Accurate Measurement of the Planck Constant

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Using a moving coil watt balance, electric power measured in terms of the Josephson and quantum Hall effects is compared with mechanical power measured in terms of the meter, kilogram, and second. We find the Planck constant $h = 6.626\,068\,91(58) \times 10^{-34}$ J s. The quoted standard uncertainty (1 standard deviation estimate) corresponds to $(8.7 \times 10^{-8})h$. Comparing this measurement to an earlier measurement places an upper limit of 2×10^{-8}/yr on the drift rate of the SI unit of mass, the kilogram. [S0031-9007(98)07164-6]

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We have measured the Planck constant h with a standard uncertainty (1 standard deviation estimate) of $(8.7 \times 10^{-8})h$. This new value has important implications for the fundamental constants, including the next least-squares adjustment of the constants [1]. This is clearly demonstrated in Table I, which lists six of the constants whose values are limited by the uncertainty of this measurement and lists the values assumed for other constants needed in the calculations. The equations used to relate these constants to h can be found in [6]. Because this measurement provides the most accurate connection between at atomic constant and the kilogram, the last artifact standard in the International System of Units (SI), it provides an improvement in the assigned uncertainties of all atomic fundamental quantities that are dependent on the kilogram, as well as providing a means of monitoring any changes in the kilogram.

The Planck constant is determined in a moving-coil watt balance experiment which realizes the electrical units as defined in the SI. The National Institute of Standards and Technology (NIST) watt balance [7] has been designed to measure the ratio of mechanical to electrical power, linking the artifact kilogram, the meter, and the second to the practical realizations of the ohm and the volt derived from the quantum Hall effect (QHE) and the Josephson effect (JE), respectively. Josephson voltages, $U_J(n, f)$, and quantized Hall resistances, $R_H(i)$, can be realized by using the following equations:

$$U_J(n, f) = nf/K_J = nf/(2e/h)$$
$$\text{and} \quad R_H(i) = R_K/i = (h/e^2)/i\,, \tag{1}$$

where n and i are integers, e is the elementary charge, f is the frequency applied to a Josephson device, K_J the Josephson constant, and R_K the von Klitzing constant. Notice that $(K_J)^2 R_K$, which is inversely proportional to electric power, is also

$$(K_J)^2 R_K = (2e/h)^2 (h/e^2) = 4/h\,. \tag{2}$$

The conventional values adopted internationally in 1990 to maintain practical electrical units of voltage and resistance are $K_{J\text{-}90}$ and $R_{K\text{-}90}$, and their values are given in Table I.

The experiment, first proposed by Kibble [8], consists of two measurement modes. In the first mode a voltage reference U is used to servo control the velocity v of a coil moving vertically in a magnetic field. In the second mode, a current I passing through the same coil, now held stationary in the same magnetic field, is used to balance

TABLE I. Fundamental constants improved by this measurement and values used to calculate them. The International Committee for Weights and Measures, CIPM, adopted the indicated values in 1990 [2]. u_r means relative standard uncertainty.

Constant	Symbol	Value	Unc. u_r (10^{-8})
Planck constant	h	$6.626\,068\,91(58) \times 10^{-34}$ J s	8.7 this work
Josephson constant (SI)	$K_J = 2e/h$	483 597.892(21) GHz/V	4.4 this work
Electron mass	m_e	$9.109\,382\,11(80) \times 10^{-31}$ kg	8.8 this work
Proton mass	m_p	$1.672\,621\,62(15) \times 10^{-27}$ kg	8.9 this work
Avogadro constant	N_A	$6.022\,141\,84(52) \times 10^{23}$ mole^{-1}	8.7 this work
Elementary charge	e	$1.602\,176\,48(7) \times 10^{-19}$ C	4.4 this work
Josephson constant	$K_{J\text{-}90}$	483 597.9 GHz/V	exact (CIPM)
von Klitzing constant	$R_{K\text{-}90}$	25 812.807 Ω	exact (CIPM)
1/(fine-structure constant)	$1/\alpha$	137.035 999 93(52)	0.38[a]
Rydberg constant	R_∞	10 973 731.568 639(91) m^{-1}	0.000 83[b]
Electron's atomic mass	m_e/m_u	0.000 548 579 911 1(12)	0.021[c]

[a]Reference [3]. [b]Reference [4]. [c]Reference [5].

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the force $F_z = mg$, where m is the mass of a standard mass and g is the local acceleration of gravity. The simple equation $F_z/I = mg/I = -\delta\Phi/\delta z = U/\nu_z$, where $\delta\Phi/\delta z$ is the vertical magnetic flux gradient in the coil, relates the two modes. This can be rewritten as

$$UI = mg\nu_z = (\boldsymbol{F} \cdot \boldsymbol{\nu})_z\,, \qquad (3)$$

which equates the electric power and the mechanical power. The reason this equation can be realized with a small uncertainty is that it is evaluated in two modes. In the velocity mode no current flows in the coils dissipating power, and in the balance mode the power dissipation from friction is negligible for the minimal motion of the balance and coil. Thus this experiment uses Eq. (3) to equate virtual power. Rewriting Eq. (3) to explicitly indicate the units used in the experiment, we obtain

$$\{UI\}_{90} W_{90} = \{mg\nu_z\}_{\mathrm{SI}}\ \mathrm{W} \quad \text{or} \quad W_{90}/\mathrm{W} = \{mg\nu\}_{\mathrm{SI}}/\{UI\}_{90}\,, \qquad (4)$$

where W_{90} and W are the units of power in their respective systems and the quantities in {} are the measured values in those units. From $\{nf/K_{\mathrm{J}}\}\mathrm{V} = \{nf/K_{\mathrm{J}\text{-}90}\}V_{90}$, $\{R_{\mathrm{K}}\}\Omega = \{R_{\mathrm{K}\text{-}90}\}\Omega_{90}$, $W_{90} = (V_{90})^2/\Omega_{90}$ (where V_{90} and Ω_{90} are the units of voltage and resistance), and using Eqs. (2) and (4), it follows that

$$h = \{4/[(K_{\mathrm{J}\text{-}90})^2 R_{\mathrm{K}\text{-}90}]\}\,(W_{90}/\mathrm{W}) = \{4/[(K_{\mathrm{J}\text{-}90})^2 R_{\mathrm{K}\text{-}90}]\}\,(\{mg\nu_z\}_{\mathrm{SI}}/\{UI\}_{90})\,. \qquad (5)$$

A measurement of h does not depend on the values chosen for $K_{\mathrm{J}\text{-}90}$ and $R_{\mathrm{K}\text{-}90}$ as long as the Josephson and QHE effect devices are used to measure $\{UI\}$. However, $K_{\mathrm{J}\text{-}90}$ was chosen using the measurements available in 1990 to make $\{mg\nu\}_{\mathrm{SI}}/\{UI\}_{90} = 1$ and any measured deviation means the conventional values $K_{\mathrm{J}\text{-}90}$ and $R_{\mathrm{K}\text{-}90}$ need adjusting to preserve this equality.

Figure 1 shows the configuration of the experiment. The axial force on a loop of wire in a purely radial field ($\boldsymbol{B} = [B_a(z)/r]\hat{\mathbf{r}}$, where $B_a(z)$ is nearly constant with z and time) is independent of the wire shape. A superconducting magnet consisting of two solenoid sections wound in opposition produces a 0.1 T radial field outside the magnet Dewar. $B_a(z)$ varies by $+385\ \mu\mathrm{T/T}$ from the center over a ± 35 mm vertical displacement, maintaining this variation over days. Because the field and not the flux must be constant, the magnet is operated in a constant current mode with 5.6 A. The magnet has 200 000 turns and an inductance of 5000 H. Two induction coils, each with 2355 turns, are located in the radial field. The lower induction coil is fixed to the support structure of the balance and the upper "moving" induction coil is attached to a wheel balance located above the Dewar. Aligning the magnetic field perpendicular to gravity, and the inductive coils to the field, is essential so that all forces and velocities measured are vertical. Thus the balance is a 31 cm radius wheel that operates like a pulley, where the inductive coils, mass standards, and countermass hang from flat bands of 50 strands of wire rolling on the wheel, allowing the coil to move strictly vertically for 100 mm as the wheel rotates $\pm 10°$. The five degrees of motion for the induction coil, other than vertical, are monitored, and excitations are damped. This monitoring of coil motion plus some mutual inductance techniques are used to align our experiment and to estimate the alignment errors [9].

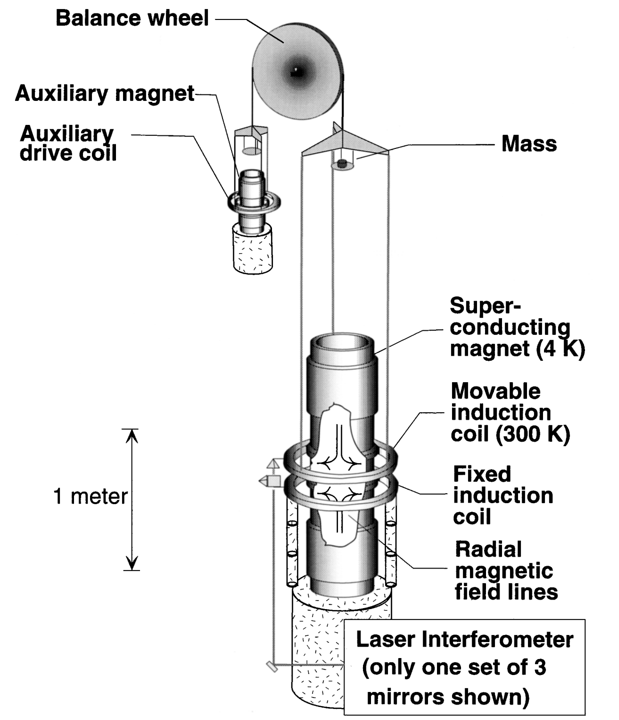

FIG. 1. Schematic of watt balance experiment. The wheel, both magnets, and the fixed induction coil are all rigidly connected. A Dewar is between the superconducting magnet and the induction coils.

For the first part of the measurement, the velocity phase, no mass standards are on the pans and a small force on the countermass side is applied, via an auxiliary coil and permanent magnet, to produce a velocity of about 2 mm/s that generates a constant voltage 1.018 ± 0.001 V across the moving induction coil. We synchronously measure the time, the voltage difference between the moving and fixed induction coils, and the distance between these coils, eliminating voltage and motion common to both. Three interferometers, spaced equally apart on the coils, record the coil center-of-mass position while three digital voltmeters integrate voltage between successive position readings with less than 200 ns dead time. With the interferometry performed in air, a refractive index is calculated from pressure, temperature, and humidity sensors. The resulting U/ν ratio has a vibration related noise of about 0.002% and must be extensively averaged. The $B_a(z)/B_a(z=0)$ variation is measured with 650 U/ν measurements timed uniformly over 85 mm travel. The field's z dependence is modeled with an eighth order orthogonal polynomial from hundreds of curves measured daily, which is then used in calculating

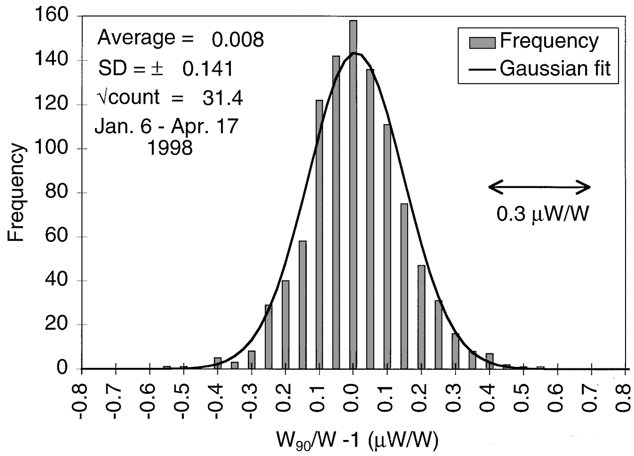

FIG. 2. Histogram of most recent 989 watt measurements.

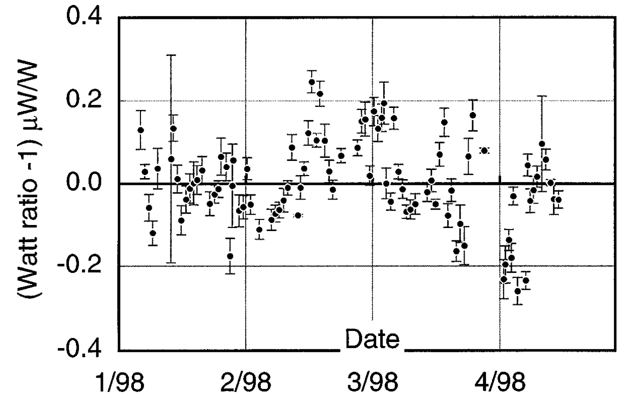

FIG. 3. The daily average of the latest watt results. The error bars are the standard deviation of the mean each day's results.

the temporal changes [± 0.2 $(\mu B_a/B_a)$/h] for each U/ν ratio at the position that the weighings are made. A set of 10 up and 10 down velocities takes 30 min.

In the second mode, the balance phase, a tare weight of 500 g placed on the countermass pan is balanced by a -10.18 mA servo current in the induction coil. As the 1 kg standard mass is placed on and off the pan, the induction coil current is reversed while allowing minimal rotation of the balance, and a ± 1.018 V reading across a 100 Ω standard resistor is recorded. Five mass "on-off" sequences take 30 min. Each F/I ratio is combined with before and after U/ν ratios for a single watt datum, where each F/I and U/ν ratio generally has equivalent relative statistical uncertainties within 0.08 μW/W. The histogram plot in Fig. 2 shows our last 989 watt readings over four months. The 0.14 μW/W standard deviation of this data shows the precision of our experimental process.

The experiment is automated and runs nightly and over holidays to reduce vibrations. Figure 3 shows the results of the last four months, where each point is a run consisting of 8 to 20 watt readings, on average. The error bars are the standard deviation of the mean. Our most recent data has day-to-day fluctuations slightly larger than expected after accounting for shorter term scatter, calibrations, or other effects that might contribute noise at low frequency. We estimate (by considering the standard deviation as a function of the "bin" size in which the data are grouped) that a 0.03 μW/W assignment for the statistical (type A) uncertainty is appropriate.

The final uncertainty is dominated by type B uncertainties, shown in Table II. A more detailed discussion of these uncertainties is submitted for publication [10]. Of the additional possible error sources that contribute to the uncertainty, the four largest contributors are as follows: (1) The index of refraction of air. Our distance measurements rely on the modified Edlen formula for calculating the index of air. However, we have constructed an absolute refractometer [11] that agrees with our Edlen calculations to about 0.05 μW/W. Air analysis from a residual gas analyzer shows no exceptional variations or contaminants. (2) Our present alignment procedures, which have resolution limits, are reasonably stable but are performed infrequently, so an error could systematically affect a few months of data. These alignment corrections are discussed in some detail in an earlier paper [9]. (3) The voltage measurements are limited by unpredictable thermal emfs associated with two volt transfers back to a Josephson array voltage standard, which is maintained in the watt laboratory. Although the estimated voltage uncertainty is actually quite reasonable (15 nV), one thermal especially is unpredictable, and, since voltage measurements occur twice in the watt calculation, the uncertainty doubles. (4) Residual knife-edge hysteresis effects during force measurements are one of the largest causes of short term fluctuations in the data. All balances have this problem, but inelastic deformation from the large knife-edge deflections in the velocity mode and 0.3 mm (1 mrad) wobbles as the mass is taken on and off limit the performance of our balance.

TABLE II. Relative standard uncertainties in the NIST watt experiment.

Uncertainty source	Value (nW/W)
Reference transfers (type B)	
Mass	20
Resistance	8
Voltage	30
Length	5
Frequency	5
Gravity	7
External effects	
Refractive index	43
Mass buoyancy	23
Alignments	40
Leakage resistance	20
Magnetic flux z-profile fit	20
Knife-edge hysteresis	20
RF noise offsets	10
RSS subtotal	82
Statistical type A	30
Combined	87

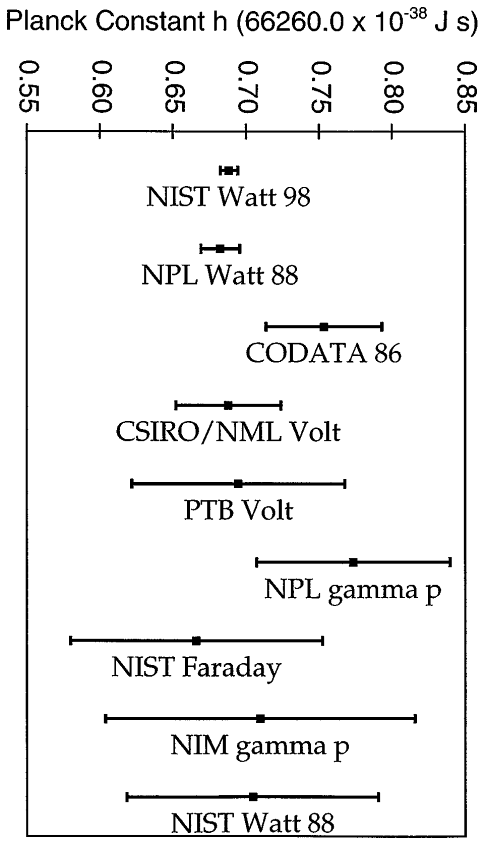

FIG. 4. Comparison with other electrical measurements of h. The National Laboratory for England is NPL; for Australia is CSIRO/NML; for Germany is PTB; for China is NIM. CODATA, the Committee on Data for Science and Technology of the International Council of Scientific Unions, Task Group on Fundamental Constants.

Using the data discussed above and shown in Figs. 2 and 3 and Table II, we obtain a relative standard uncertainty of 0.087 μW/W; the final result is

$$(W_{90}/\mathrm{W} - 1) = (+0.8 \pm 8.7) \times 10^{-8}. \quad (6)$$

Equation (6) gives for the Planck constant

$$h = 6.626\,068\,91(58) \times 10^{-34}\ \mathrm{J\,s}. \quad (7)$$

Figure 4 illustrates the relation of this value with the other electrical measurements of h. The values used in Fig. 4 are referenced in [1]. One of those, the 1988 National Physical Laboratory (NPL) watt result [12] was the most accurate with an uncertainty of 0.2 μW/W assigned as the authors recently suggest [13]. Currently the new NPL experiment has a better signal-to-noise ratio than that reported here, in part because they have their experiment in vacuum. They will likely have the most accurate result when uncertainty testing is completed. Using the 1988 NPL value for $\{W_{90}/\mathrm{W}\}_{1988}$, and our 1998 value results in a drift of the value of the watt:

$$(\{W_{90}/\mathrm{W}\}_{1998} - \{W_{90}/\mathrm{W}\}_{1988})/(1998 - 1988) = (1 \pm 2) \times 10^{-8}/\mathrm{yr}. \quad (8)$$

Since the watt is directly proportional to the Planck constant, it follows that this limit, $\pm 2 \times 10^{-8}$/yr, on the drift of the watt is a limit of the drift of the SI unit of mass, the kilogram. This is comparable with a less direct determination suggested by Davis [14] of $\pm 2 \times 10^{-8}$/yr.

By making the connection between the macroscopic unit of mass (the kilogram) to quantum standards based on the JE and QHE, this experiment provides a significant improvement in the Planck constant as well as many other constants. This 9 parts in 10^8 measurement of h meets the original design goal of this experiment and represents a factor of 15 improvement over our previous result [15]. It also allows us to set an upper limit on the drift of the last artifact in the base units of the SI. We are now implementing major modifications, including a vacuum system, that will allow a tenfold reduction in uncertainty [10]. When that new level is achieved, we will be able to monitor the kilogram and consider its possible redefinition based on defining some constant of nature, for example, either h or the atomic mass unit.

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