistic wire ($T = 1$) shows no shot noise the current in a tip which probes such a wire would fluctuate. For $T = 1$, the right-hand side of Eq. (6) does not vanish but is $1/2$.

As a second example, we investigate a metallic diffusive wire. The diffusive wire extends from $x = 0$ to $x = L$, and has a width W much smaller than its length L. For the ensemble averaged quantities, the diffusion can then be considered to be one dimensional. Furthermore, we assume that $k_F l \gg 1$ with the elastic mean free path $l \ll L$. The ensemble averaged injectivities of the two contacts of the wire are in the diffusive region [5] $\nu(x,1) = \nu_0 \frac{L-x}{L}$ and $\nu(x,2) = \nu_0 \frac{x}{L}$, where $\nu_0 = m^\star/2\pi\hbar^2$ is the two-dimensional density of states and $m^\star$ is the effective electron mass. In particular, the injectivities are independent of the transverse coordinate. Using these densities in Eq. (4) gives a parabolic dependence of the fluctuation spectrum on the tip position,

$$S_{33} \propto x(L - x)/L^2. \tag{7}$$

Note that if we average this spectrum over the entire wire (from $x = 0$ to $x = L$) this leads to a noise spectrum which is one-third of that measured at a tunneling contact of a perfect ballistic wire. Again, we have the surprising similarity to the well-known one-third reduction of the shot noise at an isolated metallic diffusive conductor [13,15,16].

Next we investigate the spectrum S_{34} of the correlations of the currents in two tips which couple at positions x and x' to a wire [Fig. 1 (tip 1 and tip 2 are present)]. Again, we can start from the general formula, Eq. (3), and expand the scattering matrix of the entire system to the lowest order in the coupling strength t. The result for the correlation spectrum depends in general on the electrochemical potentials at all four contacts of the system. Here, we specialize to three different configurations of the applied voltages. We call these configurations experiments A, B, and C. All potentials are held at the equilibrium value μ_0. In experiment A, we raise the potential of the left contact of the wire (contact 1) to the elevated value μ, so that current is injected into the system through this contact. In experiment B we raise only the potential of the right contact of the wire (contact 2) to the value μ, all others are held at the equilibrium potential μ_0. In experiment C we raise simultaneously the potentials of both sides of the wire (contacts 1 and 2) to the value μ. Comparison of the correlations of experiments A, B, and C permits one to identify the exchange correlations, i.e., the effect due to the quantum mechanical indistinguishability of particles [11]. Exchange effects in metallic diffusive conductors with wide contacts are the subject of Refs. [14] and [17]. Ballistic cavities with four tunneling contacts are investigated in Ref. [18]. An experiment by Liu *et al.* [19] measures exchange effects in an open ballistic structure. At $kT = 0$ and in linear response to the applied bias $eV = \mu - \mu_0$, we find for the correlation spectrum at the two tunneling tips the following:

$$S_{34} = 2\frac{e^2}{h} eV 16\pi^4 \nu_{\text{tip}1}\nu_{\text{tip}2}|t|^4 S_{A,B,C}, \tag{8}$$

with $\nu_{\text{tip}\alpha}$ being the LDOS in tip α and

$$S_A = -2\left|\sum_{m\in 1}\frac{1}{hv_{1m}}\psi_{1m}(x)\psi^\star_{1m}(x')\right|^2, \tag{9}$$

$$S_B = -2\left|\sum_{n\in 2}\frac{1}{hv_{2n}}\psi_{2n}(x)\psi^\star_{2n}(x')\right|^2, \tag{10}$$

$$S_C = -2\left|\sum_{\alpha=1,2}\sum_{m\in\alpha}\frac{1}{hv_{\alpha m}}\psi_{\alpha m}(x)\psi^\star_{\alpha m}(x')\right|^2$$
$$= S_A + S_B - 4\sum_{\substack{m\in 1\\ n\in 2}}\frac{1}{h^2 v_{1m}v_{2n}}$$
$$\times \mathrm{Re}\{\psi_{1m}(x)\psi^\star_{2n}(x)\psi^\star_{1m}(x')\psi_{2n}(x')\}. \tag{11}$$

These equations, together with Eq. (4), are the main results of this Letter. Here, $\psi_{\alpha m}(x)$ is the scattering state describing an incoming electron in channel m of contact α which is scattered into all channels of both contacts of the wire. The sums are over all open channels in contact 1, respectively, contact 2.

To arrive at these results which express the noise correlations in terms of scattering states, we proceed as follows: We express the scattering matrix in Eq. (3) in terms of the Greens function of the four-probe sample (wire and tips) and the coupling matrix which couples the ideal leads to the mesoscopic sample. We expand the Greens function to first order in the weak links t between the tips and the wire. The scattering states are finally related to the Greens function of the sample and the coupling matrix between the leads and the sample by a Lippmann-Schwinger equation. Note that, with the help of the injectivity operator Eq. (2), we can express Eqs. (9)–(11) in the following compact form: $S_A = -2|\nu(x,x',1)|^2$, $S_B = -2|\nu(x,x',2)|^2$, and $S_C = -2|\nu(x,x',1) + \nu(x,x',2)|^2$.

We now use the results [Eqs. (9)–(11)], to investigate the current correlations for the diffusive wire discussed above. We are interested in the correlations averaged over impurity configurations. For the averaging procedure we assume that the distance between the two tips and between each tip and the boundaries of the diffusive region is much larger than the elastic mean free path l. We express the wave functions in terms of four Greens functions and use the diagram technique to average the products of Greens functions [20]. It turns out that, for all three experiments, the strongest contribution to the averaged quantity comes from diagrams which contain four diffusons [17]. Diagrams with two and three diffusons are small as l/L, respectively, $(l/L)^2$. With the abbreviation $a(x,x') = 1/3[(x - x')^2 - 2x'(L - x)]$, the leading order terms are

$$S_A = \frac{S_C}{2} \frac{(L - x)^2 + (L - x')^2 + a(x,x')}{L^2}, \quad (12)$$

$$S_B = \frac{S_C}{2} \frac{x^2 + x'^2 + a(x,x')}{L^2}, \quad (13)$$

$$S_C = -2 \frac{(m^\star)^2}{(\pi \hbar^2)^2 N} \frac{L}{l} \frac{x(L - x')}{L^2} = -4 \frac{\nu(x,2)\nu(x',1)}{g}, \quad (14)$$

where $g = \frac{l}{2L} N$ is the Drude conductance and $N = k_F W$ is the number of channels. Here, we assumed that tip 1 is positioned to the left of tip 2. At once we see that, even after averaging over the impurity configurations, the result of experiment C is not just the addition of experiments A and B. In fact, it is interesting to determine the strength of the exchange term $S_X = S_C - S_A - S_B$. In general, this expression depends on the two coordinates x and x'. We investigate it closer for the special case where the tips are placed symmetrically around the center of the wire, $L/2$, i.e., tip 1 is placed at a distance $d/2$ to the left of the center and tip 2 is placed at the same distance $d/2$ to the right of the center. As a function of the distance d between the tips the relative strength of the exchange term is

$$\frac{S_X}{S_C} = \frac{1}{3}\left[2 + \frac{d}{L} - 2\left(\frac{d}{L}\right)^2\right]. \quad (15)$$

Interestingly, this function reaches its maximum not when the tips are closest but at the finite distance $d = L/4$ (which is still large compared to l). Its maximal value is $(S_X/S_C)_{\max} = 17/24$. For any two tip positions x and x', the exchange term S_X is always negative and therefore enhances the current correlations. An enhancement of the current correlations by the exchange term was also predicted for a chaotic cavity with four tunneling contacts [18].

In conclusion, we have shown that noise measurements at local tunnel junctions can reveal considerably more information about the electronic structure of a mesoscopic system than is accessible to pure conductance measurements. In the case of a single tip the shot noise is determined by an effective local distribution function, $\frac{\nu(x,\alpha)}{\nu(x)}$. Especially interesting are the current correlation spectra of two tips. For the three suggested experiments, Eqs. (9)–(11), they depend directly on the phase-carrying amplitudes of the wave functions. These experiments can be used to demonstrate the importance of the exchange correlation due to the indistinguishability of the electrons. We have shown that, for a metallic diffusive wire, the exchange term always gives a negative contribution to the correlation spectrum (enhances the effect) which can be as high as 70% of the total correlation spectrum.

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