Superconducting Single-Mode Contact as a Microwave-Activated Quantum Interferometer

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The dynamics of a superconducting quantum point contact biased at subgap voltages is shown to be strongly affected by a microwave electromagnetic field. Interference among a sequence of temporally localized, microwave-induced Landau-Zener transitions between current carrying Andreev levels results in energy absorption and in an increase of the subgap current by several orders of magnitude. The contact is an interferometer in the sense that the current is an oscillatory function of the inverse bias voltage. Possible applications to Andreev-level spectroscopy and microwave detection are discussed. [S0031-9007(98)07169-5]

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The classic double-slit interference experiment, where two spatially separated trajectories combine to form an interference pattern, clearly demonstrates the wavelike nature of electron propagation. For a 0-dimensional system, with no spatial structure, a completely analogous interference phenomenon may occur between two distinct trajectories in the *temporal* evolution of a quantum system. Such trajectories may appear in the presence of temporally localized nonadiabatic perturbations (rather than spatially localized slits in a screen) which scatter the system from one adiabatically evolving state to another. In this Letter we show that this type of interference phenomenon significantly controls the microscopic dynamics of a voltage-biased superconducting quantum point contact (QPC) subject to microwave irradiation.

It is well known that the Josephson current in a QPC is carried by Andreev bound states localized within the contact area. The corresponding energy levels—the Andreev levels—lie in the energy gap of the superconductor and their positions depend on the change ϕ in the phase of the condensate across the junction. Hence, the Andreev levels will move adiabatically with time within the gap if the contact is biased by a voltage V much smaller than the gap energy, Δ. With any (normal) electron scattering present in the contact the Andreev levels will, however, never cross the Fermi level; instead they will oscillate periodically with ϕ so that on the average no energy is transferred to the QPC and a purely ac current will flow through the contact (ac Josephson effect).

Microwave radiation of large frequency $\omega \sim \Delta$ represents a nonadiabatic perturbation of the QPC system. However, if the amplitude of the electromagnetic field is sufficiently small, the field will not affect the adiabatic dynamics of the system much unless the condition for resonant optical interlevel transitions is fulfilled. Such resonances will occur only at certain moments determined by the time evolution of the Andreev level spacing. They provide a mechanism for the energy transfer to the system to be nonzero when averaged over time and for a finite dc current through the junction. The rate of energy transfer is in an essential way determined by the interference between different scattering events [1], which will also lead to oscillatory features in the current-voltage characteristics of the QPC.

Although an example of the more general problem of energy level spectroscopy, the spectroscopy of Andreev levels has an important specific aspect. Because of their ability to carry electric current, detection of optical transitions between Andreev levels is possible by means of transport measurements. The appearance of a subgap current under resonant radiation can furthermore be used as a sensitive microwave detector.

For an unbiased QPC, the Andreev spectrum of each transport mode has the form $E^{\pm}(\phi) = \pm E(\phi) = \pm\Delta[1 - D\sin^2(\phi/2)]^{1/2}$, where D is the transparency of the mode and the energy is measured from the Fermi energy [2,3]. With a small bias voltage applied, the levels move along the adiabatic trajectories $E^{\pm}(t) = \pm E(\phi_0 + 2eVt/\hbar)$ in energy-time space, as shown in Fig. 1. When the criterium $\hbar\dot{\phi} = 2eV \ll 2E^2(t)/\Delta$ for adiabaticity is obeyed, the rate of interlevel transitions is exponentially small keeping the level populations constant in time [4,5]. The presence of a weak electromagnetic field [on the scale of $E(t)$] does not affect the adiabatic level trajectories except for short times close to the resonances at $t = t_{A,B}$, when $E(t_{A,B}) = \hbar\omega/2$. Here the dynamics of the system is strongly nonadiabatic with a resonant coupling which effectively mixes the adiabatic levels. This is an analog of the well known Landau-Zener transition, which describes interlevel scattering as a resonance point is passed. In our case these transitions give rise to a splitting of the quasiparticle trajectory at the

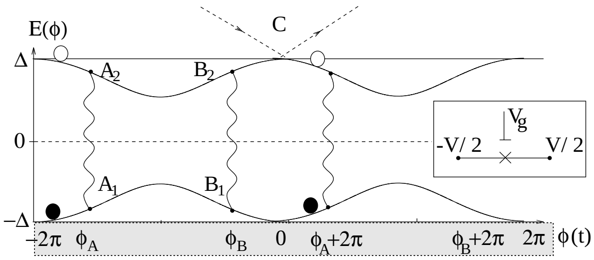

FIG. 1. Time evolution of Andreev levels (full lines) in the energy gap of a gated voltage-biased, single-mode superconducting QPC (see inset). A weak microwave field induces resonant transitions (wavy lines) between the levels at points A and B and the level above the Fermi energy becomes partly occupied to an extent determined by interference between the two transition amplitudes. Nonadiabatic interactions release the energy of quasiparticles in the (partly) occupied Andreev level into the continuum at point C, where the Andreev states and the continuum merge into each other (represented by dashed arrows; see text) and the initial conditions for the Andreev level populations are reset (filled and empty circles).

points A, B into two paths, $A_1A_2B_2$ and $A_1B_1B_2$, forming a loop in (E,t) space.

The resonant scattering opens a channel for energy absorption by the system; a populated upper level when approaching the edge of the energy gap (at point C in Fig. 1) creates real excitations in the continuum spectrum, which carry away the accumulated energy from the contact. As a result, the net rate of energy transfer to the system is finite; it consists of energy absorbed both from the electromagnetic field and from the voltage source. The confluence of the two adiabatic trajectories at B_2 (see Fig. 1) gives rise to a strong interference pattern in the probability for real excitations at the band edge, point C. The interference effect is controlled by the difference of the phases acquired by the system during propagation along the paths $A_1A_2B_2$ and $A_1B_1B_2$.

For a quantitative discussion we consider a one-mode superconducting point contact [6] with arbitrary energy-independent transparency D for normal electrons, $0 < D < 1$. The length of the junction L is small compared to the coherence length ξ_0, $L \ll \xi_0$. The contact is biased at a small applied voltage $eV \ll \Delta$, and a high frequency electromagnetic field is applied to the gate situated near the contact; see inset in Fig. 1. We will describe the evolution of the Andreev states with the time-dependent Bogoliubov–de Gennes (BdG) equation [7] for the quasiclassical envelopes $u_\pm(x,t)$ of the two-component wave function $\Psi(x,t) = u_+(x,t)e^{ik_Fx} + u_-(x,t)e^{-ik_Fx}$,

$$i\hbar\partial\mathbf{u}/\partial t = [\mathbf{H}_0 + \sigma_z V_g(x,t)]\mathbf{u}\,. \tag{1}$$

In this equation $\mathbf{u} = (u_+, u_-)$ is a four-component vector, $\mathbf{H}_0$ is the Hamiltonian of the electrons in the electrodes of the point contact,

$$\mathbf{H}_0 = -i\hbar v_F\sigma_z\tau_z\partial/\partial x + \Delta\{\cos[\phi(t)/2]\sigma_x + i\sin[\phi(t)/2]\,\mathrm{sgn}\,x\,\sigma_y\}\,, \tag{2}$$

where σ_i and τ_i denote Pauli matrices in electron-hole space and in $u_\pm$ space, respectively. The function $\mathbf{u}$, which is smooth on the scale of the Fermi wavelength, has a discontinuity at the contact which is determined by the transfer matrix of the QPC in the normal state and is described by the following boundary condition [8]: $\mathbf{u}(+0) = (1/\sqrt{D})(1 - \sqrt{R}\,\tau_y)\mathbf{u}(-0)$, $R = 1 - D$.

The gate potential $V_g(x,t) = V_\omega(x)\cos\omega t$ in Eq. (1) oscillates rapidly in time and the amplitude is assumed to be small compared to the Andreev level spacing, $V_\omega \ll E(t)$. Under this condition, the system experiences an adiabatic evolution at all times except close to the resonances (points A and B in Fig. 1) The duration δt of these resonances is short in the limit $eV \ll \Delta$ compared to the period of Josephson oscillations $T_V = \pi\hbar/eV$. Indeed, the resonant transition occurs if the deviation of the inter-level spacing from the resonance value $E(t) - \hbar\omega/2 = \dot{E}(t_{A,B})\delta t$ does not exceed the quantum mechanical resolution of the energy levels $\hbar/\delta t$. From this we can estimate δt to $\delta t \sim [\hbar/\dot{E}(t_{A,B})]^{1/2} \ll \hbar/eV$. Hence we may consider the nonadiabatic dynamics as temporally localized scattering events. By introducing a linear combination $\mathbf{u}(t) = b^+(t)e^{i\omega t/2}\mathbf{u}^+ + b^-(t)e^{-i\omega t/2}\mathbf{u}^-$ of the eigenstates $\mathbf{u}^\pm$ corresponding to the adiabatic Andreev levels $E^\pm$, we can describe the system's evolution through a resonance by letting a scattering matrix $\hat{S}$ connect the coefficients $b^\pm$ before and after the splitting points A and B. A standard analysis of the Landau-Zener interlevel transitions (see, e.g., [9]) gives the scattering matrix elements at the point A $(S_A)_{++} = (S_A)_{--} = \tau$, $(S_A)_{+-} = -(S_A)^*_{-+} = \rho$, where $|\rho|^2 = 1 - |\tau|^2 = 1 - e^{-\gamma}$ is the probability of the Landau-Zener interlevel transition. Here $\gamma = \pi|V_{+-}|^2/(dE/dt)$, where V_{+-} is the matrix element for the interlevel transitions. At the splitting point B the scattering matrix reads $\hat{S}_B = \hat{S}_A^T$. The matrix element V_{+-} was calculated for the case of a double barrier QPC structure in Ref. [9]. For a single barrier junction an analogous calculation gives us $V_{+-} = \alpha(L/\xi_0)\sqrt{DR}\,V_\omega\sin(\phi/2)$, where the constant $\alpha \sim 1$ is determined by the position of the barrier. We note that this matrix element is proportional to the *reflectivity* of the junction; reflection mixes electron states with $+k_F$ and $-k_F$ allowing optical transitions between the Andreev levels. In a perfectly transparent QPC ($D = 1$), the upper and lower Andreev levels correspond to opposite electron momenta and the effect under consideration does not exist; cf. Refs. [4,10].

By introducing the matrix $\hat{\Phi}_{j,i} = \exp[i\sigma_z\Phi(i,j)]$,

$$\Phi(i,j) = \frac{1}{2eV}\int_{\phi_i}^{\phi_j} d\phi\left(E(\phi) - \frac{\hbar\omega}{2}\right), \tag{3}$$

which describes the "ballistic" dynamics of the system between the Landau-Zener scattering events, we connect the coefficients $b^\pm$ at the end of the period of the Josephson oscillation, $\phi = 0$, with the coefficients $b_0^\pm$ at the beginning of the period, $\phi = -2\pi$,

$$\begin{pmatrix} b^+ \\ b^- \end{pmatrix} = \hat{\Phi}_{0,B}\hat{S}_B\hat{\Phi}_{B,A}\hat{S}_A\hat{\Phi}_{A,-2\pi}\begin{pmatrix} b_0^+ \\ b_0^- \end{pmatrix}. \tag{4}$$

The time-averaged current through the junction can be directly expressed through these coefficients.

The quasiclassical equation for the total time dependent current at the junction ($x = 0$) reads $I(t) = v_F\langle \mathbf{u}\tau_z\mathbf{u}\rangle$, where $\langle\ldots\rangle$ denotes a scalar product in 4-dimensional space. From Eqs. (1) and (2) it follows that

$$I(t) = \frac{2e}{\hbar}\left(\frac{d\phi}{dt}\right)^{-1}\int_{-\infty}^{\infty}dx\left[i\hbar\frac{d\langle \mathbf{u}\dot{\mathbf{u}}\rangle}{dt} - \dot{V}_g\langle \mathbf{u}\sigma_z\mathbf{u}\rangle\right]. \tag{5}$$

In the static limit, $\dot{V}_g = 0$ and $\dot{\phi} \to 0$, it equals the usual equation $I = (2e/\hbar)(dE^{\pm}/d\phi)$ for the Andreev level current. In the general nonstationary case $\mathbf{u}$ is a linear combination of $\mathbf{u}^{\pm}$ and we calculate the current averaged over the period T_V. Using the normalization condition $|b^+|^2 + |b^-|^2 = 1$ and omitting small contributions from rapidly oscillating terms, we obtain

$$I_{\mathrm{dc}} = \frac{2e}{\pi\hbar}\left(\Delta - \frac{\hbar\omega}{2}\right)[|b^+|^2 - |b_0^+|^2]. \tag{6}$$

The direct current through the contact can be viewed as resulting from photon-assisted pair tunneling or equivalently as being due to the distortion of the ac pair current due to the induced interlevel transitions. The magnitude of the current is such that the energy absorbed from the voltage source, VI_{dc}, together with the energy absorbed from the hf field corresponds to the energy necessary for creating a real continuum-state excitation.

Let us now discuss the boundary condition at $\phi = 2\pi n$ (n is an integer). In the vicinity of these points, the Andreev levels approach the continuum and the adiabatic approximation is unsatisfactory, even at small applied voltages and weak electromagnetic fields. The duration δt of the nonadiabatic interaction between the Andreev level and the continuum states can be estimated using the same argument as for the microwave-induced Landau-Zener scattering. One finds that $\delta t \sim \hbar/(\Delta e^2V^2)^{1/3}$. To derive the boundary condition, for example, at point C in Fig. 1, one needs to calculate the transition amplitude connecting the states $\mathbf{u}^+(t_1)$ at time $t_1 \ll t_C - \delta t$ and $\mathbf{u}^+(t_2)$ at time $t_2 \gg t_C + \delta t$: $\langle \mathbf{u}^+(t_2)\mathbf{U}(t_2,t_1)\mathbf{u}^+(t_1)\rangle$. Here $\mathbf{U}(t_2,t_1)$ is the exact propagator corresponding to the Hamiltonian in Eq. (1). It follows from symmetry arguments that this amplitude is exactly zero. Both the Hamiltonian (2) and the boundary condition for $\mathbf{u}$ at $x = 0$ are invariant under the simultaneous charge and parity inversion described by the unitary operator $\mathbf{\Lambda} \equiv \hat{P}\sigma_x\tau_z$, where $\hat{P}$ is the parity operator in x space. This implies that at any time any nondegenerate eigenstate of the Hamiltonian is an eigenstate of the symmetry operator $\mathbf{\Lambda}$ with eigenvalue $+1$ or -1 and that this property persists during the time evolution of the state. In particular, the static Andreev state obeys the equation $\mathbf{\Lambda}\mathbf{u}^+(\phi) = \Lambda\mathbf{u}^+(\phi)$ at any ϕ. It follows from Eq. (2) that $\mathbf{u}^+(-\phi) = \sigma_x\tau_y\mathbf{u}^+(\phi)$ so $\Lambda(-\phi) = -\Lambda(\phi)$. The immediate consequence of this property is that the static Andreev states correspond to *different* eigenvalues Λ at $\phi < 0$ and at $\phi > 0$, and therefore they are orthogonal [11], even though they belong to the *same* level. Hence the state evolving from the adiabatic state $\mathbf{u}^+(t_1)$ is orthogonal to the adiabatic state $\mathbf{u}^+(t_2)$. As a result the probability for an adiabatic Andreev state to be "scattered" into a localized state after passing the nonadiabatic region is identically zero. In reality, the Andreev state as it approaches the continuum band edge decays into the states of the continuum. Such a decay corresponds to a delocalization in real space and is the mechanism for transferring energy to the reservoir [12].

The orthogonality property shown above guarantees that the coherent evolution of our system persists during only one period of the Josephson oscillation and that the equilibrium population of the Andreev levels is reset at each point $\phi = 2\pi n$ [13]. This imposes the boundary conditions $b^+(2\pi n + 0) = 0, b^-(2\pi n + 0) = 1$ in the beginning of each period. Combining this boundary condition with Eqs. (6) and (4), we finally get

$$I_{\mathrm{dc}} = \frac{8e}{\pi\hbar}\left(\Delta - \frac{\hbar\omega}{2}\right)e^{-\gamma}(1 - e^{-\gamma})\sin^2[\Phi(A,B) + \theta], \tag{7}$$

where θ is the phase of the probability amplitude for the Landau-Zener transition, which weakly depends on V.

Equation (7) is the basis for presenting the biased QPC as a quantum interferometer. There is a clear analogy between the QPC interferometer and a standard SQUID in that they both rely on the presence of trajectories that form a closed loop. In a SQUID, which is used to measure magnetic fields, the loop is determined by the device geometry; in the QPC the voltage (analog of the magnetic field) is well defined while the "geometry" of the loop in (E,t) space can be measured. This loop is determined by the Andreev-level trajectories in (E,t) space and is controlled by the frequency of the external field. This gives us an immediate possibility to reconstruct the phase dependence of the Andreev levels from the frequency dependence of the period Π of oscillations of the current versus inverse voltage; see Fig. 2. Indeed, it follows from Eq. (3) that

$$\phi(E) = \pi \pm \frac{4\pi e}{\hbar}\left.\frac{d\Pi^{-1}}{d\omega}\right|_{\omega=2E}. \tag{8}$$

In order to be able to do interferometry it is necessary to keep phase coherence during one period of the Josephson oscillation. There are three dephasing mechanisms that impose limitations in practice: (i) deviations from an ideal voltage bias, (ii) microscopic interactions, and (iii) radiation induced transitions to continuum states. The main source of fluctuations of the applied voltage is the ac Josephson effect. In the resistively-shunted-junction model, a fixed voltage across the junction can be maintained only if the ratio between the intrinsic resistance R_i of the voltage source and the normal junction resistance R_N is small. If $R_i/R_N \ll 1$ the amplitude of the voltage

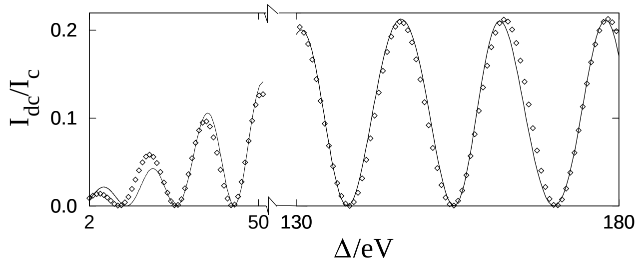

FIG. 2. Current vs inverse voltage from Eq. (7) for a biased superconducting QPC irradiated with microwaves of frequency $\omega = 1.52\Delta/\hbar$ and amplitude corresponding to a matrix element $|V_\pm| = 0.024\Delta$ for interlevel transitions. Note the cut in the inverse voltage scale. Results of the scattering approach, Eq. (5) ($\diamond$), are close to those obtained by numerically solving the BdG equation (1) with the radiation field treated in the resonance approximation (solid line). The close fit means that the scattering picture can be used to reconstruct the Andreev levels from the period of the current oscillations (Andreev level spectroscopy; see text).

R_N fluctuations δV is estimated as $\delta V \sim (R_i/R_N)\Delta$. Effects of voltage fluctuations on the accumulated phase $\Phi(A,B)$ can be neglected if $\delta\Phi(A,B) = \Phi'(A,B)\delta V \ll 2\pi$, i.e., if $eV > (R_i/R_N)^{1/2}\Delta$, which corresponds to a lower limit for the bias voltage [14].

The dephasing time due to microscopic interactions is comparable to the corresponding relaxation time [15]. This mechanism of dephasing can be neglected as soon as the relaxation time exceeds the Josephson oscillation period, $\tau_i \gg \hbar/eV$. Taking electron-phonon interaction as the leading mechanism of inelastic relaxation, we estimate τ_i to be of the order of the electron-phonon mean free time at the critical temperature, $\tau_{\mathrm{ph}}(T_c)$, since the large deviations from equilibrium in our case occur in the energy interval $E < \Delta$. This gives [16] another limitation on how small the applied voltage can be, $eV > 10^{-2}\Delta$.

The third mechanism of dephasing becomes important when the Andreev levels are closer than $\hbar\omega$ to the continuum band edge. One can estimate the corresponding relaxation time as $\tau_\omega \sim \hbar\Delta/V_\omega^2$. For small radiation amplitudes τ_ω exceeds T_V, while for optimal amplitudes they are about equal. The effect of the level-continuum transitions on the interference oscillations depends on the frequency. If $\hbar\omega < 2\Delta/3$ the "loop region" $[E(t) < \hbar\omega/2]$ is optically disconnected from the continuum, and transitions cannot destroy interference. Possible level-continuum transitions at times outside the loop will only decrease the amplitude of the effect by a factor of $\exp(-\alpha V_\omega^2/eV\Delta)$, where $\alpha < 1$ is the relative fraction of the period T_V during which transitions to the continuum are possible. Accordingly, this factor is of the order of unity for the voltages that correspond to the maximum amplitude of oscillations. If $\hbar\omega > 2\Delta/3$, the interference is impeded by the optical transitions into the continuum, and the current oscillations decrease. Still, a nonzero average current through the junction will persist.

The interference effect presented here can also be applied for detecting weak electromagnetic signals up to the gap frequency. Because of the resonant character of the phenomenon, the current response is proportional to the ratio between the amplitude of the applied field and the applied voltage, $I \sim |V_\pm|^2/\Delta eV$. For common superconductor-insulator-superconductor detectors a nonresonant current response is proportional to the ratio between the amplitude and the frequency of the applied radiation [17], $I \sim |V_\pm/\omega|^2$; i.e., it depends entirely on the parameters of the external signal and cannot be improved.

In conclusion, we have shown that irradiation of a voltage biased superconducting QPC at frequencies $\omega \sim \Delta$ can remove the suppression of subgap dc transport through Andreev levels. Because of the resonant nature of the photon-induced interlevel scattering the phenomenon can be used for sensitive microwave detection. Quantum interference among the resonant scattering events can be used for microwave spectroscopy of the Andreev levels.

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[12] This conclusion is in agreement with the analysis of multiple Andreev reflections in Ref. [5].
[13] This result is obvious for a pure contact ($D = 1$) since the two Andreev states involved in the transition at the band edge have momenta k_F and $-k_F$ pointing in opposite directions. See also [4].
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