

Comment on “Quantum Statistical Mechanics of an Ideal Gas with Fractional Exclusion Statistics in Arbitrary Dimension”

In a recent Letter on quantum statistical mechanics of an ideal gas with fractional exclusion statistics, Iguchi [1] proved an interesting property that anyons can be regarded as composites of fermions and boson in arbitrary dimension, i.e., in an N -anyon system the thermodynamic potential $Q(a, N)$ can be factorized in terms characteristic

of the ideal boson ($a = 0$) and fermion ($a = 1$) gases by the relation $Q(a, N) = (1 - a)Q(0, N_b) + aQ(1, N_f)$, in which $N = N_f + N_b$. In this Comment I first use high-temperature perturbation theory to point out that this relation does not exist in an arbitrary dimension. Next, I prove that in the two-dimensional N -anyon system the thermodynamic potential can be factorized as $Q(a, N) = (1 - a)Q(0, N) + aQ(1, N)$.

First, from the anyon distribution function $n = 1/(W + a)$, and the relation $W^a(1 + W)^{(1-a)} = e^{\beta(\varepsilon - \mu)}$ [2], the high-temperature expansion of the energy becomes

$$E(a, N) = \frac{D}{2} NkT \left[1 + \frac{N}{V} \lambda^D (-1 + 2a)/2^{(1+D/2)} + \left(\frac{N}{V} \lambda^D \right)^2 \times [1/2^D - 2/3^{(1+D/2)} - 4a/2^D + 3a/3^{D/2} + 4a^2/2^D - 3a^2/3^{D/2}] + \dots \right],$$

in which λ is the conventional thermal wavelength, and D is the space dimension. From this result we see that the relation $E(a, N) = (1 - a)E(0, N_b) + aE(1, N_f)$ cannot be satisfied for any value of $\{N_f, N_b\}$ with the condition $N = N_f + N_b$. Thus the theorem 3 in [1] is incorrect. The mistake can be traced to the fact that the definitions $W(\varepsilon) = e^{\beta(\varepsilon - \mu_f)}$ and $1 + W(\varepsilon) = e^{\beta(\varepsilon - \mu_b)}$ used in [1] can be satisfied only for $\varepsilon = 0$ but not for other values of ε .

Next, I have evaluated the high-temperature expansion of the energy $E(a, N)$ to order $(\lambda^D)^7$, and the result shows that *only in two dimensions* can it be linear in a . Let me now prove that energy $E(a, N)$ is linear in a . Using the definition of W we have two useful relations: $d[\ln(1 + W^{-1})]/d\varepsilon = -\beta n$ and $W^{-1}(1 + W)^{-1}dW/da = n[-\beta d\mu/da + \ln(1 + W^{-1})]$. Then, as the density of state N_d is constant in two dimensions we have the relation

$$\begin{aligned} d[PV\beta]/da &= \int d\varepsilon N_d d[\ln(1 + W^{-1})]/da = - \int d\varepsilon N_d W^{-1}(1 + W)^{-1} dW/da \\ &= - \int d\varepsilon N_d n [-\beta d\mu/da + \ln(1 + W^{-1})] \\ &= N\beta d\mu/da - kT \int d\varepsilon N_d \ln(1 + W^{-1}) d[\ln(1 + W^{-1})]/d\varepsilon \\ &= N\beta d\mu/da + \frac{kT}{2} N_d \{\ln[1 + W(0)^{-1}]\}^2. \end{aligned}$$

Now, because μ is linear in a [2] and $W(0) = e^{-\beta\mu_b}$, we thus see that $d[PV\beta]/da$ does not depend on a . This means that the thermodynamic potential $Q(a, N)$ is linear in a and can be factorized as $Q(a, N) = (1 - a)Q(0, N) + aQ(1, N)$.

In summary, the relation $Q(a, N) = (1 - a) \times Q(0, N_b) + aQ(1, N_f)$, where $N = N_f + N_b$, does not hold in an arbitrary dimension anyon system, while in a two-dimensional anyon system the thermodynamic potential can be factorized as $Q(a, N) = (1 - a)Q(0, N) + aQ(1, N)$.

Wung-Hong Huang
Department of Physics
National Cheng Kung University
Tainan, 70101, Taiwan

Received 6 April 1998
PACS numbers: 05.30.-d

[S0031-9007(98)07154-3]

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