Breakdown of Fourier's Law near the Superfluid Transition in ^{4}He

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The behavior of the superfluid–normal fluid interface in ^{4}He has been studied in the presence of a heat flux, $\vec{Q}$, in the range $0.1 < Q < 6$ erg s^{-1} cm^{-2}. Fourier's Law, $\vec{Q} = -\kappa \nabla T$, has been observed, for the first time, to break down very close to the interface. In the nonlinear region, κ depends on Q. Theoretical predictions, which do not take gravity into account, are compared with these experimental results. The predicted extent of the nonlinear region agrees well with the data; however, at lower Q, a systematic discrepancy is found that may be ascribed to the effect of gravity on the interface. [S0031-9007(98)07125-7]

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The thermal conductivity of liquid ^{4}He diverges as the superfluid transition temperature T_λ is approached from above [1], due to fluctuations in the superfluid order parameter [2]. If a heat flux, $\vec{Q}$, flows in the upward direction through a sample of liquid ^{4}He containing an interface between the normal and superfluid phases, the resulting temperature gradient in the normal fluid tapers to zero at the interface. The superfluid transition is a critical transition with a divergent correlation length, $\xi = \xi_0 |t|^{-\nu}$, where $t = (T - T_\lambda)/T_\lambda$ is the reduced temperature, $\nu = 0.672$, and $\xi_0 \approx 3.4 \times 10^{-8}$ cm [3]. The linear response relationship, $\vec{Q} = -\kappa \nabla T$, is expected to break down due to the interactions of the heat flux, carried in part by superfluid counterflow, with the fluctuations in the superfluid order parameter [4]. This nonlinearity becomes apparent only near the superfluid transition, since these fluctations become large and, hence, relatively long-lived as the superfluid transition is approached [4]. This nonlinear region results when the temperature difference over a correlation length becomes comparable to the reduced temperature itself [5]. The criterion $\xi |\nabla t| \ll t$ may be used to estimate the extent of the linear regime. If the approximation $\kappa = \kappa_0 |t|^{-x}$, where $\kappa_0 = 120$ erg s^{-1} cm^{-1} K^{-1} and $x = 0.44$ [6], is used for the thermal conductivity, the linear response criterion becomes $t \gg (\xi_0 Q/\kappa_0 T_\lambda)^{1/(1-x+\nu)} \approx 9.2 \times 10^{-9} Q^{0.813}$ for Q expressed in units of erg s^{-1} cm^{-2}. This inequality suggests that nonlinear effects become important in a temperature range that may be resolved experimentally, and that the temperature width of the nonlinear region increases with Q. Haussmann and Dohm (HD) [7] have calculated the function $\kappa(Q, t)$ for $T > T_\lambda$ using renormalization group methods to systematically account for the effects of fluctuations. They find that κ is depressed below the zero-Q value as Q is increased, and define $T_{\rm nl}$ as the temperature at which the effective Q-dependent conductivity is depressed by 5% from the zero-Q result. According to their calculation, $t_{\rm nl}(Q) = [T_{\rm nl}(Q) - T_\lambda]/T_\lambda \approx 8.3 \times 10^{-9} Q^{0.744}$, as displayed in Fig. 1.

In addition to lowering the thermal conductivity in the vicinity of the transition, a heat flux will create a temperature gradient in the region immediately below T_λ [10,11]. This behavior has already been observed experimentally for $Q > 5$ erg s^{-1} cm^{-2} [8,9,12]. The temperature corresponding to the onset of dissipation may be designated $T_{\rm c}(Q)$. At present, little is known theoretically about the region $T_{\rm c}(Q) < T < T_\lambda$, although an upper bound on $T_{\rm c}(Q)$ has been calculated independently by Onuki [10] and by HD [11]. We will refer to the entire area between $T_{\rm c}(Q)$ and $T_{\rm nl}(Q)$ as the nonlinear region (Fig. 1).

Measurements in the nonlinear region are hampered by its small spatial extent. Although this region's temperature boundaries widen with increasing Q, they do so with an exponent less than one. The temperature gradient is expected to increase with Q more strongly than does the temperature width, narrowing the region's spatial extent as Q is increased. In a gravitational field, the nonlinear region is also compressed at low Q by the transition

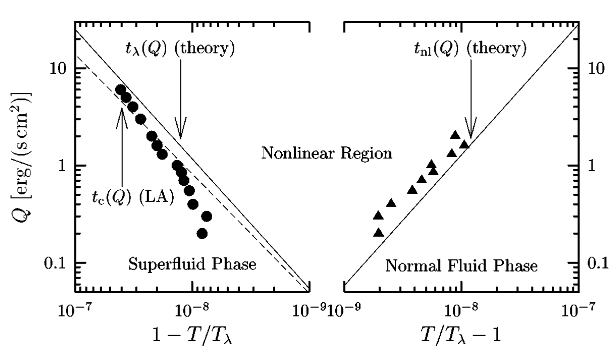

FIG. 1. Boundary of the nonlinear region in the Q-T plane. Solid lines are the theoretical predictions of Ref. [7], the dashed line is the extrapolation to low Q of the $t_{\rm c}(Q)$ results from Liu and Ahlers (LA) Ref. [12], and solid circles and triangles are the $t_{\rm c}(Q)$ and $t_{\rm nl}(Q)$ results of this work.

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temperature gradient along the sample created by the pressure gradient along the sample's height [5,10,13]. Assuming the HD result for the thermal conductivity [7] for $T > T_\lambda$, and an extrapolation down to $T_c(Q)$ [12], we estimate that the maximum spatial width of the nonlinear region under gravity is about 100 μm, and it occurs at about $Q = 1$ erg s^{-1} cm^{-2}. Hence, our measurements are centered around this value.

Our measurement technique was optimized to resolve the temperature at a precise vertical position in a column of helium. Sidewall probes were employed for temperature measurement to avoid boundary effects in a manner that is analogous to the four-wire technique for measuring electrical resistance. The sidewall of the sample cell was constructed from cylindrical sections of Vespel [14], a thermally resistive material. Glued in place between these sections were thin foils of annealed copper that served as the sidewall temperature probes. The end caps of the cell, on which heaters were mounted, were made from pure aluminum. More details of the cell construction have been presented in a prior publication [15]. The copper foils were soldered to paramagnetic salt thermometers of the type described by Lipa, Leslie, and Walstrom [16]. Using these thermometers, we have achieved a temperature resolution better than 10^{-10} K with a drift rate of about 10^{-15} K/s [17]. Heat was supplied to the cell bottom, while the top of the cell was cooled through a conductive link to a separate thermal stage. Two sidewall probes were located near the bottom of the cell. The first, located 1.5 mm from the cell bottom, was 76 μm thick; the second, 2.5 mm from the cell bottom, was 25 μm thick. Foils of different thickness were used so that probe-induced effects on the data could be evaluated. A third probe, located 5.5 mm above the cell bottom, was used to sample the superfluid temperature.

While it would have been ideal to measure the steady-state temperature profile with a closely spaced array of thermometers along the cell's length, such instrumentation was impractical. Instead, we swept the interface by a thermometry probe at a rate which was slow enough to assure steady-state conditions. The temperature profile read on the sidewall thermometer as the interface was swept past provided a quasi-steady-state representation of the thermal gradient near the interface. We slowly increased the superfluid temperature at a rate of about 40 pK/s by controlling the temperature read by the top thermometer, using a heater located on the cell top. Fluctuations in the control power supplied to the cell top were always very small compared with the total heat flux flowing through the cell, which was supplied by a heater on the cell bottom. In this way, the interface and the temperature profile through the normal fluid could be swept past the lower probes at a rate of about 0.3 μm per second. The temperature gradient in the cell at any instant could be reconstructed by plotting the temperature difference between a lower probe and the topmost probe against the temperature of the superfluid, which may be linearly related to changes in interface position by $dz = (1.273 \text{ K/cm})^{-1} \, dT_{sf}$ [13] (Fig. 2). The thermal conductivity was inferred by dividing Q by the local thermal gradient, which was derived by differentiating the temperature profile numerically.

A correction to the data was required to compensate for the sidewall thermal conductivity. The sidewall of the cell conducts a small amount of heat, some of which flows back into the helium through the thermometry probe, creating a predictable temperature offset as this heat flows across the boundary resistance into the helium. Figure 2 displays the raw and corrected temperature profiles for $Q = 0.85$ erg s^{-1} cm^{-2}. Numerical solutions of the steady-state heat flow equation near the sidewall probes predicted the exponential rise displayed in Fig. 2. This exponential rise was predictably larger in the 25 μm probe than in the 76 μm probe, confirming that the rise was a sidewall probe effect, and not an actual temperature variation within the helium. The experimental determination of $T_c(Q)$ was taken to be the first departure from the exponential rise displayed in Fig. 2. $T_c(Q)$ was determined to within two nanoKelvins by observing the sudden change in slope when the data, such as that in Fig. 2, was displayed on a semilog plot.

Figure 3 shows thermal resistivity results derived from the corrected 76 μm probe temperature profiles for different values of Q. The temperature T_λ was determined by fitting the high-temperature portions of the data to a power law, allowing T_λ to vary. Well above the transition, the data for different values of Q are observed to collapse as expected and agree well with the $Q = 0$ fit. Very near the transition, the data depart from the power law at a temperature that depends on Q. Below T_λ, the onset of resistance is observed to occur at $T_c(Q)$, which decreases with Q. Above T_λ, the solid line in the figure is the prediction of HD at a heat flux of 1.6 erg s^{-1} cm^{-2} [7]. Below T_λ, where HD make no prediction, an extrapolation

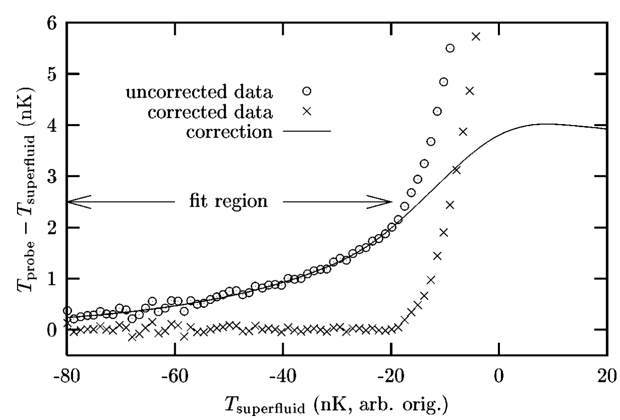

FIG. 2. Raw (circles) and corrected (crosses) temperature profiles for $Q = 0.85$ erg s^{-1} cm^{-2}. The line is the correction calculated from the model.

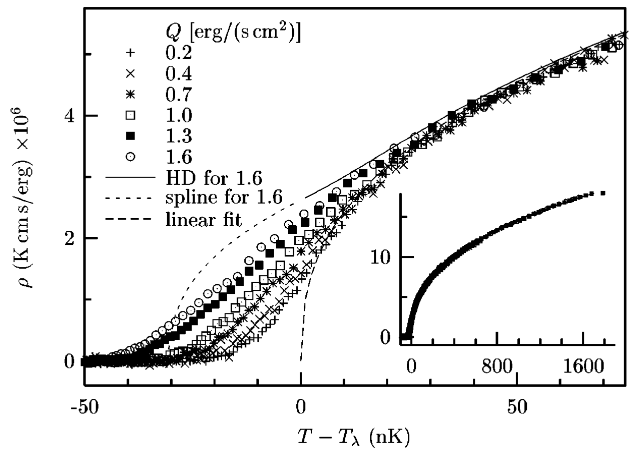

FIG. 3. Thermal resistivity ρ at six different values of Q. The inset shows the same data over a much greater temperature range. The solid line is the prediction of HD [7].

is shown [12]. Although the theory seems to predict T_c and T_{n1} reasonably well, the data lie significantly below the prediction in both regions $T_c < T < T_\lambda$ and $T_\lambda < T < T_{n1}$. At low heat currents, and for $T > T_\lambda$, a discrepancy is expected because the theory does not account for the effect of gravity. However, as Q increases, finite-Q effects are expected to outweigh the effect of gravity [5,7,10].

Figure 4 illustrates the effect of the sidewall heat flow correction on the thermal resistivity data. Plotted in the figure are the differences in the thermal resistivity derived from the 76 μm probe data before and after application of the correction procedure. Also plotted are differences between the corrected 76 μm probe data and the corrected 25 μm probe data and between the corrected 76 μm probe data and the theory, all for $Q = 1$ erg s^{-1} cm^{-2}.

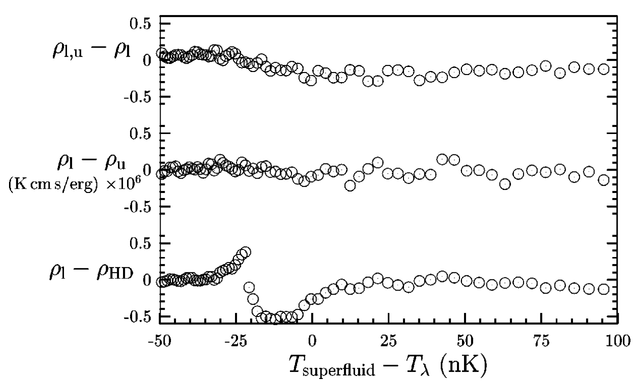

FIG. 4. Thermal resistivity differences for $Q = 1$ erg s^{-1} cm^{-2}. The top graph shows the difference between the resistivity determined by using lower probe data without correcting for the sidewall artifact, $\rho_{l,u}$, and the resistivity from the lower probe data, ρ_l. The middle graph shows the difference between ρ_l and the resistivity determined from the upper probe data, ρ_u. The bottom graph shows the difference between ρ_l and the theory of HD [7], ρ_{HD}.

Close to the transition, we find that the correction produces a smaller change in the thermal resistivity results than the difference existing between the data and the theory, so an error in the sidewall correction would have only a minor impact on our results. Furthermore, the discrepancy between the different sized probes is less than what exists between the data and theory, suggesting that probe effects are not responsible for the disagreement with theory.

Several possible sources of error in the measurement were evaluated. One potential error source was a radial heat flux in the cell resulting from stray heating of the temperature probes. A small heat current was found to flow from the sidewall probe into the helium with a magnitude that decayed over time to less than 20 pW. That level was found to be too small to affect the measurements. We found the effect of ambient vibrations on the measurements to be negligible by shaking the apparatus at several times the ambient level and noting very little change in the measured temperature profiles. A significant source of error was found to arise from imperfect leveling of the cell. Although care was taken to assure that the cell was mounted level with the top plate of the cryostat, releveling at low temperature was conducted using the measured data itself. It was found that tilting the cryostat would result in an apparent broadening of the transition, so the cryostat level was adjusted slightly until the transition's width was minimized. An intentional tilt of 5 mrad from level created a rounding of about 1 nK over a range of about 15 nK above and below T_λ, which was readily detectable and correctable experimentally. Hence, we estimate that the cell was level to within 3 mrad.

The boundaries of the nonlinear region taken from the experimental data are plotted in Fig. 1. The data for $t_{n1}(Q)$ (Fig. 1) agree reasonably well with the theoretical prediction over the entire range of Q. The experimental $t_c(Q)$, on the other hand, fall below the theoretical upper bound. The values of t_c and t_{n1} depend on the identification of T_λ. We preferred to identify T_λ as the temperature at which the linear thermal resistivity data extrapolate to zero. Another way of identifying T_λ would be to extrapolate the $t_c(Q)$ data to zero Q, assuming a power-law behavior. If that method was used, the $t_c(Q)$ data would naturally lie closer to the prediction, and the extrapolation of the higher-Q data, but the fit to the thermal resistivity data would worsen and the $t_{n1}(Q)$ data would lie further from the prediction. In either case, the temperature difference $T_\lambda[t_{n1}(Q) - t_c(Q)]$ remains nonzero as Q goes to zero, and approximately equal to 15 nK experimentally. Recently Haussmann calculated this difference using renormalization group techniques [18]. He predicted that this difference should be 19 nK, and that it will vanish in microgravity.

The presence of gravity is a limiting factor in many measurements of divergent thermodynamic properties very close to the λ transition. In most of these studies, where a bulk quantity such as the heat capacity is to be measured, the effect of gravity is to spread each measurement point over a range of reduced temperatures corresponding to the height of the sample. The minimum usable cell height is constrained by boundary and finite-size effects. The measurement described in this paper is local in character and may be conducted much closer to the transition before the type of gravity-induced distortion experienced in the heat capacity measurements becomes important [19]. Nevertheless, the presence of gravity is expected to influence the structure of the interface. For $Q = 0$, Onuki [20] defines a local correlation length, $\xi_{\mathrm{local}} = \xi_g \tanh(\xi/\xi_g)$, where ξ is the zero-g correlation length and

$$\xi_g = \xi_0^{1/(1+\nu)} \left(\frac{1}{T_\lambda} \frac{dT_\lambda}{dz} \right)^{-\nu/(1+\nu)} \approx 110\ \mu\mathrm{m}.$$

Instead of diverging at T_λ, the local correlation length is limited to ξ_g under gravity. A heat flux is predicted to further limit the divergence of ξ_{local} at T_λ to $\xi_Q = (g_0 k_B T_\lambda / Q)^{1/2}$, where $g_0 = 2 \times 10^{11}\ \mathrm{s}^{-1}$ [7]. The crossover heat flux, Q_{c}, below which gravity imposes the stricter limit on ξ_{local}, is approximately $0.7\ \mathrm{erg\,s^{-1}\,cm^{-2}}$ [7,10], so we may assume that, below that heat flux, the influence of gravity outweighs that of the heat flux on the behavior of the interface. The data plotted in Fig. 3 demonstrate a significant departure from the zero-g, finite-Q theory of Haussmann and Dohm [7] (solid line). For example, the data show an onset of thermal resistance at a lower temperature, $T_{\mathrm{c}}^{\mathrm{exp}}(Q)$, than the $T_{\mathrm{c}}(Q)$ predicted by the theory. We note that the difference $T_{\mathrm{c}}(Q) - T_{\mathrm{c}}^{\mathrm{exp}}(Q)$ is roughly constant with Q and becomes equal to $T_\lambda - T_{\mathrm{c}}(Q)$ at $Q \approx 0.5\ \mathrm{erg\,s^{-1}\,cm^{-2}} \approx Q_{\mathrm{c}}$, suggesting that the departure of the data from the theoretical prediction may be attributable to the presence of gravity.

Although these data demonstrate the existence of a nonlinear region near T_λ of roughly the dimensions predicted by theory, a quantitative test of the theory is not presently possible because of the influence of gravity. A theory including the effects of gravity at arbitrarily low-Q appears to be prohibitively difficult [18], and conducting the measurement at higher Q is experimentally impractical because of the spatial narrowing of the nonlinear region with Q. We suspect that a quantitative test of the theory will only be possible in an environment without gravity.

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