Magnetic Anisotropy in Quantum Hall Ferromagnets

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We show that the sign of magnetic anisotropy energy in quantum Hall ferromagnets is determined by a competition between electrostatic and exchange energies. Easy-axis ferromagnets tend to occur when Landau levels whose states have similar spatial profiles cross. We report measurements of integer quantum Hall effect (QHE) evolution with magnetic-field tilt. Reentrant behavior observed for the $\nu = 4$ QHE at high tilt angles is attributed to easy-axis anisotropy. This interpretation is supported by a detailed calculation of the magnetic anisotropy energy. [S0031-9007(98)06972-5]

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In the quantum Hall effect (QHE) regime, two-dimensional electron systems (2DES) can have ferromagnetic ground states in which electronic spins are completely aligned by an arbitrarily weak Zeeman coupling [1]. However, spin independence of the electron-electron interaction leads to isotropic Heisenberg ferromagnetism, and therefore to loss of ferromagnetic order at any finite temperature [2]. Richer physics occurs when the two Landau levels (LL's) that are nearly degenerate differ by more than a spin index. For example, double-layer QHE systems can be regarded as easy-plane (XY) two-dimensional ferromagnets [3] and exhibit a variety of effects which have received considerable experimental [4] and theoretical [5] attention in recent years. Idealized single-layer QHE systems have a phase transition [6] in tilted magnetic fields between unpolarized and spin-polarized states, and as we show below, can be regarded as easy-axis (Ising) ferromagnets. In this Letter we report experimental data for a 43 nm wide unbalanced GaAs quantum well in which a loss of the QHE at $\nu = 4$ is observed over a finite range of magnetic-field tilt-angles. We derive a general expression for the magnetic anisotropy energy and propose that its sign is responsible for this observation.

In a strong magnetic field, the single-particle states of a 2DES are grouped into LL's with orbital degeneracy $N_\phi = AB/\Phi_0$, where A is the system area, B is the field strength, and Φ_0 is the magnetic flux quantum. We consider the case where the LL filling factor $\nu \equiv N/N_\phi$ is an integer [7] and two different groups of N_ϕ orbitals are close to degeneracy. We assume that other LL's are far enough from the Fermi energy to justify their neglect [8]. Using a *pseudospin* language [3] to represent the LL index degree of freedom, the Hamiltonians we consider can be expressed in the form

$$H = -b\sigma(\vec{q}=0) + \frac{1}{2A}\sum_{\vec{q}}\{V_{\rho,\rho}(\vec{q})\rho(-\vec{q})\rho(\vec{q}) + V_{\sigma,\sigma}(\vec{q})\sigma(-\vec{q})\sigma(\vec{q}) + V_{\rho,\sigma}(\vec{q})[\rho(-\vec{q})\sigma(\vec{q}) + \sigma(-\vec{q})\rho(\vec{q})]\}. \tag{1}$$

In Eq. (1), b is half the energy separation between the nearly degenerate LL's, and $\rho(\vec{q})$ and $\sigma(\vec{q})$ are, respectively, the sum and difference of the density operators [9] projected onto the up and down pseudospin LL's. Note that b is half the *single-particle* energy difference and does not include mean-field contributions from Coulomb or exchange interactions with electrons in the LL's of interest. We have limited the present discussion to cases for which the total number of electrons with each pseudospin index is conserved. The effective interactions that appear in Eq. (1) are related to the effective interactions between pseudospins by the following relations: $V_{\rho,\rho} = (V_{\uparrow,\uparrow} + V_{\downarrow,\downarrow} + 2V_{\uparrow,\downarrow})/4$, $V_{\sigma,\sigma} = (V_{\uparrow,\uparrow} + V_{\downarrow,\downarrow} - 2V_{\uparrow,\downarrow})/4$, and $V_{\rho,\sigma} = (V_{\uparrow,\uparrow} - V_{\downarrow,\downarrow})/4$.

Our calculation of the pseudospin anisotropy energy is based on the following single Slater determinant wave function:

$$|\Psi[\hat{n}]\rangle = \prod_{m=1}^{N_\phi} c^\dagger_{m,\hat{n}}|0\rangle. \tag{2}$$

Here m labels the orbital states within a LL and $\hat{n}$ denotes the pseudospinor aligned in the $\hat{n} = [\sin(\theta)\cos(\phi), \sin(\theta)\sin(\phi), \cos(\theta)]$ direction. This many-particle state is fully pseudospin polarized [10,11]. For the dependence of energy on pseudospin orientation, we find that

$$\frac{\langle\Psi[\hat{n}]|H|\Psi[\hat{n}]\rangle}{N} = -b^*\cos(\theta) + \frac{U_{\sigma,\sigma}}{2}\cos^2(\theta). \tag{3}$$

Here $b^* = b - U_{\rho,\sigma}$ and for all indices

$$U_{s,s'} = \int\frac{d\vec{q}}{(2\pi)^2}[V_{s,s'}(\vec{q}=0) - V_{s,s'}(\vec{q})] \times \exp(-q^2\ell^2/2), \tag{4}$$

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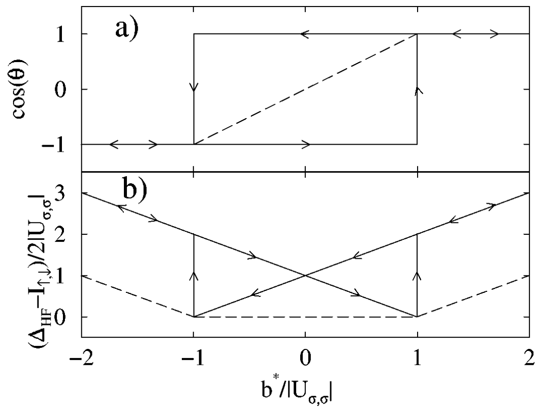

FIG. 1. Pseudospin orientation (a) and Hartree-Fock quasiparticle gap minus exchange energy $I_{\uparrow\downarrow}$ in units of $2|U_{\sigma,\sigma}|$ (b) as a function of the effective field $b^*/|U_{\sigma,\sigma}|$ for the easy-axis (full line) and easy-plane (dashed line) broken symmetry states.

where $\ell = \sqrt{\hbar c/eB}$ is the magnetic length. In Eq. (3) we have dropped terms in the energy that are independent of pseudospin orientation. The right-hand side of this equation is independent of ϕ because the $\hat{z}$ component of total pseudospin is a good quantum number.

For each effective field strength b^*, the pseudospin orientation is determined by minimizing the total energy. For $U_{\sigma,\sigma} > 0$, easy-plane anisotropy, $\cos(\theta) = 0$ at $b^* = 0$ and the pseudospin evolves continuously with effective field as illustrated in Fig. 1(a), reaching alignment for $|b^*| > U_{\sigma,\sigma}$. For $U_{\sigma,\sigma} < 0$, easy-axis anisotropy, local minima occur at both $\cos(\theta) = 1$ and $\cos(\theta) = -1$ for $|b^*| < |U_{\sigma,\sigma}|$. If only global pseudospin rotation processes were possible, macroscopic energy barriers would separate these two locally stable states, resulting in hysteretic behavior [see Fig. 1(a)]. The sign of $U_{\sigma,\sigma}$ is determined by competition between the two terms in square brackets on the right-hand side of Eq. (4). The $V_{\sigma,\sigma}(\vec{q} = 0)$ term is an electrostatic energy which is present when the two pseudospin states have different charge density profiles perpendicular to the electron layers. This term favors easy-plane anisotropy. The $V_{\sigma,\sigma}(\vec{q})$ term is the exchange energy which favors easy-axis anisotropy. Easy-axis anisotropy will always occur when $V_{\sigma,\sigma}(\vec{q})$ is an increasing function of wave vector.

Transport measurements in the QHE regime are extremely sensitive to the energy gap for charged excitations. Generally, large energy gaps give rise to well developed Hall plateaus and deep minima in the dissipative resistivity. In the Hartree-Fock approximation, the quasiparticle energy gap of anisotropic QHE ferromagnets can be written as [12]

$$\Delta_{\rm HF} = I_{\uparrow\downarrow} - 2U_{\sigma,\sigma} + \frac{2b^*}{\cos(\theta)}, \qquad (5)$$

where $I_{\uparrow\downarrow} = \int \frac{dq^2}{(2\pi)^2} \exp(-q^2\ell^2/2)\,(V_{\rho,\rho} - V_{\sigma,\sigma})$. For the easy-plane case, $\Delta_{\rm HF}$ is a continuous function of the effective field b^*, decreasing linearly for $b^*/U_{\sigma,\sigma} < -1$, constant for $|b^*|/U_{\sigma,\sigma} < 1$, and increasing linearly for $b^*/U_{\sigma,\sigma} > 1$. In contrast, if the system has easy-axis anisotropy, $\Delta_{\rm HF}$ decreases to $I_{\uparrow\downarrow}$ at the extremes of the hysteresis loop ($b^*/U_{\sigma,\sigma} = \pm 1$) before jumping to $I_{\uparrow\downarrow} + 4|U_{\sigma,\sigma}|$ when the pseudospin magnetization reverses. In Fig. 1(b) we summarize the above results by plotting $(\Delta_{\rm HF} - I_{\uparrow\downarrow})/2|U_{\sigma,\sigma}|$ as a function of $b^*/|U_{\sigma,\sigma}|$. In the Hartree-Fock approximation this quantity depends only on the sign of the anisotropy energy.

For concreteness, we now mention idealized models which have easy-plane and easy-axis anisotropy. For two arbitrarily narrow quantum wells separated by a distance d with full polarization of the true electron spin, we let pseudospin represent the layer index. The "pseudospin" Zeeman field b is then proportional to the bias electric field E_g, created by a gate external to the electron system: $b = eE_g d/2$. On the other hand, for a single arbitrarily narrow quantum well with $\nu = 2m$ in which the real-spin Zeeman coupling has been increased [6] so as to bring the up-spin $n = m$ LL close to degeneracy with the down-spin $n = m - 1$ LL, we let the pseudospin represent the spin index of the LL close to the Fermi energy. The pseudospin Zeeman coupling for this model is $b = (g^*\mu_B B - \hbar\omega_c + I_0)/2$. Here the first term is the real-spin Zeeman coupling, the second term is the cyclotron energy, and the last term is the contribution to b from exchange interactions with frozen LL's lying well below the Fermi energy [$I_0/(\sqrt{\pi/2}\,e^2/\epsilon\ell) = 1/2$, $5/16$, and $31/128$ for $m = 0$, 1, and 2, respectively [6,13]]. The effective Coulomb interaction energies for the two models are summarized in Table I. For the ideal double-layer model, the electrostatic term $V_{\sigma,\sigma}(q = 0)$ dominates, $V_{\sigma,\sigma}(q)$ is a monotonically decreasing function of q, and $U_{\sigma,\sigma}$ is positive. On the other hand, for the ideal tilted-field model, the pseudospin wave functions differ only in the plane of the 2DES, the electrostatic term is consequently absent, and the exchange term produces easy-axis anisotropy [$U_{\sigma,\sigma}/(\sqrt{\pi/2}\,e^2/\epsilon\ell) = -3/16$, $-33/256$, and $-107/1024$ for $m = 0$, 1, and 2, respectively].

Now we turn to the discussion of the measured QHE evolution with tilted field, shown in Fig. 2 [14]. In finite width quantum wells, the large tilt angles necessary to

TABLE I. Effective Coulomb interactions in units of $2\pi e^2\ell/\epsilon$ as a function of wave vector q in units of ℓ^{-1} for ideal double-layer and tilted-field models. $L_n(x)$ is the Laquerre polynomial.

Model	$V_{\rho\rho}$	$V_{\sigma\sigma}$	$V_{\rho\sigma}$
Double-layer	$(1 + e^{-qd})/2q$	$(1 - e^{-qd})/2q$	0
Tilted-field	$\frac{[L_m(q^2/2)+L_{m-1}(q^2/2)]^2}{4q}$	$\frac{[L_m(q^2/2)-L_{m-1}(q^2/2)]^2}{4q}$	$\frac{[L_m(q^2/2)]^2-[L_{m-1}(q^2/2)]^2}{4q}$

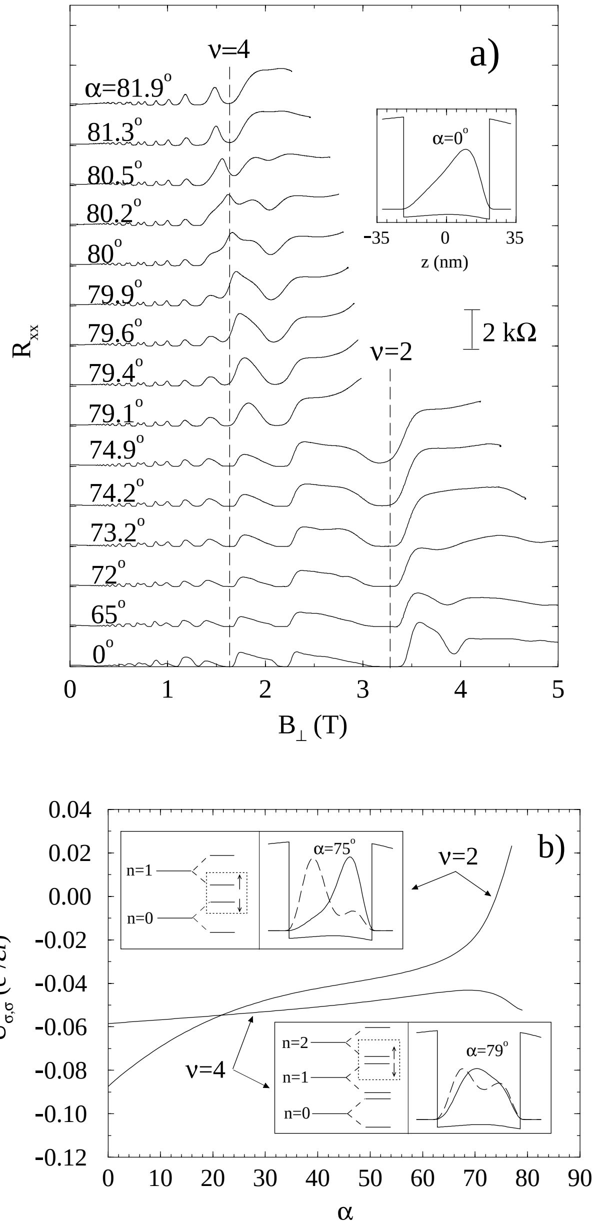

FIG. 2. (a) Measured longitudinal resistance vs perpendicular component of the magnetic field for different tilt angles (α is measured from the normal to the plane of the 2DES) at $T =$ 300 mK. The $\nu = 4$ QHE is lost at $\alpha \approx 80°$ and reappears at $\alpha \approx 80.5°$ while for $\nu = 2$ no loss of the QHE is observed near $\alpha = 72°$. The inset shows calculated charge distribution at $\alpha = 0$ for the unbalanced GaAs quantum well studied here. The front-gate and back-gate voltages and the 2DES density ($N = 1.57 \times 10^{11}$ cm^{-2}) were fixed during the experiment. (b) Anisotropy energies calculated for the geometry of this sample at $\nu = 2$ and 4. The two Landau levels which are brought close to degeneracy by applying in-plane component of the magnetic field are indicated in the insets together with calculated density profiles for up (dashed line) and down (solid line) pseudospin orbitals at high tilt angles.

bring the up and down pseudospin LL's close to degeneracy result in substantial coupling of the in-plane component of the magnetic field to orbital degrees of freedom [15]. These orbital effects can be incorporated [11] by adjusting the effective interactions appropriately. In particular, for real finite-width quantum wells, the perpendicular charge density profiles of the two pseudospin LL's differ, and the electrostatic contribution to $U_{\sigma,\sigma}$ is no longer zero. The sign of the anisotropy energy depends in detail on the quantum well geometry, the tilt angle, and the filling factor. The insets in Fig. 2(b) show charge-density profiles in the quantum well studied for the relevant orbitals at high tilt angles α obtained from self-consistent LDA calculations [15]: $n = 0, \downarrow$ and $n = 1, \uparrow$ at $\nu = 2$; $n = 1, \downarrow$ and $n = 2, \uparrow$ at $\nu = 4$. From these orbitals we see that, for $\nu = 2$, the Hartree contribution to $U_{\sigma,\sigma}$ is significant at high α. Hence, $U_{\sigma,\sigma}$ increases substantially with tilt angle and becomes positive at large α [see Fig. 2(b)]. This result demonstrates that easy-plane anisotropy can occur in realistic single quantum wells. If so, referring to the quasiparticle gap predictions summarized in Fig. 1, a strong QHE may be expected throughout the region of tilt angles where the relevant LL's are close to degeneracy. The experimental data of Fig. 2 show a strong minimum at $\nu = 2$ at all angles near $\alpha = 72°$ where $b^* = 0$ occurs. No clear evidence for the disappearance of the QHE is observed up to the highest accessible tilt angles for $\nu = 2$, consistent with easy-plane anisotropy. We note that the calculated $U_{\sigma,\sigma}$ becomes positive at α slightly higher than 72°; we attribute this small discrepancy to the local density approximation for exchange and correlation, used in the self-consistent field calculations for the LL orbitals.

At $\nu = 4$, our calculations predict that $b^* = 0$ occurs at $\alpha = 79°$, and that the density profiles of the two pseudospin states are similar even at high tilt angles, as illustrated in Fig. 2. Hence, $U_{\sigma,\sigma}$ is only weakly angle dependent and is still strongly *negative* around $\alpha = 79°$. We attribute the clear degradation of the measured QHE at $\nu = 4$ to phenomena associated with easy-axis anisotropy. The tilt angle $\alpha = 80°$ where the $\nu = 4$ QHE disappears is in a good quantitative agreement with the theoretically predicted angle $\alpha = 79°$ at which the pseudospin Zeeman field b^* vanishes. We expect transport properties inside the hysteresis loop in the easy-axis case, to have a complicated disorder dependence. Spatially random potentials couple differently to different LL's and will produce a random pseudospin magnetic field. This is expected [16] to lead to the formation of large domains with particular pseudospin orientations. The dynamics of pseudospin reorientation is likely to be controlled by barriers to domain wall motion. If these are comparable to $k_B T$, the pseudospin will achieve alignment with the effective field on laboratory time scales, $\cos(\theta)$ will change from -1 to 1 at $b^* = 0$, and the energy gap will have a cusp. This scenario appears to apply for recent experiments which study analogous LL crossings in the valence band of GaAs [17] and to some tilted field driven transitions at fractional LL filling factors [14]. On the other hand, when some domain wall motion barriers are much larger than $k_B T$ we expect that all physical properties will exhibit hysteretic behavior, and that the electronic state will have domain structure for b^* close to zero. Dissipation due to mobile charges created in domain walls [11] can then lead to a breakdown of the QHE, observed in our data at $\nu = 4$ (Fig. 2). We expect that dissipative and Hall resistances will then depend on measuring current and sample history, as well as on temperature. In our experiment, however, we have not found clear evidence of hysteresis, possibly because the base temperature (300 mK) is too large.

In the disorder free limit, easy-axis anisotropy in two dimensions leads to a finite temperature continuous phase transition in the Ising universality class and stronger thermodynamic anomalies than for the Kosterlitz-Thouless phase transition of easy-plane systems. The transition temperature can be estimated [11] by balancing energy and entropy terms in the free energy of long domain walls:

$$k_B T_c \sim U_{\sigma,\sigma}(wR/\ell^2), \tag{6}$$

where w is the domain wall width and R is the domain wall orientation correlation length. The domain wall physics of these easy-axis ferromagnets is unconventional because the spin stiffness is negative [11]. Preliminary results from work presently in progress suggest that wR/ℓ^2 is substantially larger than one and that the critical temperature should typically exceed $\sim$1 K.

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Note added.—A recent experimental study [18] we learned of after this work was completed finds hysteretic behavior in a narrow (25 nm) GaAs quantum well in vicinity of $\nu = 2/5$ and $4/9$ fractional QHE's which correspond to integer QHE's at composite fermion filling factors $\nu = 2$ and 4, respectively. In these experiments, Zeeman coupling strength was controlled both by applying hydrostatic pressure and by tilting the field. We believe that the theory developed in this paper explains the origin of the hysteresis found in Ref. [18] at very low temperatures ($T \lesssim 200$ mK).

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