Relationship between the Superconducting Energy Gap and the Critical Temperature in High-T_c Superconductors

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(Received 9 March 1998; revised manuscript received 30 April 1998)

The relationship between the superconducting energy gap and the critical temperature T_c in high-T_c superconductors is discussed. By examining carefully some of the most recent low temperature data of penetration depth, angle resolved photoemission, and tunneling spectroscopy measurements, we conclude that the gap amplitude at the d-wave nodes scales with T_c. This scaling behavior holds independent of carrier concentration although in the underdoped regime the maximum gap does not scale with T_c. [S0031-9007(98)07106-3]

PACS numbers: 74.20.Fg, 74.72.–h

High-T_c superconductors (HTS) are layered materials in which the superconducting pairing is believed to occur in the CuO_2 planes. It has now been established that the high-T_c superconducting pairing has $d_{x^2-y^2}$-wave symmetry, namely, the gap parameter

$$\Delta_k = \Delta_0(\phi)\cos 2\phi \qquad (1)$$

resembles a four leaf clover with nodes and a change in sign every 45° [1]. (Another commonly used expression for the $d_{x^2-y^2}$ gap function with nearest neighbor pairing on a lattice system is $\Delta_k = \Delta_0(\phi)(\cos k_x - \cos k_y)/2$.) Here ϕ is the angle between the momentum k of the paired electrons and the horizontal axis of a CuO_2 plane, i.e., $\phi = \arctan(k_y/k_x)$, and $\Delta_0(\phi)$ (>0) is the superconducting gap amplitude. In most theoretical and experimental analyses of the properties of d-wave superconductors, $\Delta_0(\phi)$ is assumed to be ϕ independent and generally denoted as Δ_0.

In the mean-field Bardeen-Cooper-Schrieffer (BCS) theory for d-wave superconductors, Δ_0 is proportional to T_c. This scaling behavior of Δ_0 with T_c has been used to explain satisfactorily many of the physical properties of HTS. However, some recent experiments seem to show that this mean-field BCS result is strongly violated, particularly in underdoped cuprates. This violation imposes serious constraints to the theory of high-T_c superconductivity and is worth examining very carefully before a conclusion can be drawn.

Since the discovery of HTS various experimental techniques have been employed to investigate the physical properties of the superconducting energy gap in these materials. One of these is the magnetic penetration depth measurement. The temperature dependence of the penetration depth is determined by the thermal excitations of unpaired electrons. The presence of a linear term in the low temperature in-plane penetration depth, λ_{ab}, of clean HTS has now been established [2–10] and was actually one of the earliest pieces of evidence for the d-wave symmetry of Δ_k. Recently we have found that the normalized superfluid density $[\lambda_{ab}(0)/\lambda_{ab}(T)]^2$ scales approximately with T_c [Fig. 1(a)] [3,6,7,9,10]. These results were obtained by the low field ac-susceptibility technique. If we use the standard BCS result for the low temperature $\lambda_{ab}(T)$ of a d-wave superconductor [11],

$$\frac{\lambda_{ab}^2(0)}{\lambda_{ab}^2(T)} \approx 1 - \frac{(2\ln 2)T}{\Delta_0}, \qquad (2)$$

to fit the experimental data shown in Fig. 1(a), we find that Δ_0 also scales approximately with T_c [Fig. 1(b)]. Values of Δ_0 for optimally doped $Bi_2Sr_2CaCu_2O_{8+\delta}$ [4,5] and $Tl_2Ba_2CuO_{6+\delta}$ [8], deduced from the experimental data of $\lambda_{ab}(T)$ measured by the microwave technique are also included in Fig. 1(b). Δ_0/T_c for $YBa_2Cu_3O_{7-\delta}$ (YBCO) estimated from the a-axis penetration depth data [12] measured by the University of British Columbia group is about 3, which is slightly higher than but within the error bars of the corresponding data shown in Fig. 1(b). Equation (2) is obtained by assuming the gap amplitude is ϕ independent. If Δ_0 is ϕ dependent, then Δ_0 in Eq. (2) should be replaced by $\Delta_0(\pi/4)$, since at low temperatures only low energy excitations around the gap nodes contribute to the linear T term in Eq. (2). Thus strictly speaking it is $\Delta_0(\pi/4)$ which scales approximately with T_c.

The error in Δ_0 arises entirely from the uncertainty in determining $\lambda_{ab}(0)$ since the error in the temperature dependence of the measured $\lambda_{ab}(T)$ from both the ac-susceptibility and the microwave techniques is very small. The maximum possible error for $\lambda_{ab}(0)$ obtained by ac-susceptibility measurements is estimated to be $\pm 20\%$ [6,7] which in turn yields approximately a $\pm 20\%$ error in Δ_0. The microwave technique cannot measure $\lambda_{ab}(0)$ directly. The values of $\lambda_{ab}(0)$ used for $Bi_2Sr_2CaCu_2O_{8+\delta}$ and $Tl_2Ba_2CuO_{6+\delta}$ are determined by muon spin relaxation (μSR) measurements for corresponding polycrystalline samples with similar T_c [4,5,8,13]. In this case it is difficult to estimate the error in $\lambda_{ab}(0)$ since different samples are used in the microwave and μSR experiments. The error bars of Δ_0 for the $Bi_2Sr_2CaCu_2O_{8+\delta}$

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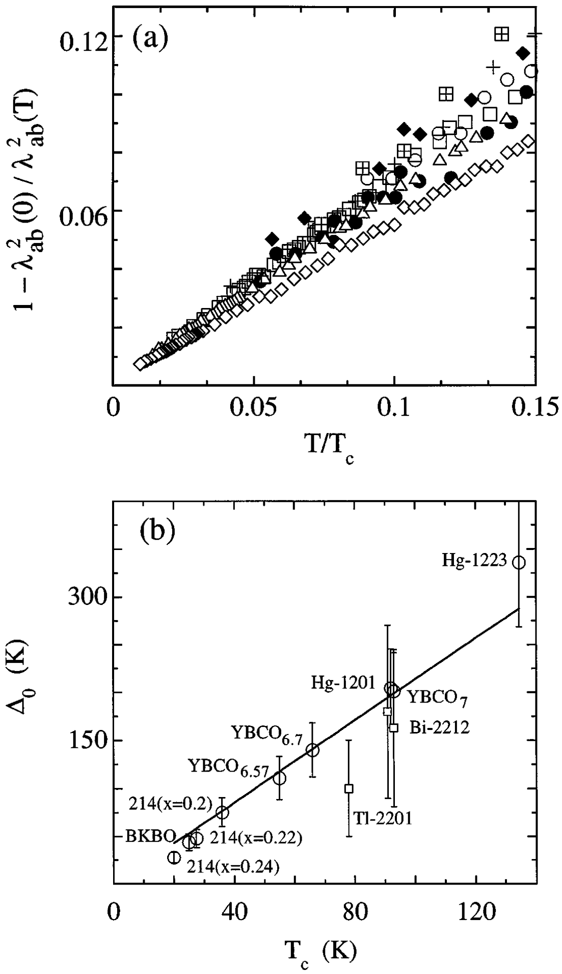

FIG. 1. (a) Normalized superfluid density $1-[\lambda_{ab}(0)/\lambda_{ab}(T)]^2$ as a function of T/T_c for $HgBa_2Ca_2Cu_3O_{8+\delta}$ (Hg-1223, open diamonds) [6], $HgBa_2CuO_{4+\delta}$ (Hg-1201, triangles) [6], $YBa_2Cu_3O_7$ ($YBCO_7$, closed circles) [3,9], $YBCO_{6.7}$ (open circles) [9], $YBCO_{6.57}$ (squares) [9], $La_{2-x}Sr_xCuO_4$ (214) [$x = 0.2$ (crosses), 0.22 (closed diamonds), and 0.24 (crossed squares)] [10]. (b) The gap amplitude Δ_0 [or strictly speaking $\Delta_0(\pi/4)$] as a function of T_c for all the materials depicted in panel (a), and for two $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Bi-2212) [4,5] and one $Tl_2Ba_2CuO_{6+\delta}$ (Tl-2201) samples [8]. The value of Δ_0 for the s-wave superconductor $Ba_{0.6}K_{0.4}BiO_3$ (BKBO) [7] is included for comparison. The solid line is the standard BCS result for a d-wave superconductor with $\Delta_0 = 2.14T_c$. Hg-1223, Hg-1201, $YBCO_7$, Bi-2212, and Tl-2201 are almost optimally doped. $YBCO_{6.7}$ and $YBCO_{6.57}$ are underdoped, and all three 214 samples are overdoped.

and $Tl_2Ba_2CuO_{6+\delta}$ data shown in Fig. 1(b) are obtained based on the assumption that the error for $\lambda_{ab}(0)$ is as high as $\pm 50\%$ [13].

If only the data for the underdoped materials are considered, the scaling behavior of $[\lambda_{ab}(0)/\lambda_{ab}(T)]^2$ with T/T_c in Fig. 1(a) can be understood also from the linear temperature behavior of $\lambda_{ab}(T)$ combined with the empirical Uemura relation $n_s(0) \propto T_c$, where $n_s(0) \propto [1/\lambda_{ab}(0)]^2$ is the zero-temperature superfluid density [14,15]. However, this scenario cannot account for the experimental results of overdoped $La_{2-x}Sr_xCuO_4$ ($x = 0.20, 0.22, 0.24$) [10,15] and optimally doped $YBa_2Cu_3O_{7-\delta}$ [3], which do not follow the Uemura relation.

The superconducting energy gap has also been extensively investigated by angle-resolved photoemission (ARPES) [16–20] and tunneling spectroscopy [21] measurements. ARPES can in principle measure the gap value at any momentum point on the Brillouin zone and determine directly the variation of the energy gap on the whole Fermi surface. However, the superconducting gap usually cited in the literature referring to ARPES data is the maximum energy gap, that is, the gap at $\phi = 0$ or $\pi/2$. In tunneling experiments, the gap value is phenomenologically defined as the half distance between the conductance peaks. The gap such obtained is not necessarily the same as Δ_0 around the nodes ($\phi = \pi/4$). Both spectroscopic techniques show that for overdoped HTS the measured gap is approximately proportional to T_c, consistent with the penetration depth data. However, in the underdoped region the measured gap is found to be greater than that determined by penetration depth and seems to increase (while T_c falls) with decreasing carrier concentration in contrast to the standard BCS result.

In addition to the superconducting energy gap, ARPES, tunneling as well as many other measurements have also found the presence of a normal state gap Δ_N in the spin and charge excitation spectrum at temperatures much higher than T_c. This normal state gap has pronounced effects on the normal state transport and thermodynamic properties of HTS [22]. In the phase fluctuation [23] and some other theories [24], Δ_N is interpreted as the precursor of the superconducting gap, namely, the binding energy of preformed superconducting pairs in the normal state.

The discrepancy in the values of Δ_0 between the penetration depth and the ARPES and tunneling measurements indicates that the gap amplitude Δ_0 in the d-wave gap function is not simply a constant but instead strongly ϕ dependent. Hence the energy gap determined by ARPES and tunneling measurements (or strictly speaking, the gap values which are generally used for demonstrating the failure of the BCS scaling law of Δ_0 with T_c) is the gap maximum which is different from $\Delta_0(\pi/4)$ determined by low temperature penetration depth measurements. We believe it is the interplay between the superconducting gap and its normal state partner which is responsible for the unconventional ϕ dependence of Δ_0.

The above explanation is in fact consistent with recent ARPES measurements [20] which show that in underdoped HTS, Δ_N develops first around $\phi = 0$ or $\pi/2$ at high temperatures and then spreads towards the d-wave gap nodes $\phi = \pi/4$ as the temperature decreases. This implies that the physical effect responsible for Δ_N has stronger influence on $\Delta_0(0)$ and $\Delta_0(\pi/2)$ but weaker on $\Delta_0(\pi/4)$. In other words, if we assume that the quasiparticle excitations around the gap nodes are still governed by the mean-field BCS theory $\Delta_0(\pi/4)/T_c$ should be weakly doping dependent even though $\Delta_0(0)/T_c$ and $\Delta_0(\pi/2)/T_c$ are strongly doping dependent. This in fact explains quite naturally why in overdoped HTS, where Δ_N is small if not completely absent, the superconducting gap obtained by ARPES and tunneling spectroscopy is approximately proportional to T_c.

To test our model we have reexamined the low energy data of ARPES [16–19] and tunneling [21] experiments and found that the existing data do support our picture. We have calculated $\Delta_0(\phi)$ from the published ARPES data of Δ_k according to Eq. (1) or $\Delta_k = \Delta_0(\phi)(\cos k_x - \cos k_y)/2$, which has been used to describe the ARPES data of several HTS [16–19]. The results of $\Delta_0(\phi)$, after being normalized by T_c, as a function of $\cos 2\phi$ or $(\cos k_x - \cos k_y)/2$ are shown in Fig. 2. Only the error bars of $\Delta_0(\phi)/T_c$ for the optimally doped $Bi_2Sr_2CaCu_2O_{8+\delta}$ data are shown for clarity. As ϕ moves towards the gap node, $\cos 2\phi$ decreases and the error bars of $\Delta_0(\phi)/T_c$ increase. For other data the magnitude of the error bars is similar. We note that for optimally doped $Bi_2Sr_2CaCu_2O_{8+\delta}$ the value of $\Delta_0(\phi)$ between $\phi < \pi/4$ and $\phi > \pi/4$ is not perfectly symmetric probably due to experimental errors. (For the other compounds depicted in Fig. 2 only data for $\phi < \pi/4$ are available.) Even with the large error bars associated with $\Delta_0(\phi)/T_c$ near $\phi = \pi/4$, there is a clear tendency for Δ_0/T_c, particularly for the underdoped samples, to decrease as we approach the gap node. Also $\Delta_0(0)/T_c$, the value usually quoted in the literature as evidence against the scaling of Δ_0 with T_c, is significantly larger and very sensitive to doping. This is in contrast to $\Delta_0(\phi)/T_c$ near $\phi = \pi/4$ which is smaller and seems to be insensitive to doping. Of course considering the large error bars in the ARPES data near $\pi/4$ one cannot draw conclusions regarding the absolute value and the scaling of Δ_0, near $\phi = \pi/4$, with T_c. Nevertheless, on the basis of the existing ARPES data and associated experimental errors it is reasonable to say that Δ_0/T_c is a function of ϕ and $\Delta_0(\pi/4)/T_c < 4$. More ARPES measurements around the gap nodes are needed for a more accurate determination of $\Delta_0(\pi/4)/T_c$.

Further evidence for the scaling behavior of the superconducting energy gap with T_c comes from the low energy data of recent tunneling spectroscopy measurements for $Bi_2Sr_2CaCu_2O_{8+\delta}$ [21]. From Fig. 1 of Ref. [21], it can be seen that the slope of the low energy normalized differential tunneling conductance dI/dV [which for a clean d-wave superconductor is inversely proportional to $\Delta_0(\pi/4)$] decreases with increasing T_c, for both underdoped and overdoped samples. This striking correlation

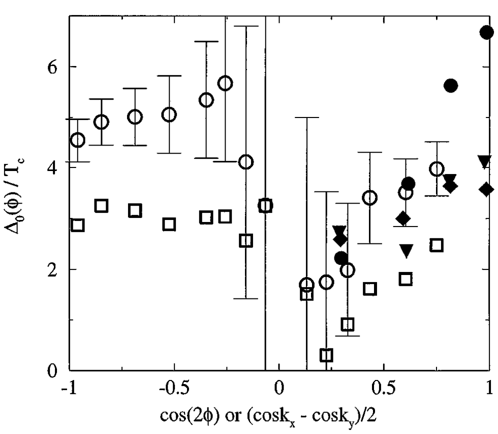

FIG. 2. ARPES data of $\Delta_0(\phi)/T_c$ as a function of $\cos 2\phi$ or $(\cos k_x - \cos k_y)/2$ for optimally doped $Bi_2Sr_2CaCu_2O_{8+\delta}$ (open circles and squares, $T_c = 87$ K) [18], optimally doped $Bi_2Sr_{1.65}La_{0.35}CuO_{6+\delta}$ (diamonds, $T_c = 29$ K) [17], underdoped $Bi_2Sr_2Ca_{1-x}Dy_xCu_2O_{8+\delta}$ (triangles, $T_c = 78$ K) [16], and heavily underdoped $Bi_2Sr_2Ca_{1-x}Dy_xCu_2O_{8+\delta}$ (closed circles, $T_c = 46$ K) [16]. All data, except those shown as circles, were obtained from the leading edge midpoint shifts from the Fermi energy. Open circles are for the same data depicted with squares but obtained from a theoretical fit as discussed in Ref. [18]. Error bars are those given in Ref. [18].

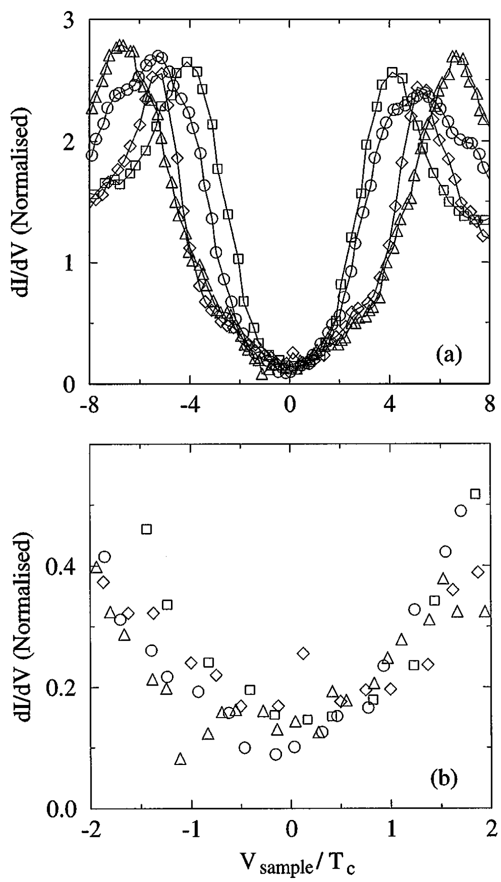

FIG. 3. (a) Normalized tunneling conductance versus V_{sample}/T_c for four $Bi_2Sr_2CaCu_2O_{8+\delta}$ single crystals measured at 4.2 K [21]: circles ($T_c = 74.3$ K; overdoped), squares ($T_c = 56$ K; overdoped), triangles ($T_c = 83$ K; underdoped), and diamonds ($T_c = 92.2$ K; optimally doped). The solid lines are drawn as guides to the eye. (b) Low energy region of the data shown in panel (a).

between the low energy slopes of dI/dV and T_c is more evident by plotting the dI/dV curves against V_{sample}/T_c [Figs. 3(a) and 3(b)], where V_{sample} is the bias voltage. Clearly at higher voltages, dI/dV behaves very differently for the underdoped and overdoped samples, probably due to the effect of Δ_N, whereas at low bias all data fall, within the measurement errors, onto a single curve as expected from the scaling behavior of $\Delta_0(\pi/4)$ with T_c.

Another set of experimental results which has been systematically used to study the properties of the superconducting and normal state gaps comes from specific heat measurements [22,25,26]. The low temperature specific heat data for overdoped materials also suggest that the superconducting energy gap is proportional to T_c. However, in the underdoped regime the experimental results strongly fluctuate due to disorder effects and a reliable analysis for the behavior of the superconducting gap is still not available.

In summary, from recent penetration depth, ARPES and tunneling spectroscopy data we found that the superconducting energy gap amplitude at the gap nodes $\Delta_0(\pi/4)$ is approximately proportional to T_c for both underdoped and overdoped HTS. This result is consistent with the mean-field BCS theory and reveals very important information on the high-T_c pairing mechanism. More accurate ARPES, tunneling, and specific heat measurements are needed to further clarify the low energy behavior of the superconducting gap.

We are grateful to J. R. Cooper for useful discussions, and to Ch. Renner for providing the original tunneling data shown in Fig. 3. C. P. thanks Trinity College, Cambridge for financial support.

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