Approach to Perturbative Results in the N-Δ Transition

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We show that constraints from perturbative QCD (PQCD) calculations play a role in the nucleon to $\Delta(1232)$ electromagnetic transition even at moderate momentum transfer scales. The PQCD constraints, tied to real photoproduction data and unseparated resonance response functions, lead to explicit forms for the helicity amplitudes wherein the $E2/M1$ ratio remains small at moderately large momentum transfer. [S0031-9007(98)07140-3]

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The nucleon-$\Delta(1232)$ electromagnetic transition at very low momentum transfer is mainly a magnetic dipole ($M1$ or M_{1+}) transition, with a small electric quadrupole ($E2$ or E_{1+}) component in addition [1]. The so-called electromagnetic ratio (EMR), which is the ratio E_{1+}/M_{1+} at the Δ peak, is a few percent in magnitude and negative in this region. In contrast, at very high momentum transfer, perturbative QCD (PQCD) should be valid and demands that this ratio approach unity [2]. How should the $M1$ and $E2$ amplitudes interpolate between these two extremes? In particular, can the PQCD results be in any way relevant at momentum transfers experimentally reachable now or in the near future? That is the concern of this paper.

One clear issue is that the existing experiments, with momentum transfers up to 3.2 GeV^2 [3], do not show any hint that $E2/M1$ is becoming positive and large, when one examines what are considered the most sophisticated analyses [4]. The two analyses of Ref. [4] give $E_{1+}/M_{1+} = 0.06 \pm 0.02 \pm 0.03$ and 0.0 ± 0.14, respectively, at 3.2 GeV^2. Indeed, there is a school of thought holding that the PQCD limit is not experimentally reachable in exclusive reactions [5]. On the other hand, PQCD predictions for scaling [6–8] and normalization [9,10] are mirrored by the data in many cases.

We shall here examine the point of view that PQCD results can be relevant to moderate momentum transfer squared (q^2) exclusive reactions, and that considering the approach to the PQCD limit can be useful in understanding lower q^2 data being studied at present. We shall interpolate between the very low and very high q^2 domains using analytic functions motivated by simplicity, and using what is known about the threshold behavior, the $q^2 = 0$ point, and the asymptotic PQCD limit as anchor points. The latter includes the known scaling laws [2,11] and normalized leading twist calculations [12–14], where possible. Future data will throw further light on the validity of our interpolations.

Our choice to study the Δ transition amplitudes is motivated by their special characteristic. Δ electromagnetic production falls relative to continuum with increasing q^2, in contrast to other resonances where the resonance signal relative to background is roughly constant. This is mirrored in the normalized PQCD calculation of the leading twist helicity amplitudes, which show that asymptotic N-Δ transition amplitude is anomalously small.

Before showing our interpolations, let us review the notations. The transverse electromagnetic helicity amplitudes $A_{1/2}$ and $A_{3/2}$ for the N-Δ transition are related to the multipole amplitudes by [15]

$$M1 = -\frac{1}{2} A_{1/2} - \frac{\sqrt{3}}{2} A_{3/2} ,$$
$$E2 = -\frac{1}{2} A_{1/2} + \frac{1}{2\sqrt{3}} A_{3/2} . \tag{1}$$

At very high $Q^2 = -q^2$, PQCD predicts the scaling behavior of the helicity amplitudes to be [2,12]

$$A_{1/2} \propto \frac{1}{Q^3} ,$$
$$A_{3/2} \propto \frac{1}{Q^5} , \tag{2}$$

modulo $\log(Q^2)$ factors. Hence, the asymptotic ($Q^2 \to \infty$) prediction that $E2/M1 \to 1$ follows.

We have already mentioned that the asymptotic N-Δ transition amplitude is small. In order to realize how small the asymptotic N-Δ transition amplitude actually is [6,16], one should quote the electromagnetic transition amplitudes for the elastic and quasielastic cases in comparable fashions. One can translate the leading amplitude in all cases into the helicity amplitude G_+,

$$G_+ = \frac{1}{2m_N} \left\langle R, \lambda' = \frac{1}{2} \left| \epsilon_\mu^{(+)} \cdot j^\mu(0) \right| N, \lambda = \frac{1}{2} \right\rangle , \tag{3}$$

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where $\epsilon_\mu^{(+)}$ is the photon polarization vector for photon helicity $+1$, j^μ is the electromagnetic current, λ refers to helicity, and the overall mass factor is included to make G_+ dimensionless. Asymptotically, G_+ scales the same way as $A_{1/2}$.

For the elastic case we relate G_+ to G_M by

$$Q^3 G_+(p \to p) = \frac{1}{m_N\sqrt{2}} Q^4 G_M \approx 0.75 \text{ GeV}^3, \quad (4)$$

where the last part is valid at large q^2 and the numerical value comes both from data and from calculations using any of the standard nucleon distribution amplitudes mentioned below. For nucleon-resonance transitions, the result is most commonly quoted in terms of the helicity amplitude $A_{1/2}$ [17,18] which has some factors of charge and momentum multiplied in. In terms of G_+,

$$Q^3 G_+(N \to R) = \frac{1}{e}\sqrt{\frac{m_R^2 - m_N^2}{m_N}}\, Q^3 A_{1/2}, \quad (5)$$

where e is the elementary charge.

For the $\Delta(1232)$, the calculations show a small asymptotic $A_{1/2}$, and in terms of G_+

$$Q^3 G_+(N \to \Delta) = \begin{cases} 0.05 \text{ GeV}^3 & (\text{CZ}), \\ 0.08 \text{ GeV}^3 & (\text{KS}), \end{cases} \quad (6)$$

where the calculations used the Δ distribution amplitude calculated in QCD sum rule calculations in Ref. [12] and the nucleon distribution amplitude calculated by either Chernyak and Zhitnitsky (CZ [19]) or King and Sachrajda (KS [20]) [21]. The uncertainties in the QCD sum rule determination of the Δ distribution amplitude are sufficient that the correct answer could be 2 or 3 times larger or smaller than the previously quoted results. Nonetheless, the leading N-Δ transition amplitude appears to be truly small. This is underscored by comparing to the N-$N^*(1535)$ transition, for which the normalized PQCD calculation is also possible [12] and leads to

$$Q^3 G_+[p \to N^*(1535)] = \begin{cases} 0.46 \text{ GeV}^3 & (\text{CZ}), \\ 0.58 \text{ GeV}^3 & (\text{KS}). \end{cases} \quad (7)$$

This brings us to our main question: What functional form shall we choose to interpolate between the low q^2 and asymptotic domains? We can receive guidance from the nucleon elastic case. The helicity amplitude G_+ for the nucleon has a kinematic zero at $Q = 0$, and after noting that $G_+ \propto Q G_M$, it is known that the magnetic form factor G_M is decently fit with a dipole form. For resonance production the kinematic zero moves to the "pseudothreshold" or "no-recoil" point, where in the resonance rest frame the nucleon is also at rest. This point is the threshold for Dalitz decay of the resonance, $R \to N + \gamma^* \to N e^+ e^-$. The kinematic zero is proportional to powers of $|\vec{q}^{\,*}|$, the momentum of the photon in the resonance rest frame, and the number of powers is one for both $G_\pm$ in Δ electroproduction [22]. Further, both the nucleon and Δ are in the ground state 56-plet of the approximate SU(6) spin-flavor symmetry, so we feel it is a good *ansatz* to also using a simple dipole form for the $A_{1/2}$ amplitude for the N-Δ transition, and a similar form with one more asymptotic power of Q^{-2} for $A_{3/2}$. The controlling factor in $|\vec{q}^{\,*}|$ is [23]

$$Q^* = \sqrt{Q^2 + (m_R - m_N)^2}. \quad (8)$$

Hence we shall take our interpolating forms to be

$$\begin{aligned} A_{1/2}(Q^2) &= \frac{Q^*}{m_\Delta - m_N}\frac{A_{1/2}(0)}{(1 + Q^2/\Lambda_1^2)^2}, \\ A_{3/2}(Q^2) &= \frac{Q^*}{m_\Delta - m_N}\frac{A_{3/2}(0)}{(1 + Q^2/\Lambda_3^2)^3}, \end{aligned} \quad (9)$$

where Λ_1 and Λ_3 are parameters.

At low q^2 the multipole amplitudes are more natural, and one expects as well as sees a dominance of the $M1$ amplitude. If the dominance is complete, one expects from Eq. (1)

$$A_{3/2}(0) = \sqrt{3}\, A_{1/2}(0). \quad (10)$$

Since $E2$ is not quite zero, we shall use real photon helicity amplitudes given by the data [1].

At the high q^2 end, we have

$$\lim_{Q^2\to\infty} Q^3 A_{1/2} = A_{1/2}(0)\frac{\Lambda_1^4}{m_\Delta - m_N}. \quad (11)$$

The parameter Λ_1 can now be constrained by the calculated asymptotic values of the left-hand side, Eq. (6). The data may indicate a somewhat different value. In any case, since $Q^3 A_{1/2}$ is asymptotically small, we have reason to expect Λ_1 to be small compared to the typical scale exemplified by the mass parameter of the nucleon form factor. This means that the helicity amplitude $A_{1/2}(N \to \Delta)$ will show an *anomalously rapid* falloff.

Regarding Λ_3, we have no special guidance from PQCD but also no reason to think that $A_{3/2}(N \to \Delta)$ should be anomalous at high q^2. So we might choose, for example, the value of the mass parameter used in the dipole approximation to the nucleon electromagnetic form factor, suggesting $\Lambda_3^2 = 0.71$ GeV^2, or the corresponding parameter in the fit to the axial form factor, which would give $\Lambda_3^2 \approx 1.1$ GeV^2 [24]. Alternatively, we could simply fit Λ_3^2 to intermediate q^2 data. That is what we shall eventually do, but we shall begin with a naive fit using the 0.71 GeV^2 value.

Figure 1 illustrates the expected $E2/M1$ ratio under several specific assumptions of hadron dynamics. The dashed curve is a naive fit with $Q^3 A_{1/2}$ in the asymptotically large Q^2 limit given by the KS amplitudes, with the parameter Λ_3 given by the dipole scale of the electromagnetic form factor of the nucleon. We shall return to the solid and dotted curves shortly. We note that the falloff of the helicity amplitudes as functions of Q^2 is rapid. This is especially true for $A_{1/2}$, explaining why the EMR stays negative to several GeV^2.

It is here that we can benefit from the existing unpolarized data on the quantity G_T which is proportional to the

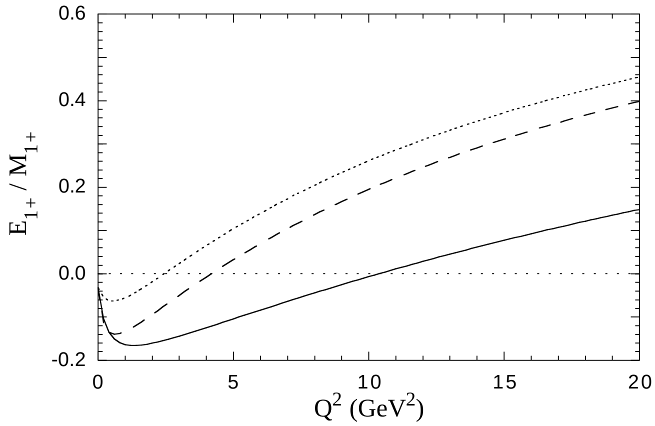

FIG. 1. The $E2/M1$ ratio for the N-Δ transition as a function of Q^2. The dashed curve is the naive fit described in the text, the solid curve is our preferred parametrization taking into account a number of constraints, and the dotted curve is another parametrization with a larger asymptotic value of $A_{1/2}$. The amplitudes in the last two parametrizations fit the unseparated data well.

sum of the squares of the helicity amplitudes, compiled by Stoler [6],

$$|G_T(Q^2)|^2 = \frac{2m_N^2}{Q^2}(|G_+|^2 + |G_-|^2), \qquad (12)$$

and compared to the dipole form,

$$G_{\mathrm{dipole}} = \frac{2.79}{(1 + Q^2/0.71\ \mathrm{GeV}^2)^2}. \qquad (13)$$

The naive fit does not reproduce this data set in the Δ region (Fig. 2, dashed curve). A tuning of the asymptotic value of $Q^3A_{1/2}$ to 0.08 GeV$^{5/2}$ (or Q^3G_+ to 0.22 GeV^3), along with the parameter Λ_3 adjusted to 1.14 GeV (closer to the axial form factor mass parameter), describes this data a lot better, as illustrated by the solid curve of Fig. 2. This forms our sounder basis for predicting the EMR behavior as a function of Q^2. This is shown by the solid curve in Fig. 1. Current experiments, under analysis at CEBAF in the Jefferson Lab, will test this prediction in the near future.

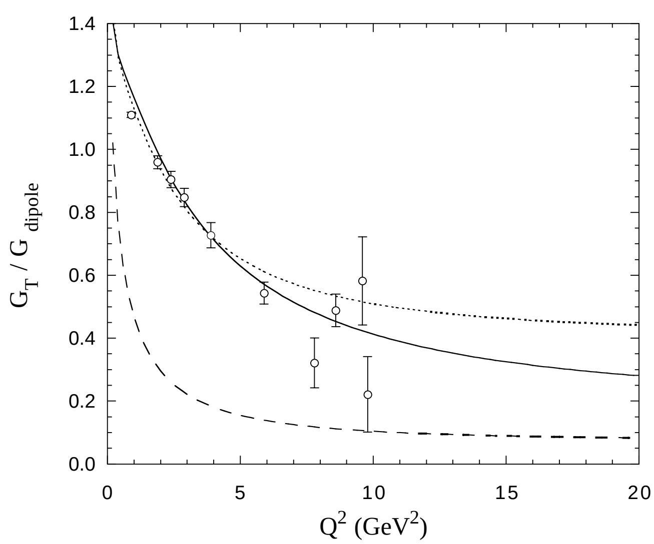

FIG. 2. Our parametrizations compared to the unseparated data for $A_{1/2}^2 + A_{1/2}^2$ presented as the quantity G_T/G_{dipole}, defined in the text. The curves match Fig. 1, and the data is from Table 5 of the second paper of Ref. [6].

As another possibility, we show in Fig. 1 the prediction using a larger asymptotic value $Q^3A_{1/2} = 0.17$ GeV$^{5/2}$ (which happens to be what is obtained using the Gari-Stefanis nucleon distribution amplitude [21,25] and the Ref. [12] Δ distribution amplitude) and the Λ_3 shrunken slightly to 1.10 GeV. This change in $A_{1/2}$ has little effect on Fig. 2 below 5 GeV^2.

Considerations of the longitudinal helicity amplitudes are also possible in the same spirit. They do require considerations of pseudothreshold constraints [26], and lie outside the scope of the present paper.

In summary, it is important to question how one should interpolate between the constituent quark model at low momentum transfers and perturbative QCD at high momentum transfers. The nucleon to Δ electromagnetic transition may be particularly instructive because the asymptotic PQCD prediction for the EMR is far from what is seen at the highest momentum transfers for which there are data reported [3]. One may even entertain the idea that PQCD is irrelevant at feasibly measurable momentum transfers. Our considerations lead to the opposite conclusion.

We take interpolating functions that have the correct behavior near the no-recoil point, have the correct asymptotic PQCD behavior, are normalized at the photon point by the constituent quark model or by data, and have an asymptotic normalization that is guided when possible by normalized PQCD calculations (tuned by data). Such functions are simple, plausible, and fit the data well. Further, despite the fact that their Q^2 dependence and large Q^2 normalization are in accord with PQCD, they lead directly to having the EMR remain negative to momentum transfers of many, though fewer than 10, GeV^2.

In conclusion, our present study in the Δ resonance region has thrown light on the role of perturbative physics as constrained in the value of the quantity $Q^3A_{1/2}$ for asymptotically large Q^2 in influencing the behavior of the $E2/M1$ ratio for the nucleon to Δ transition. Given the abnormal suppression of the normally dominant amplitude $A_{1/2}$ in the case of the Δ excitation, anticipated by the nucleon and Δ amplitudes inferred from the QCD sum rule approaches, the parameter controlling the falloff of the subleading amplitude $A_{3/2}$ also plays a crucial role in determining this ratio. New experimental results on this from electron factories will test this rigorously, and they are eagerly awaited.

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$$(2m_R)^{-1}\{[Q^2 + (m_R - m_N)^2][Q^2 + (m_R + m_N)^2]\}^{1/2},$$

which might suggest another kinematic zero at another value of Q^2. However, the argument that leads to kinematic zeros [22] as powers of $|\vec{q}^{\,*}|$ applies only at pseudothreshold, and not at the $\gamma^* \rightarrow R + N$ production threshold. An easy case to check explicitly is an imagined elementary nucleon, where G_+ has a factor Q but no factor $\sqrt{Q^2 + 4m_N^2}$.
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