Reversible Formation of a Bose-Einstein Condensate

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We present a method of adiabatically changing the local phase-space density of an ultracold gas using a combination of magnetic and optical forces. Applying this method, we observe phase-space density increases in a gas of sodium atoms by as much as 50-fold. The transition to Bose-Einstein condensation was crossed reversibly, attaining condensate fractions of up to 30%. Measurements of the condensate fraction reveal its reduction due to interactions. [S0031-9007(98)07066-5]

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The physical properties of atomic gases change dramatically when quantum degeneracy is reached, i.e., when the ground state population approaches unity [1]. Recent successes in reaching quantum degeneracy with Bose gases [2–4] have relied on nonadiabatic, irreversible methods such as laser and evaporative cooling. The possibility of changing the ground state population by an adiabatic (i.e., isentropic) change in the trapping potential had been overlooked for quite some time [5]. Indeed, in the case of an ideal gas, adiabatic changes in the *strength* of the trapping potential do not change the ground state population [6,7]. However, Pinkse and collaborators [8] recently showed, both theoretically and experimentally, that by changing the *form* of the trapping potential, the population in the ground state can be changed without changing the entropy. For a nondegenerate gas the ground state population is identical to the phase-space density $\Gamma = n\lambda_T^3$, where n is the density of the gas and λ_T is the thermal de Broglie wavelength. Within the type of trap deformations considered in Ref. [8] the maximum increase of phase-space density is limited to a factor of 20.

In this Letter, we show that a more general deformation of the trapping potential can increase the phase-space density by an arbitrary factor, and we implement this scheme using a combination of magnetic and optical forces. Furthermore, we demonstrate the ability to cross the Bose-Einstein condensation (BEC) phase transition reversibly.

Adiabatic increase in phase-space density.—The type of trap deformations which we study can be understood with the following "two-box" model. Consider a classical gas of N atoms confined in a box of volume $V_0 = V_1 + V_2$ with an initial phase-space density Γ_0. Suppose that the potential within a subvolume V_2 of the box is lowered to a final well depth U. In this final potential, the gas equilibrates at a temperature T_f, and the density in V_2 will be higher than that in V_1 by the Boltzmann factor $e^{U/k_B T_f}$. Using the condition of constant entropy and constant particle number, one obtains the relative increase of phase-space density in V_2 compared to that in V_0 before compression:

$$\ln(\Gamma_2/\Gamma_0) = \frac{U/k_B T_f}{1 + (V_2/V_1)e^{U/k_B T_f}}. \qquad (1)$$

For deep potential wells, where $U/k_B T_f \gg \ln(V_1/V_2)$, there is no increase in phase-space density since all of the gas becomes confined in V_2, and the adiabatic deformation corresponds simply to a uniform compression of the gas. For shallow potential wells [$U/k_B T_f \ll \ln(V_1/V_2)$], the phase-space density in V_2 increases as $e^{U/k_B T_f}$. As U is varied between these limits, the phase-space density increase reaches a maximum which is greater than $(V_1/V_2)^{1/2}$. Thus, by choosing an extreme ratio of volumes V_1/V_2, an arbitrarily large increase in phase-space density Γ_2/Γ_0 is possible.

To demonstrate this phase-space density increase in a gas of trapped atoms, a narrow potential well (analogous to V_2) was added to a broad harmonic potential (corresponding to V_1) by focusing a single infrared laser beam at the center of a magnetic trap. First, a gas of atomic sodium was evaporatively cooled to a temperature higher than the BEC phase transition temperature in the cloverleaf magnetic trap [9]. The number of atoms and their temperature were adjusted by varying the final radio frequency (rf) used in the rf-evaporative-cooling stage [10]. Afterwards, the magnetically trapped cloud was decompressed by slowly reducing the currents in the magnetic trapping coils. Time-of-flight absorption imaging was used to characterize the cloud. The total number of atoms N was determined by integrating the column density across the cloud, and the initial temperature T_0 was determined by one-dimensional Gaussian fits to the wings of the density distribution. From these, we determined the fugacity z of the gas by the relation $g_3(z) = N(\hbar\bar{\omega}/k_B T_0)^3$, and then its phase-space density by $\Gamma_0 = g_{3/2}(z)$, where $g_n(z) = \sum_{i=1}^{\infty} z^i/i^n$ [11]. Here, $\bar{\omega}$ is the geometric-mean trapping frequency of the magnetic trap, as determined by *in situ* measurements [12]. These phase-space density measurements were calibrated with images from magnetically trapped clouds at the phase transition.

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The optical setup was similar to that used in Ref. [13]. The infrared laser power was gradually ramped-up from zero to a power P_c at which the onset of BEC was seen in time-of-flight images of clouds released from the deformed trap; this implied $\Gamma_f = g_{3/2}(1) = 2.612$ for the final phase-space density. These time-of-flight images allowed us to determine the final temperature T_f. The depth of the optical potential well was given by $U_c/k_B = (37\ \mu\mathrm{K})P_c/w_0^2\ (\mu\mathrm{m}^2/\mathrm{mW})$, where w_0 is the $1/e^2$ beam-waist radius at the focus. The ramp-up time was made long enough to ensure that the trap deformation was adiabatic, but also short enough to minimize heating and trap loss. Ramp-up times of up to 10 s were used.

Figure 1a shows the increases in phase-space density which were measured at three different settings of the trap parameters. A maximum increase by a factor of 50 was obtained. Condensates were observed in clouds with temperatures as high as 5 μK. Further increases were hindered by limitations in laser power and by limits to the ramp-up time set by the various heating and loss processes in the deformed trap. The scatter in the data is primarily due to statistical errors at the level of 30% in our measurements of U_c/k_BT_f, and in the determination of the transition point.

The well depth U_c required to reach BEC can be understood by a simple model depicted in Fig. 1. We begin with a harmonically trapped gas with a chemical potential $\mu < 0$ (Fig. 1b). Then, by lowering a potential well to a depth U_c at which BEC occurs, one essentially measures μ as $\mu = -U_c$ (Fig. 1c). However, this simple picture neglects the change in the chemical potential and the temperature of the gas during the adiabatic compression. Thus, let us consider instead the condition of constant entropy. The entropy per noncondensed particle in a harmonically confined Bose gas is determined uniquely by its fugacity z [8]:

$$\frac{S}{N}(z) = 4\,\frac{g_4(z)}{g_3(z)} - \ln z\,. \tag{2}$$

This equation describes the entropy of the gas before compression, with the fugacity given by z_0. After compression, because of the small volume of the potential well, the entropy per particle is approximately that of a harmonically trapped gas [Eq. (2)] with fugacity $z_f = e^{-U_c/k_BT_f}$ [14]. Here T_f is the final temperature of the gas. Constant entropy then implies $z_0 = z_f$. The initial phase-space density of the gas is $\Gamma_0 = g_{3/2}(z_0)$ and its final phase-space density is $\Gamma_f = g_{3/2}(1)$. Thus we obtain

$$\frac{\Gamma_f}{\Gamma_0} = \frac{g_{3/2}(1)}{g_{3/2}[\exp(-U_c/k_BT_f)]}\,. \tag{3}$$

This prediction, shown in Fig. 1a, describes our data well, and accounts for the universal behavior of our measurements over a wide range of temperatures and well depths.

One may also consider the process of adiabatically increasing the phase-space density as a change in the density of states $D(\epsilon)$ of the system. By increasing the well depth in a small region of the trap, we lowered only the energy of the ground state and a few excited states. Thus Γ, a local quantity, increases as the ground state energy is brought closer to the chemical potential, while the entropy, a global property of the gas, is unchanged by the minimal modification of $D(\epsilon)$.

The fact that global properties of the gas are not affected by the trap deformation can also be seen in the momentum distributions probed by time-of-flight imaging [9,15]. The onset of BEC in the combined optical and magnetic trap is signaled only by the formation of a condensate peak. The remaining thermal cloud is well fit by a Maxwell-Boltzmann distribution, which describes a magnetically trapped cloud far from condensation (Fig. 2a). In contrast, at the BEC transition in the harmonic magnetic trap, the momentum distribution of the thermal cloud is clearly Bose enhanced at low momenta (Fig. 2b).

Adiabatic condensation.—We now turn to the studies of adiabatic, i.e., reversible, condensate formation. A cloud

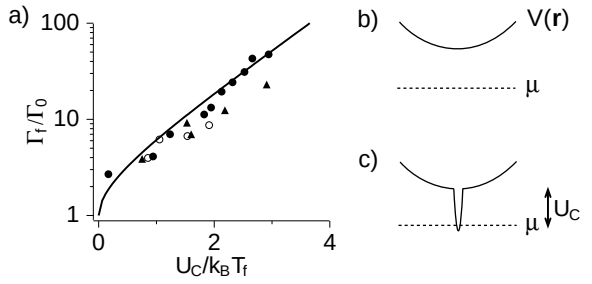

FIG. 1. Phase-space density increase to reach BEC vs normalized well depth (a). Various trap settings were used: $\bar{\omega} = 2\pi \times 100$ Hz, $w_0 = 9\ \mu$m (triangles); $\bar{\omega} = 2\pi \times 100$ Hz, $w_0 = 18\ \mu$m (open circles); and $\bar{\omega} = 2\pi \times 33$ Hz, $w_0 = 18\ \mu$m (closed circles). The solid line gives the prediction of Eq. (3). (b) Prior to deformation, the harmonic trapping potential $V(r)$ holds a cloud above the BEC transition temperature ($\mu < 0$). (c) When a potential well with depth $U_c = -\mu$ is added, a small condensate forms.

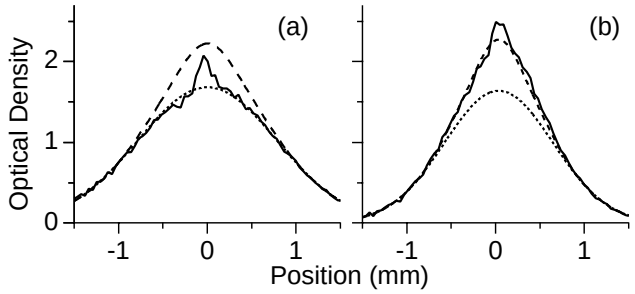

FIG. 2. Momentum distributions of the thermal cloud at the BEC phase transition show no Bose enhancement for the deformed trap (a), but a clear Bose enhancement for the purely magnetic trap (b). Both distributions show a small condensate peak. Lines show a Maxwell-Boltzmann distribution (dotted line) and a Bose-Einstein distribution (for $z = 1$) (dashed line) for clouds in a harmonic trap, which were fit to the thermal wings. The momentum distribution is shown as a profile across absorption images taken after 40 ms of ballistic expansion.

of about 50×10^6 atoms was cooled to the transition temperature in the magnetic trap, at trap frequencies of $\omega_r = 2\pi \times 20$ Hz and $\omega_z = 2\pi \times 13$ Hz in the radial and the axial direction, respectively. The power of the infrared laser beam (of radius $w_0 = 20$ μm) was ramped up over 1 s and held at a constant power for a dwell time of 1.5 s. Condensate fractions as small as 1% could be distinguished from the normal fraction by their anisotropic expansion in time-of-flight images [2,3]. The condensate number N_0 was determined by subtracting out the thermal cloud background using Gaussian fits to the thermal cloud in regions where the condensate was clearly absent.

As shown in Fig. 3, the adiabatic trap deformation yielded condensate fractions of up to 15%; we observed condensate fractions of 30% with different trap settings. By varying the dwell time, we confirmed that clouds for which $U/k_BT_f < 1.5$ suffered no significant losses of condensate number due to three-body decay or heating, whereas those points with higher values of U/k_BT_f were affected by such losses.

For an ideal Bose gas, the result of this adiabatic change can be understood as follows. Before compression, the cloud of N particles at the BEC transition has an entropy S_i given by Eq. (2) as $S_i = N \times 4g_4(1)/g_3(1)$. After compression, the situation is similar to that indicated in Fig. 1c, i.e., because of the small volume of the attractive well, the cloud is well described as a harmonically trapped gas with $\mu = -U$. Thus, accounting for the fact that condensate particles carry no entropy and again using Eq. (2), which gives the entropy per *noncondensed* particle, we equate the entropy before and after compression and obtain

$$\frac{N_0}{N} = 1 - \frac{4g_4(1)/g_3(1)}{4g_4(e^{-U/k_BT_f})/g_3(e^{-U/k_BT_f}) + U/k_BT_f}. \quad (4)$$

However, this simple prediction does not describe our findings well. The theory described above has two shortcomings. First, the approximation of using Eq. (2) for the deformed trap is not strictly valid. However, calculations which accounted for the true shape of the deformed potential changed the prediction of Eq. (4) only for $U/k_BT_f > 1$, and only slightly improved the fit to our data.

A second shortcoming is the neglect of interactions. It has been shown that the condensate fraction of harmonically trapped Bose gases with repulsive interactions is reduced in comparison to that of an ideal gas [16,17]. To estimate this effect in our system, we use the "semi-ideal" model of Ref. [17]. The thermal cloud is described as an ideal gas for which the chemical potential is raised by $gn_0 = 4\pi\hbar^2 a n_0/m$, where n_0 is the maximum condensate density, m the mass of sodium, and $a = 2.75$ nm its scattering length [18]. This simply corresponds to using Eq. (4) with the substitution $U \rightarrow U - gn_0$. We determined n_0 using the Gross-Pitaevskii equation in the Thomas-Fermi limit [19] and a harmonic approximation for the deformed trap potential at its center. Both approximations are valid for all data in Fig. 3.

This approach predicts a significant reduction of the condensate fraction (Fig. 3, dashed line), and the improved agreement with our data is strong evidence for this effect. In contrast to related studies in purely harmonic traps [9,15], which did not show evidence for interaction effects, this depletion is strongly enhanced by the shape of the deformed potential. The mean-field energy of the condensate gn_0 is large because the condensate forms in the tight optical potential, while the transition temperature T_c is small since it is determined by the weak magnetic potential. The remaining discrepancy between the semi-ideal prediction and our data may be due to our approximation of the entropy of the interacting gas, which fails for large values of gn_0/k_BT_f [20].

The reversibility of crossing the BEC phase transition was demonstrated by preparing a magnetically trapped cloud just above T_c. We then sinusoidally modulated the power of the infrared light at 1 Hz, between 0 and 7 mW. This modulation frequency was significantly smaller than the magnetic trap frequencies ($\omega_r = 2\pi \times 48$ Hz and $\omega_z = 2\pi \times 16$ Hz). These low frequencies and a large optical focus ($w_0 = 18$ μm) were used to minimize trap loss due to inelastic collisions.

During the first seven condensation cycles the condensate fraction oscillated between zero and 6% (Fig. 4); later probing showed repeated condensation for at least 15 cycles. The peak of these oscillations decreased slowly in time. The temperature oscillated with an amplitude of about 100 nK, while gradually rising by about 10 nK/s. This heating and the decrease of the peak condensate fraction are consistent with similar behavior in clouds held at a constant infrared power [13]. Thus, within the stability limitations of our optical setup, the repeated crossing of the BEC phase transition appears fully adiabatic.

This method of creating condensates provides insight into their formation, which was recently studied experimentally [21] and theoretically [22]. For example, in the experiment described above, the condensate fraction was found to lag about 70 ms behind the modulation of the laser power, which is a measure for the formation time.

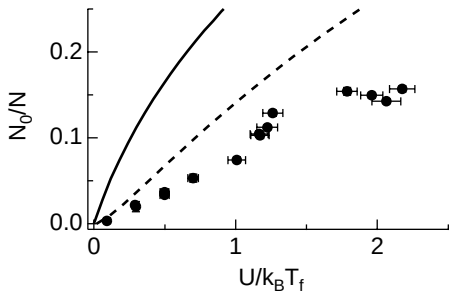

FIG. 3. Condensate formation by adiabatic trap deformation. The condensate fraction is plotted against the (normalized) well depth. Lines show predictions for an ideal gas (solid line) and for an interacting gas (dashed line).

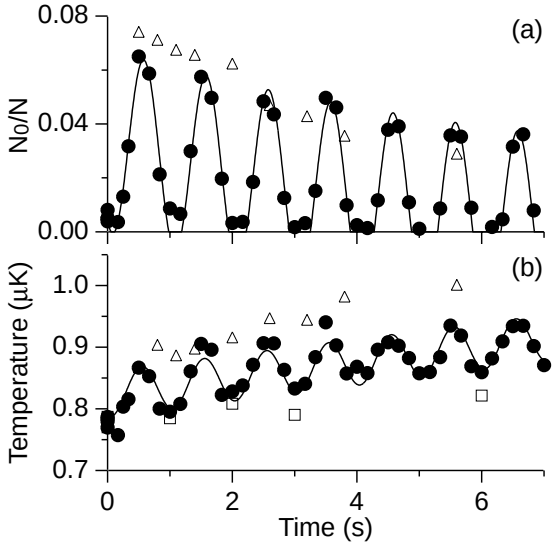

FIG. 4. Adiabatic cycling through the phase transition. Shown is the condensate fraction (a) and the temperature (b) vs time for the case of a modulated infrared beam (closed circles), an infrared beam ramped up to a constant power (open triangles), and no infrared light (open squares). The solid lines are guides to the eye.

That this lag time is much shorter than the time scale for deforming the potential ($\approx$1 s) confirms that the repeated condensation is performed adiabatically.

In other experiments, by switching on the infrared light instantly, we observed condensation on time scales much faster than the oscillation periods in the magnetic trap and along the weakly confining axis of the optical trap. The resulting condensates showed striations in time-of-flight images, indicating that the condensates formed into excited states of the deformed potential. Such studies of shock condensation might give new insight into the formation of quasicondensates and condensation into excited states [23,24].

In conclusion, we have demonstrated the adiabatic Bose-Einstein condensation of an ultracold gas of atomic sodium. Changes in the trapping potential resulted in large phase-space density increases and allowed for repeated crossings of the BEC phase transition. This method allows for detailed studies of condensate formation and the phase transition. The combined trapping potential widens the range of trap parameters over which BEC can be studied. This was used to strikingly enhance the role of interactions and led to higher transition temperatures (up to 5 μK) than achieved in purely magnetic traps.

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