Anomalous Resistance Induced by Chaos of Electron Motion and its Application to Plasma Production

Z. Yoshida,[1] H. Asakura,[1] H. Kakuno,[1] J. Morikawa,[1] K. Takemura,[1] S. Takizawa,[1] and T. Uchida[2]
[1]*University of Tokyo, Hongo, Tokyo 113-8656, Japan*
[2]*ULVAC Japan, Ltd., Chigasaki, Kanagawa 253-8543, Japan*
(Received 25 February 1998)

Null points in magnetic fields destroy the adiabatic invariants of charged particle motion, resulting in a chaotic motion. The mixing effect of the chaos produces efficient collisionless heating of electrons. The entropy production is represented by an effective resistance in a macroscopic description. This "chaos-induced resistance" enables plasma production at a low gas pressure suitable for ultrafine plasma etching. [S0031-9007(98)07155-5]

PACS numbers: 52.80.Pi, 05.45.+b, 52.50.Dg, 52.75.Rx

Nonlinearity stems in the equation of motion of a charged particle from the spatial inhomogeneity of electromagnetic fields. The "magnetic null point" yields a strong enough nonlinearity to generate "chaos" of the particle motion [1]. The chaotic motion of electrons brings about rapid production of entropy, resulting in efficient heating of electrons at a low-collisionality regime. This nonlinear process can be applied to plasma production that meets the increasing demand for a low-gas-pressure plasma source suitable for use in ultrafine etching of semiconductors [2,3]. Moreover, this effect may play an important role in high-temperature plasmas such as solar corona, neutral sheet, and fusion plasmas. At the magnetic null point, magnetic field lines can reconnect if there is a finite resistivity (magnetic diffusivity). In many different examples, the classical collisional resistivity, which is due to the scattering of current-carrying electrons by field particles, is very small and it cannot account for the realistic reconnection rates [4]. Some different mechanisms have been proposed to explore the "anomalous resistivity," including the effect of the chaos.

The aim of this Letter is to elucidate the chaos-induced resistivity for radio-frequency (rf) driven currents in a magnetic-null region. The conventional analysis of the plasma conductivity [5] does not apply when the electron motion is nonintegrable. In a chaotic system, microscopic analyses of the orbit of a test particle do not provide us with any useful information about the collective behavior of the system of particles. By studying statistical properties of the system, we explore how the chaos, a microscopic process of generating complexity, can bring about the macroscopic effect of generating heat.

We will start with a pure collisionless model, and will show that the mixing effect of the chaos yields a rapid production of the (kinetic) entropy. This process, however, is transient, and the heating saturates after a short time. In the second step, we study the effect of collisions with neutral particles. Electrons lose energy through inelastic collisions. This process opens a "sink" of the energy in a high-energy region of the velocity space. A steady state is achieved when the same number of electrons are supplied from a low-energy region, and they "cascade" towards the sink. The intermediate energy range can be approximated by the collisionless model. The chaos accelerates the cascade process, and enhances the energy dissipation into the sink. The following numerical analysis gives a proof for the above-mentioned scenario, and derives a quantitative estimate of the resistivity.

We consider an electron that obeys Newton's equation of motion:

$$m\frac{d^2}{dt^2}\boldsymbol{X} = -e\left[\boldsymbol{E} + \left(\frac{d}{dt}\boldsymbol{X}\right)\times\boldsymbol{B}\right], \tag{1}$$

where m is the electron mass, e is the elementary charge, and $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric field and magnetic field, respectively. If $\boldsymbol{E}$ and $\boldsymbol{B}$ are spatially homogeneous fields, (1) is a linear equation with respect to $\boldsymbol{X}$. For example, let us assume that $\boldsymbol{B} = \text{const}$ and $\boldsymbol{E} = \Re e^{i\omega_0 t}\boldsymbol{E}_0$ ($\boldsymbol{E}_0 =$ const). If the frequency ω_0 is not resonant with the cyclotron frequency $\omega_c = eB/m$, the particle motion is periodic, and hence "heating" cannot occur [6]. If we can introduce "disorder" to the system, we can heat electrons. Collisions randomize the phase of oscillations of particles, resulting in a nonzero average of energy transfer from the electric field to particles. The other possible mechanism is the chaos that is a deterministic dynamics producing complex orbits of particles. Here, we consider a strongly inhomogeneous magnetic field that makes (1) nonlinear with respect to $\boldsymbol{X}$. When the adiabatic invariance of the magnetic moment is destroyed in an inhomogeneous field, the degree of freedom increases enough to generate chaotic motion of electrons.

We formulate a slab plasma model by assuming, in Cartesian coordinates,

$$\boldsymbol{B} = \begin{pmatrix} 0 \\ 0 \\ B_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ Jx \end{pmatrix}, \tag{2}$$

$$\boldsymbol{E} = \begin{pmatrix} E_x \\ E_y \\ 0 \end{pmatrix} = \Re e^{i\omega_0 t}\begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \\ 0 \end{pmatrix}, \tag{3}$$

where J is a real constant that characterizes the gradient of the magnetic field strength, and $\mathcal{E}_x$ and $\mathcal{E}_y$ are complex constants. The length scale is defined as follows. For a given ω_0, the cyclotron-resonant magnetic field B_c is given by solving $\omega_0 = eB_c/m$ for B_c. This B_c occurs at $x = L = m\omega_0/(eJ)$. We define the normalized time and coordinates by

$$\hat{t} = \omega_0 t, \qquad \hat{\boldsymbol{X}} = \frac{\boldsymbol{X}}{L}.$$

The normalized temporal derivative $(d/d\hat{t})$ will be denoted by $'$. The normalized electric field is

$$\hat{\boldsymbol{E}} = \frac{\boldsymbol{E}}{L\omega_0 B_c}.$$

The y component of (1) yields

$$\hat{y}' = \frac{\hat{x}^2}{2} - \hat{F}_y \qquad \left(\hat{F}_y = \int \hat{E}_y d\hat{t} + C\right),$$

where C is a constant number determined by the initial velocity $\hat{y}'$. The x component of (1) now reads

$$\hat{x}'' = -\frac{\hat{x}^3}{2} + \hat{F}_y(\hat{t})\hat{x} - \hat{E}_x(\hat{t}). \tag{4}$$

In the z direction, the particle moves with a constant velocity.

When $\hat{\boldsymbol{E}} = 0$, the energy (Hamiltonian) of the particle conserves, and, hence, the nonlinear Eq. (4), which involves only one degree of freedom, is integrable. The particle describes a "meandering" orbit in the magnetic null region ($|\hat{x}| \lesssim 1$). A finite electric field ($\hat{\boldsymbol{E}} \neq 0$) changes the energy to yield nonintegrable (chaotic) orbits (Fig. 1). The equation of motion (4) becomes "most nonlinear" when all terms have the same order of magnitude [7–9]. For example, let us consider an rf electric field with $\omega_0/2\pi = 13.56 \times 10^6\ \mathrm{sec}^{-1}$ and $|\boldsymbol{E}| = 10^3$ V/m. For $L = 2.4 \times 10^{-2}$ m, we obtain $|\hat{\boldsymbol{E}}| = 1$.

The maximum Lyapunov exponent of the chaotic meandering orbit is about 0.2 for $|\hat{\boldsymbol{E}}| = O(1)$. In Fig. 2, we show the Poincaré plot on the phase space $\hat{x}$-$\hat{x}'$. The orbit moves almost densely over a region in the phase space. The kinetic entropy is defined by $S = -\sum_\ell p_\ell \ln p_\ell$, where p_ℓ is the probability of realization of a certain state (cell in the phase space) denoted by ℓ (the absolute value of S depends on how we divide the phase space into cells). Figure 3 shows the time evolution of S in the chaotic dynamics (solid line), and compares it with a periodic case of $\boldsymbol{B} = 0$ (dashed line). The rapid increase in S is due to the mixing process of the chaos. The rate of the mixing is of the order of the Lyapunov exponent. In a short time, the mixing process saturates. In this slow phase, the domain, over which the orbit moves, does not expand appreciably, and the average energy of particles saturates. In this macroscopic equilibrium, the averages of chaotic accelerations and decelerations of particles cancel out.

Figure 4 shows the velocity distribution of 10^4 particles. Each particle has different initial conditions, but obeys the same equation of motion (4); viz., there are no mutual interactions such as collisions or collective motions. We observe that small variations in the initial state expand to create almost Gaussian distribution. Although the oscillating electric field is constantly applied, the total energy of particles approaches a constant vale. The system may be regarded as a canonical ensemble. The rf electric field interacts with particles, while the total energy transfer cancels out in the approximate equilibrium state; i.e., the macroscopic current is out of phase with the rf electric field, so that the average of the Poynting flux vanishes. Because of the almost ergodic motion of particles (Fig. 2), the Gibbs distribution can be deduced from the equal probability in the phase space.

In the approximate equilibrium state, the macroscopic current produces approximately periodic oscillations, although the response of the macroscopic system to the driving electric field is apparently nonlinear. Figure 5 shows the Fourier spectrum of the macroscopic current waveform. We observe that the harmonics of the frequency ω_0 dominate the oscillations.

In what follows, we discuss the role of a sink of the energy (or the entropy) in the velocity space that is introduced by including the effect of inelastic collisions into the model. Electrons lose energy through inelastic collisions of excitation and ionization, which have threshold energies. For a low temperature plasma, the sink appears in the superthermal region of the velocity space. We simulate collisions by a random process of removing particles from the sink region with a given probability that is consistent with the relevant inelastic collisions.

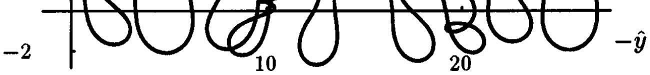

FIG. 1. The chaotic meandering motion of an electron ($|\hat{E}_y| = 1$, $|\hat{E}_x| = 0$).

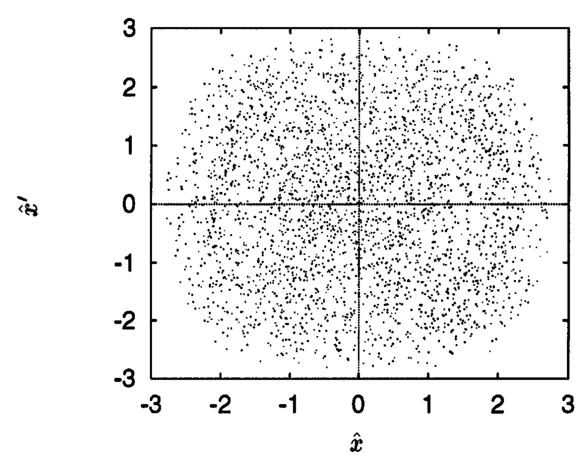

FIG. 2. The Poincaré plot of the chaotic orbit ($|\hat{E}_y| = 1$, $|\hat{E}_x| = 0$).

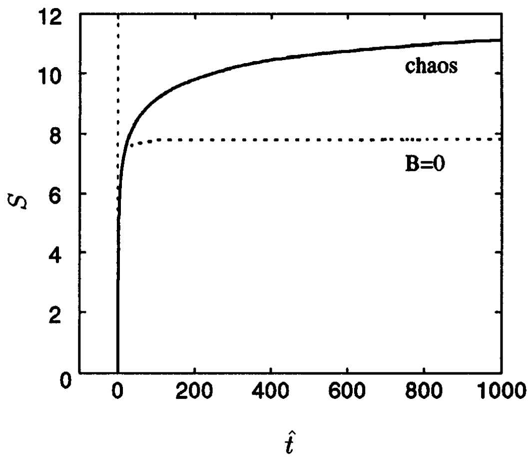

FIG. 3. Increase of the kinetic entropy S. Solid line: chaotic motion ($\hat{E}_y = 1$, $\hat{E}_x = 0$). Dashed line: periodic motion.

Particles are supplied with zero initial velocity to conserve the total number of particles. In a macroscopic steady state, the energy removed by the particles leaking into the sink balances with the energy gained by the supplied particles. To model collisions, we need parameters to evaluate the physical time and length scales. Here, we assume $\omega_0/2\pi = 13.56 \times 10^6\ \mathrm{sec}^{-1}$ and $L = O(10^{-2})$ m [see the discussion after Eq. (4)].

The mixing effect drives the cascade in the velocity space to broaden the distribution function, and enhances the energy dissipation in the sink region. We estimate the

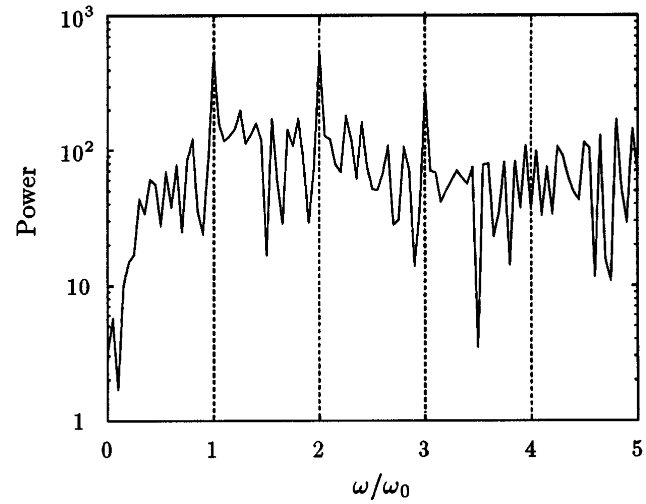

FIG. 5. The frequency spectrum of the macroscopic current ($\langle v_x \rangle$ driven by $\hat{E}_y$; integrated over $-1.6 < \hat{x} < 1.6$).

effective resistivity by the total change of the energy. In Fig. 6, we show that the effective resistivity is enhanced by the mixing effect of the chaos by a factor of $10-10^2$ in comparison with the case of $\boldsymbol{B} = 0$. The effective collision induced by the chaos is comparable to the scattering in a neutral gas of 0.1 Pa. These estimates are consistent with experimental observations [2,3].

The electron heating occurs in a narrow region around the null point. Figure 7 shows the spatial distribution of the dissipative current density (the current density that oscillates in the same phase of the rf electric field). We observe that the heat production is concentrated in the region $|\hat{x}| < 5$. Because of the localized power absorption (plasma production), a plasma source based on this heating mechanism has a potential advantage in reducing contamination due to plasma-wall interactions.

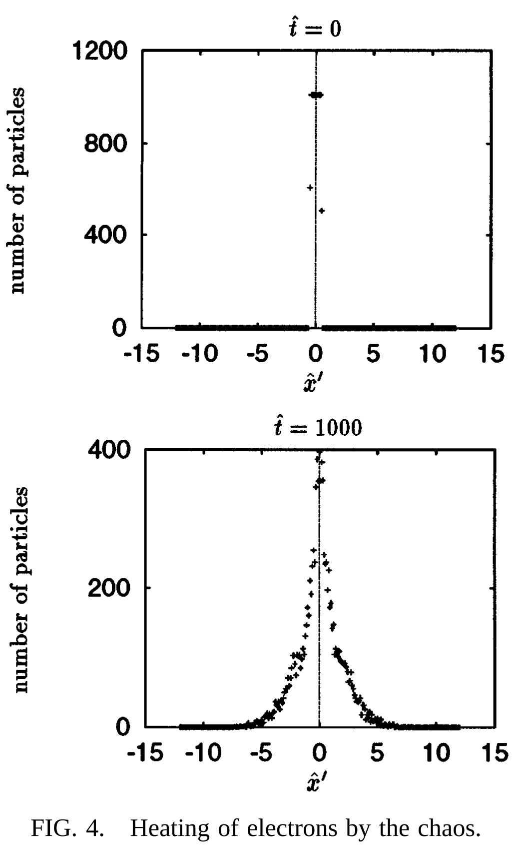

FIG. 4. Heating of electrons by the chaos.

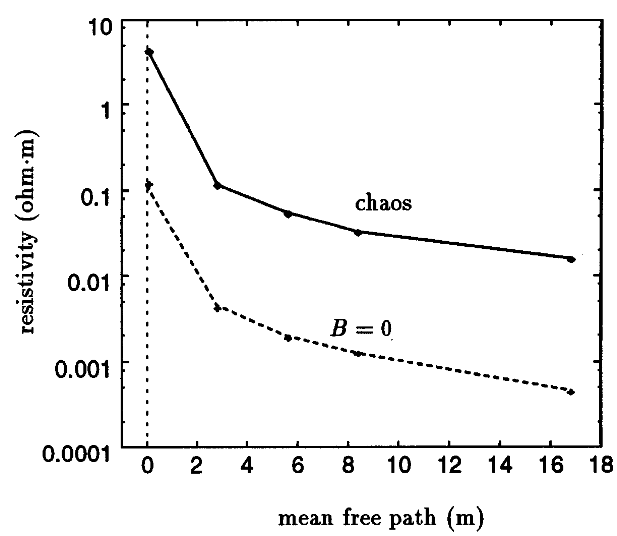

FIG. 6. Effective resistivity (solid curve) enhanced by the chaos, in comparison with the classical resistivity (dashed curve).

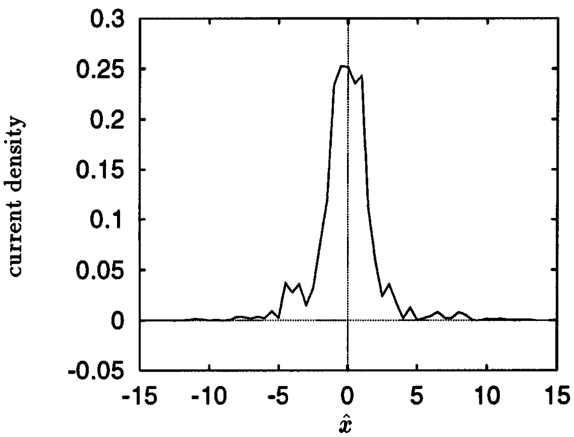

FIG. 7. Spatial distribution of the current density (the resistive component).

In summary, we have studied both collisionless and collisional models of chaos-induced resistance. In the collisionless system, an approximate canonical equilibrium is achieved after the rapid mixing phase that produces the entropy. There is an essential difference between the present collisionless chaotic system and the usual collisional system. The former system can absorb energy from the rf electric field in the first mixing phase, while the energy saturates and an approximate equilibrium state appears. In the latter case, however, unceasing heating must occur when we continue to apply an rf electric field without assuming an energy loss mechanism. The combination of the chaos effect due to the inhomogeneous magnetic field and the inelastic collision effect yields an enhanced resistance. Inelastic collisions open a sink of energy (entropy) in the high-energy region of the velocity space. This nonequilibrium system is characterized by the cascade process driven by the mixing effect. The energy dissipation is determined by the speed of the cascade, which is scaled by the Lyapunov exponent, and the energy removal rate in the sink region.

This work was supported by Grant-in-Aid for Scientific Research from the Japanese Ministry of Education, Science and Culture No. 09308011.

[1] The history of research on the particle motion in the neighborhood of a null point goes back to the early studies on cusps and neutral sheets; see J. Berkowitz and C. S. Gardner, Commun. Pure Appl. Math. **12**, 501 (1959); H. Grad and R. van Norton, Nucl. Fusion Suppl. **1**, 61 (1962); G. Schmidt, Phys. Fluids **5**, 994 (1962); S. I. Syrovatskii, Sov. Phys. JETP **23**, 754 (1965). For laboratory experiments on plasma phenomena in the vicinity of a null point, see K. Kawahata *et al.*, Nucl. Fusion **12**, 727 (1972); N. Ohyabu, S. Okamura, and N. Kawashima, Phys. Fluids **17**, 2009 (1974).

[2] T. Uchida, Jpn. J. Appl. Phys. **33**, L43 (1994); H. Tsuboi *et al.*, Jpn. J. Appl. Phys. **34**, 2476 (1995).

[3] T. Sakoda *et al.*, Jpn. J. Appl. Phys. **36**, 6981 (1997).

[4] R. Horiuchi and T. Sato, Phys. Plasmas **1**, 3587 (1994).

[5] For example, see F. F. Chen, *Introduction to Plasma Physics and Controlled Fusion* (Plenum Press, New York, 1984), 2nd ed., Chap. 4.

[6] Even if a homogeneous rf electric field is applied to a plasma, the "anomalous skin effect" modifies the distribution of the rf electric field and enables a collisionless heating; see M. M. Turner, Phys. Rev. Lett. **71**, 1844 (1993).

[7] Chaotic motion occurs in the magnetic null region ($|\hat{x}| \lesssim 1$), where B_z varies from zero to the order of B_c. This strong inhomogeneity of B_z yields a strong nonlinearity of electron motion. In the magnetic null region, the velocity $\hat{x}'$, which is amenable to the electric field of $|\hat{\boldsymbol{E}}| = O(1)$, is of order unity, and, hence, the length scale of the orbit must be of order unity. Such a particle cannot describe a cyclotron orbit because of the strong inhomogeneity of the magnetic field, and it is unmagnetized (Fig. 1). The local cyclotron frequency varies along the orbit, and, hence, the particle motion does not have a definite frequency. The cyclotron resonance cannot occur. The heating mechanism considered here is different from the cyclotron heating. Random sequences of acceleration and deceleration yield complex changes of the particle energy, which lead to the collisionless heating; see Z. Yoshida and T. Uchida, Jpn. J. Appl. Phys. **34**, 4213 (1995); H. Asakura *et al.*, Jpn. J. Appl. Phys. **36**, 4493 (1997). When the electron cyclotron resonance layer ($|\hat{x}| = 1$) is removed, the heating is decreased but still significant.

[8] For a small electric field ($|\hat{\boldsymbol{E}}| \ll 1$) and small initial velocities ($|\hat{x}'|, |\hat{y}'| \ll 1$), we can linearize the equation of motion (4) in the magnetized region. Then, the conventional theory of particle motion applies. Let $\hat{x} = \hat{x}_0(1 + \tilde{x})$ with $\hat{x}_0 = \text{const}$. We assume that $\tilde{x}$ is small. With choosing $C = \hat{x}_0^2/2$, which corresponds to the "gradient B drift velocity," we can linearize (4) as $\tilde{x}'' = -\hat{x}_0^2\tilde{x} + a\exp(i\hat{t})$, where $a = -(\hat{\mathcal{E}}_x + i\hat{\mathcal{E}}_y)/\hat{x}_0$. We obtain $\tilde{x}(\hat{t}) = a\exp(i\hat{t})/(\hat{x}_0^2 - 1)$. For many particles with random phases, the ensemble average of the energy achieves equilibrium, if the cyclotron resonance ($|\hat{x}_0| = 1$) is avoided. Resonant particles absorb the electric field energy constantly. An increase in $|\hat{\boldsymbol{E}}|$ brings about complexity of the motion. For a moderate $|\hat{\boldsymbol{E}}|$, bifurcation theories apply; see A. J. Lichtenberg and M. A. Lieberman, *Regular and Chaotic Dynamics* (Springer-Verlag, New York, 1992), 2nd ed., Chap. 4 and Sect. A.4. When $|\hat{\boldsymbol{E}}| = O(1)$, (4) is strongly nonlinear, and the particle motion becomes totally chaotic in the magnetic null region

[9] For a mathematical background, see J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems and Bifurcation of Vector Fields* (Springer-Verlag, New York, 1983).