Measurement of the Running of Effective QED Coupling at Large Momentum Transfer in the Spacelike Region

S. Odaka,[2,*] K. Abe,[1] K. Amako,[2] Y. Arai,[2] Y. Asano,[3] M. Chiba,[4] Y. Chiba,[5] M. Daigo,[6,†] M. Fukawa,[2,‡] Y. Fukushima,[2] J. Haba,[2] H. Hamasaki,[3] Y. Hemmi,[7] M. Higuchi,[8] T. Hirose,[4] Y. Homma,[9] N. Ishihara,[2] Y. Iwata,[10] J. Kanzaki,[2] R. Kikuchi,[7] T. Kondo,[2] T. T. Korhonen,[2,11,§] H. Kurashige,[7] E. K. Matsuda,[12] T. Matsui,[2] K. Miyake,[7] S. Mori,[3] Y. Nagashima,[13] Y. Nakagawa,[14,**] T. Nakamura,[12,††] I. Nakano,[15] K. Ogawa,[2,‡‡] T. Ohama,[2] T. Ohsugi,[10] H. Ohyama,[16] K. Okabe,[15] A. Okamoto,[7] A. Ono,[17] J. Pennanen,[2,11] H. Sakamoto,[7] M. Sakuda,[2] M. Sato,[8] N. Sato,[2] M. Shioden,[18] J. Shirai,[2,§§] T. Sumiyoshi,[2] Y. Takada,[3] F. Takasaki,[2] M. Takita,[13] N. Tamura,[19] K. Tobimatsu,[20] T. Tsuboyama,[2] S. Uehara,[2] Y. Unno,[2] T. Watanabe,[21] Y. Watase,[2] F. Yabuki,[4] Y. Yamada,[2] T. Yamagata,[14] Y. Yonezawa,[22] and H. Yoshida[23]

(VENUS Collaboration)

[1]*Department of Physics, Tohoku University, Sendai 980, Japan*
[2]*High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801, Japan*
[3]*Institute of Applied Physics, University of Tsukuba, Tsukuba 305-8573, Japan*
[4]*Department of Physics, Tokyo Metropolitan University, Hachioji 192-03, Japan*
[5]*Yasuda Women's Junior College, Hiroshima 731-01, Japan*
[6]*Faculty of Economics, Toyama University, Toyama 930, Japan*
[7]*Department of Physics, Kyoto University, Kyoto 606, Japan*
[8]*Department of Applied Physics, Tohoku-Gakuin University, Tagajo 985, Japan*
[9]*Faculty of Engineering, Kobe University, Kobe 657, Japan*
[10]*Department of Physics, Hiroshima University, Higashi-Hiroshima 724, Japan*
[11]*Research Institute for High Energy Physics, Helsinki University, SF-00170 Helsinki, Finland*
[12]*Faculty of Engineering, Miyazaki University, Miyazaki 889-2192, Japan*
[13]*Department of Physics, Osaka University, Toyonaka 560, Japan*
[14]*International Christian University, Mitaka 181, Japan*
[15]*Department of Physics, Okayama University, Okayama 700, Japan*
[16]*Hiroshima National College of Maritime Technology, Higashino 725-0200, Japan*
[17]*Faculty of Cross-Cultural Studies, Kobe University, Kobe 657, Japan*
[18]*Ibaraki College of Technology, Katsuta 312, Japan*
[19]*Department of Physics, Niigata University, Niigata 950-2181, Japan*
[20]*Center for Information Science, Kogakuin University, Tokyo 163-91, Japan*
[21]*Department of Physics, Kogakuin University, Hachioji 192, Japan*
[22]*Tsukuba College of Technology, Tsukuba 305, Japan*
[23]*Naruto University of Education, Naruto 772, Japan*
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The q^2 running of effective QED coupling $\alpha(q^2)$ is measured in a spacelike region, $(10\ \text{GeV})^2 \leq -q^2 \leq (54\ \text{GeV})^2$, using Bhabha-scattering and muon-pair production data at e^+e^- collisions with a center-of-mass energy of 57.77 GeV. The result verifies the prediction of a theoretical estimation with a precision of better than 1% in terms of $\alpha(q^2)/\alpha(t_0)$ with $t_0 = -(10\ \text{GeV})^2$. [S0031-9007(98)07120-8]

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The vacuum polarization (VP) of the photon field mediating charged-particle interactions is one of the most remarkable consequences from the quantum-field nature of the quantum electrodynamics (QED). The VP effect is expected to depend on the momentum transfer q^2 of the interaction and to be able to be interpreted in terms of a q^2 running of the effective QED coupling $\alpha(q^2)$ [1]. The effective coupling equals the fine-structure constant α at the Thomson limit ($q^2 = 0$) and is expected to increase logarithmically as $|q^2|$ increases at large $|q^2|$.

The VP effect is known to be appreciable and necessary to be taken into account in calculating reliable expectations for reactions in high-energy e^+e^- collisions [2]. The effect leads to corrections of more than 10% to the cross sections for reactions with $|q^2| \gtrsim (30\ \text{GeV})^2$. These reactions are, therefore, expected to provide good places for an experimental evaluation of the VP effect.

It should be noted here that, in order to fully describe e^+e^- reactions at high energies, it is necessary to use the electroweak theory instead of QED. The electroweak theory predicts the participation of the Z^0 boson together with the photon. Hence, uncertainties on Z^0 properties may hide the VP effect from an experimental evaluation.

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Recent precise measurements on the Z^0 resonance [3] have enabled an evaluation in the off-resonance region.

Theoretically, the VP effect can be estimated by adding the contributions of all possible virtual states (loops) that can be embedded in the photon propagator mediating charged-particle interactions. While the contribution of leptonic loops can be reliably estimated from perturbative calculations, hadronic contributions are usually estimated with a dispersion integral based on the experimental data concerning the reaction $e^+e^- \to$ hadrons [2,4], because of the presence of nonperturbative strong interactions. Thus, the estimation suffers from ambiguities arising from techniques used for combining the results of many experiments, as well as those from experimental uncertainties. Being motivated by the need for a precise determination of weak parameters at the CERN Large Electron-Positron Collider (LEP) experiments, the estimation has been improved [5–7] and is now being considered to reach a precision at the level of 0.1% in terms of $\alpha(q^2)/\alpha$ [8].

Concerning the effect in the timelike region ($q^2 = s > 0$), an experimental evaluation was carried out by the TOPAZ group at TRISTAN [9,10] and the OPAL group at the LEP collider [11] using data on the reactions $e^+e^- \to$ hadrons and $e^+e^- \to \mu^+\mu^-$. Although they observed a significant contribution of the VP effect, the precision of the evaluation is limited by the data statistics and is still worse than 1%.

It has been pointed out for a long time that the VP effect in the spacelike region ($q^2 = t < 0$) can be probed in Bhabha scattering, $e^+e^- \to e^+e^-$ [4]. This reaction is expected to be dominated by the exchange of a spacelike (t-channel) photon, for which the momentum transfer is correlated to the scattering angle θ as $t = -s(1 - \cos\theta)/2$. Therefore, in principle, the effect can be observed in the scattering-angle dependence of the reaction. Although the required measurement is very simple, no evaluation has yet been made, due to the limited precision of measurements and a lack of suitable theoretical frameworks.

As reported [12], we measured Bhabha scattering at a center-of-mass energy ($\sqrt{s}$) of 57.77 GeV with good precision, based on high-statistics data collected with the VENUS detector at the TRISTAN e^+e^- collider of KEK. As a result of this measurement, we have observed a significant running effect of $\alpha(q^2)$ for the first time in this reaction. However, we could not carry out any direct measurement of $\alpha(q^2)$, because of the presence of a non-negligible contribution of the timelike (s-channel) photon exchange. We needed to impose an assumption on the q^2 dependence in order to evaluate the significance of the observation.

Recently we have completed an analysis on the muon-pair production, $e^+e^- \to \mu^+\mu^-$, based on the same data sample [13]. This measurement provides information on the s-channel photon exchange. Combining this result with the Bhabha-scattering result, we can make an assumption-free evaluation of the VP effect in the spacelike region.

Along with the experimental improvements, the existence of a suitable theoretical framework is crucial for studies to probe the VP effect. It has been shown [14,15] that external photon-radiation corrections can be treated separately from short-range hard (effective-Born) processes including internal propagator and nonphotonic vertex corrections. This allows us to separate the VP effect from complicated experiment-dependent corrections due to photon radiation. Note that the separation was not clear in traditional treatments of the radiative correction [16]. Furthermore, based on the above argument, precise calculations of the photon-radiation corrections have been made available for analyses of experimental data [14,15].

In addition, it has been established that the internal corrections within the framework of the standard electroweak theory can be implemented with good accuracy by a simple replacement of fundamental constants in tree-level (Born) formulas with q^2-dependent effective parameters [improved-Born (IB) approximation] [17]. The approximation includes the replacement of α with $\alpha(q^2)$ with the same definition that has been introduced within the framework of QED.

Based on the improved-Born approximation, the effective-Born (EB) cross section for Bhabha scattering is described by the sum of the ten terms given in Eq. (20) of Ref. [12]. The muon-pair production is described by the sum of three s-channel terms indicated with the following suffices: $\gamma_s\gamma_s$, $\gamma_s Z_s$, and $Z_s Z_s$. Since the Z^0-boson properties are known precisely [3], we can evaluate $\alpha(q^2)$ by fitting the formulas to the experimental results.

In the following discussion the mass and total decay width of Z^0 is fixed to 91.19 GeV$/c^2$ and 2.49 GeV, respectively, with the effective weak mixing-angle $\sin^2\theta_W^{\rm eff}$ for leptons of 0.232. The ρ parameter in the effective Z^0 coupling is fixed to 1.01, taking into account a correction due to the heavy top-quark mass (174 GeV$/c^2$). The influence of the errors in these parameters is negligible, since the interactions are dominated by photon exchanges at our energy.

As a measurement for Bhabha scattering, we use the result on the EB cross section normalized to the cross section in the end-cap (EC) region ($0.822 \le \cos\theta \le 0.968$),

$$\frac{dR_i^{\rm EB}}{d\Omega} = \frac{d\sigma_i^{\rm EB}/d\Omega}{\sigma_{\rm EC}^{\rm EB}}\,. \qquad (1)$$

The result is obtained for 12 scattering angles ($i = 1$–12), as presented in Table V of Ref. [12].

As for the muon-pair production, we use the total cross section ($\sigma_{\rm TOT}^{\rm EB}$) and the forward-backward (FB) asymmetry ($A_{\rm FB}^{\rm EB}$), obtained from a fit to the differential EB cross section [13]. In order to make a uniform treatment, we convert $\sigma_{\rm TOT}^{\rm EB}$ to a ratio to the EC Bhabha scattering, defined as

$$R^{\mathrm{EB}}_{\mu\mu} \equiv \frac{N^{\mathrm{EB}}_{\mathrm{TOT}}(\mu\mu)}{N^{\mathrm{EB}}_{\mathrm{EC}}} = \frac{\sigma^{\mathrm{EB}}_{\mathrm{TOT}}(\mu\mu)}{\sigma^{\mathrm{EB}}_{\mathrm{EC}}}. \tag{2}$$

The converted result is

$$R^{\mathrm{EB}}_{\mu\mu} = (7.38 \pm 0.13) \times 10^{-3}, \quad A^{\mathrm{EB}}_{\mathrm{FB}} = -0.350 \pm 0.017, \tag{3}$$

with an error correlation of -0.036. The error from the luminosity determination (0.9%) is not included in this result. The conversion was done by refitting the angular distribution after excluding the luminosity error.

The error matrix for these 14 measurement results is given by the quoted errors, together with the correlation matrix shown in Table IV of Ref. [12] and the error correlation between $R^{\mathrm{EB}}_{\mu\mu}$ and $A^{\mathrm{EB}}_{\mathrm{FB}}$. The experimental error in the measurement of the EC Bhabha (0.7%) is added to the error matrix as a common normalization error for $dR^{\mathrm{EB}}_i/d\Omega$ and $R^{\mathrm{EB}}_{\mu\mu}$.

The experimental results used in the present discussion are all EB quantities, for which experiment-dependent photon-radiation effects are unfolded. The unfolding parameters were determined by using a computer program ALIBABA [15]. The precision of the estimation by ALIBABA is anticipated to be at the level of 0.5% or better for individual cross sections. Indeed, the precision for the EC Bhabha scattering is estimated to be 0.47% from our study [13]. The influence of this error is expected to be smaller for the quantities that we use in the present discussion, because the major part of the error would be common to the measured reactions. However, the details are hard to discuss here. Instead, we conservatively add an error of 0.3% as a normalization error for $dR^{\mathrm{EB}}_i/d\Omega$, and an error of 0.5% for $R^{\mathrm{EB}}_{\mu\mu}$. In addition, another error of 0.3% is assigned as a common error for $dR^{\mathrm{EB}}_i/d\Omega$ and $R^{\mathrm{EB}}_{\mu\mu}$.

Parameters allowed to vary in the fit must be chosen so that they can determine the predictions for the 14 measured quantities simultaneously. In addition, the correlation between them is desired to be small. Roughly speaking, the result for Bhabha scattering is determined by $\alpha(t)/\alpha(t_0)$ predominantly, and $R^{\mathrm{EB}}_{\mu\mu}$ must be sensitive to $\alpha(s)/\alpha(t_0)$, where t_0 represents a typical t for Bhabha scattering in the EC region.

Based on these thoughts, we choose the fitting parameters to be

$$\frac{\alpha(t_0)}{\alpha}, \qquad \frac{\alpha(t_m)}{\alpha(t_0)}\ (m = 1\text{–}5), \qquad \frac{\alpha(s)}{\alpha(t_0)}. \tag{4}$$

From an average weighted by the angular distribution,we choose as

$$t_0 = -(10.0\ \mathrm{GeV})^2. \tag{5}$$

The first parameter in Eq. (4) is introduced only for a technical reason. We don't expect any good measurement for it, since the experimental results are sensitive to it only indirectly through the Z^0 exchange and γ-Z interference terms. These terms have relatively smaller contributions at our energy. The effective coupling in the spacelike region $\alpha(t)$ is assumed to be constant in certain ranges of t (denoted by t_m) at large scattering angles. The assumed ranges are shown in Table I. The boundaries are determined from those for the $\cos\theta$ bins in the Bhabha-scattering measurement. The ranges in t_m include several bins in $\cos\theta_i$, except for the smallest-$|t|$ bin.

The result of the fit is presented in Table I. The result for $\alpha(t_0)/\alpha$ has a large error, as expected. The experimental results are mainly sensitive to the other parameters, which represent the running from $q^2 = t_0$. The parameter $\alpha(t_m)/\alpha(t_0)$ has been determined with a precision appreciably better than 1% in small $|t|$ bins, where the precision is dominantly determined by normalization errors. Thus, the error correlation is large in these bins. Normalization errors are gradually overwhelmed by the statistical error as $|t|$ becomes larger. Accordingly, the error correlation becomes smaller. The precision is still close to 1% even in the largest $|t|$ bin, where the error is dominated by data statistics.

In the above fit we assumed the running of $\alpha(q^2)$ estimated in Ref. [5] for calculating the Bhabha cross section integrated over the end-cap region ($\sigma^{\mathrm{EB}}_{\mathrm{EC}}$). We confirmed that the dependence of the result on this assumption is very small. Even if no running is assumed in the end-cap region, the change in the fit result is smaller than 0.0001 for all parameters. This is a result of a proper choice of the fitting parameters and the t_0 value. The dependence becomes larger if we choose other values for t_0.

The results for $\alpha(t_m)/\alpha(t_0)$ and $\alpha(s)/\alpha(t_0)$ are plotted in Fig. 1 as a function of $Q = \sqrt{|q^2|}$. We can obviously see a systematic enhancement of the results from unity, corresponding to an observation of the running. In the

TABLE I. Result of the fit to the experimental results. The best fit is presented together with the correlation matrix for the estimated error.

	$Q = \sqrt{\lvert q^2\rvert}$ (GeV)	Fit	Error correlation						
$\alpha(t_0)/\alpha$	10.0	1.013 ± 0.052	1	0.046	0.036	−0.035	−0.101	0.015	0.101
$\alpha(t_m)/\alpha(t_0)$	20.7−25.2	1.0067 ± 0.0047		1	0.696	0.531	0.423	0.264	0.368
	25.2−32.4	1.0099 ± 0.0049			1	0.518	0.411	0.246	0.374
	32.4−38.2	1.0099 ± 0.0061				1	0.338	0.217	0.285
	38.2−45.6	1.0144 ± 0.0074					1	0.248	0.138
	45.6−53.9	1.0328 ± 0.0114						1	−0.119
$\alpha(s)/\alpha(t_0)$	57.77	1.0063 ± 0.0095							1

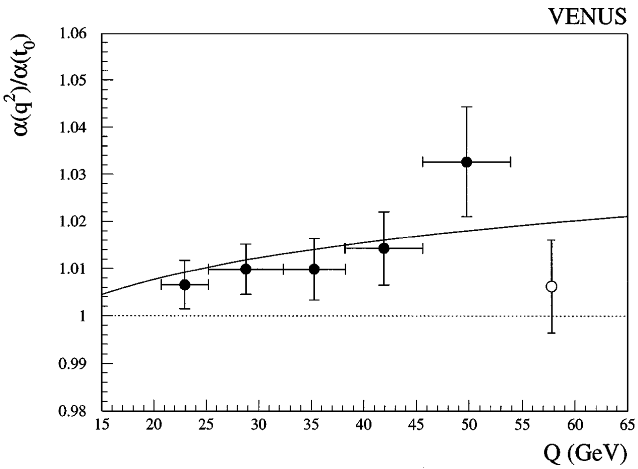

FIG. 1. Plot of the results relevant to the running of the effective QED coupling from $q^2 = t_0$, shown in Table I. The filled circles show the results for the spacelike region ($q^2 = t$), and the open circle for the timelike region ($q^2 = s$). The curve indicates the prediction from the functional result by Burkhardt *et al.* [5].

figure, the results are compared with a prediction from a functional result given in Ref. [5]. The prediction well describes the experimental result. Namely, the validity of the theoretical calculation has been experimentally proved with a precision at the level of 1% or better in the studied q^2 region.

We have obtained an assumption-free result on the q^2 running of the effective QED coupling $\alpha(q^2)$ in the spacelike region using Bhabha scattering and muon-pair production data. This result indicates the possibility for an experimental verification close to the level that is anticipated for the precision of recent theoretical estimates, although such a verification cannot be made with the present result alone. A compilation of similar measurements by on-going and near-future experiments at various energies is desired. Along with experimental improvements, it is also necessary to improve the knowledge of photon-radiation effects in order to achieve the desired precision.

The studied q^2 range is rather limited in the present measurement. In order to verify the running more clearly, the range is desired to be expanded by future measurements, especially to smaller $|q^2|$ values. An expansion can be made by using Bhabha scattering at smaller angles for the normalization. Another possible way is to use the $e^+e^- \to e^+e^-\mu^+\mu^-$ reaction, which the exchange of nearly on-shell ($q^2 \approx 0$) photons dominates. This method was used by the TOPAZ group for evaluating the running in the timelike region [10]. However, since the photon-radiation effect has been studied only up to dominant parts of the first-order correction for this reaction [18], more detailed theoretical studies are necessary in order to achieve a verification precision better than 1%.

*Electronic address: shigeru.odaka@kek.jp
†Present address: Faculty of Economics, Doshisha University, Kyoto 602-8580, Japan.
‡Present address: Naruto University of Education, Naruto 772, Japan.
§Present address: Paul Scherrer Institute, SLS, CH-5232 Villigen PSI, Switzerland.
**Present address: Faculty of Science, Ehime University, Matsuyama 790, Japan.
††Deceased.
‡‡Deceased.
§§Present address: Department of Physics, Tohoku University, Sendai 980, Japan.

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