Interaction Constants and Dynamic Conductance of a Gated Wire

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We show that the interaction constant governing the long-range electron-electron interaction in a quantum wire coupled to two reservoirs and capacitively coupled to a gate can be determined by a low-frequency measurement. We present a self-consistent, charge and current conserving, theory of the full conductance matrix. The collective excitation spectrum consists of plasma modes with a relaxation rate which increases with the interaction strength and is inversely proportional to the length of the wire. [S0031-9007(98)06991-9]

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The comparison of interacting electron theories and experiments often suffers from the fact that the interaction constants are not known. In particular, this is true for interacting quantum wires, where Luttinger models with a wide range of coupling parameters are discussed [1,2]. Moreover, a single experiment is often not sufficient to determine the coupling constant. Thus, as will be shown, the capacitance (per unit length) of a wire above a back gate is related to the interaction parameter g via

$$c_\mu = g^2 e^2 \nu_F \,. \tag{1}$$

Since the density of states $\nu_F = 2/h v_F$ (evaluated at constant interaction potential [3]) is generally not known, a capacitance measurement alone cannot determine the interaction constant. Here we propose to investigate the frequency dependence of the current induced into the gate. Compared to the measurement of a frequency dependent conductance at a direct contact, the measurement of the gate current can be performed at relatively small frequencies in the kHz range, since the frequency response is not on top of a possibly large dc conductance. Recently considerable advances have been made in the high precision frequency measurements [4] in mesoscopic conductors. In this Letter we consider a simple model system—a perfect ballistic wire coupled capacitively to a gate and connected to two electron reservoirs—and calculate the dynamic conductance matrix. While the dc conductance in this system is quantized and thus provides no information on the interaction, the dynamic conductance is a sensitive function of the interaction strength.

A conceptually important point which needs to be addressed in solving this problem is the coupling of an interacting wire to electron reservoirs. Previous works [5] have proposed a purely one-dimensional model in which the interaction changes from a value $g < 1$ (characteristic of interactions) to a value $g = 1$ (characterizing a system without interactions) at the transition from the ballistic wire to the electron reservoir. Another more recent proposal [6] consists of a radiative boundary condition in which the electron density is proportional to the applied voltage. In this work we use a different concept of reservoirs based on the *electrochemical* nature of electric transport [3]. The electron density in the wire is the sum of two terms: a chemical density, which follows the chemical potential of the reservoir from which the carriers are injected into the wire, and an induced density, which results from the (long range) Coulomb screening of the injected charge. Indeed, from a screening point of view electrons in a reservoir are not free: an increase of the electrochemical potential is followed by an equal increase in the electrostatic potential, and the local density is left invariant. That corresponds to strong interaction (very effective three-dimensional screening) in a reservoir rather than to a noninteracting one-dimensional (1D) model with $g = 1$. These divergent views arise from the fact that interactions play a role on very different length scales [7]. Different interaction parameters must be used to describe long-range and short-range effects [8]. Conceptually, Ref. [9], which describes the reservoirs by the charges, conjugate to the chemical reservoir potentials, is closest to our approach.

Ballistic single mode wires [1,2] coupled to reservoirs are the simplest model system in which these questions are significant. The ac response of 1D interacting systems has been investigated previously in the framework of the Luttinger model with short-ranged interactions [10–13]. A drawback of these papers is that they calculate the response to an external field of a specific form; another choice of the field profile would lead to different results. Furthermore, the results of Refs. [12,13] are not charge and current conserving (gauge invariant) [14]. The reason for this is that in 1D quantum wire interactions are generally not short ranged. Below we present results for the ac conductance of a system *with* short-ranged interactions—quantum wire, connected to two reservoirs and capacitively coupled to a gate. On a length scale large compared to the distance between the wire and the gate the interactions can be treated as short ranged. Our discussion explicitly includes the effect of the gate and provides conservation of the total current; the particle current in the wire alone

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is not conserved. We calculate the potential profile rather than postulate it. Our results for the conductance matrix can be obtained from Ref. [10] (where a different concept of reservoirs is used, and the current conservation is postulated *ad hoc* rather than checked independently) and disagree with Refs. [11–13]. Moreover, we propose to measure the capacitive (wire to gate) conductances in order to determine the interaction constant.

Consider the system depicted in Fig. 1, consisting of a 1D quantum wire of length L, connected to two reservoirs at $x = 0$ and $x = L$. The potential in the left (1) reservoir is modulated in time, $V_1(t) = V_{1,\omega} e^{-i\omega t}$, whereas the potential in the right (2) reservoir is kept constant. We treat the interactions in random phase approximation (RPA), which is known to capture the long-range response. The $2k_F$ oscillations are not essential in our model, since the interaction is pointlike only at length scales of the wire-gate distance d, and $k_F d \gg 1$. Our results are valid up to frequencies much lower than the Fermi energy.

Self-consistent potential.—In the absence of interactions, a potential modulation in the left reservoir injects a bare charge density $\rho_{0,\omega}(x)$ into the wire,

$$\rho_{0,\omega}(x) = \frac{\nu_F V_{1,\omega}}{2} e^{iq_F x}, \tag{2}$$

where $\nu_F = 2/h\upsilon_F$ is the density of states at the Fermi level (υ_F is the Fermi velocity), and $q_F = \omega/\upsilon_F$.

In the presence of an external potential $\phi_\omega(x)/e$, the true charge density is

$$\rho_\omega(x) = \rho_{\omega,0}(x) - \int_0^L dx' \Pi_\omega(x, x') \phi_\omega(x'), \tag{3}$$

where the polarization kernel is given by (see, e.g., [15])

$$\Pi_\omega(x, x') = \nu_F \delta(x - x') + \frac{iq_F \nu_F}{2} e^{iq_F|x-x'|}. \tag{4}$$

Equation (3) gives the charge density as a sum of two contributions: a chemical one, proportional to the potential $V_{1,\omega}$ of the reservoir, and an induced component, proportional to the electrostatic potential.

We now take electron-electron interactions into account by determining the actual potential $\phi_\omega(x)$ in the wire self-consistently, i.e., by relating the total charge ρ_ω on the left-hand side of Eq. (3) to the potential ϕ_ω. In order to do this, we need to specify the electron-electron interaction. In the case of bare Coulomb interactions, the required relation is the Poisson equation $\Delta\phi_\omega = -4\pi e^2 \rho_\omega$. For short-ranged interactions this relation is different and is found as follows: Generally, the interaction potential $V(\boldsymbol{r}, \boldsymbol{r}')$ is the Green's function for the operator equation $\hat{K} V(\boldsymbol{r}, \boldsymbol{r}') = 4\pi e^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$. The same operator $\hat{K}$ also connects charge ρ and potential ϕ via $\hat{K}\phi(\boldsymbol{r}) = 4\pi e^2 \rho(\boldsymbol{r})$. For bare Coulomb interactions, $\hat{K}$ is the Laplace operator Δ, and the Poisson equation follows. Here we consider short-range interactions characterized by the interaction strength V_0, $V(x - x') = V_0 \delta(x - x')$. In this case the operator $\hat{K}$ is just a multiplication with a constant factor $4\pi e^2/V_0$. Thus, the potential and charge are connected via

$$\phi_\omega(x) = V_0 \rho_\omega(x), \tag{5}$$

instead of the Poisson equation. At this point it is quite natural to introduce the capacitance of the wire per unit length, $c = e^2/V_0$. Physically, this corresponds to a single mode quantum wire, formed by depletion induced by a back gate with a capacitance c per unit length, parallel to the wire (see Fig. 1). This capacitance incorporates the geometric arrangement, and originates from long-ranged Coulomb interactions between the wire and the gate. The well-known interaction parameter g of the Luttinger liquid is then related to c [16] via

$$g^2 = \frac{1}{1 + e^2 \nu_F / c}. \tag{6}$$

In particular, the case of the locally charge neutral wire [15], corresponds to $c = 0$ or infinitely strong pointlike interactions ($g = 0$). Indeed, the single-channel results of Ref. [15] are obtained in the $g \to 0$ limit of the formulas derived below.

Now we generalize our approach to the case when the back gate is modulated by a potential $V_3(t) = V_{3,\omega} e^{-i\omega t}$ as well. Then the total density of the wire contains in addition to the density injected from reservoir 1 an induced density due to the modulation of the gate. The self-consistent potential distribution $\phi_\omega(x)$ along the wire must now be found from the equation

$$\frac{c}{e^2 \nu_F} [\phi_\omega(x) - V_{3,\omega}] = \frac{V_{1,\omega}}{2} e^{iq_F x} - \phi_\omega(x) - \frac{iq_F}{2} \int_0^L dx' e^{iq_F|x-x'|} \phi_\omega(x'), \tag{7}$$

which is obtained by substituting $\phi_\omega(x) - V_{3,\omega}$ in the left-hand side of Eq. (5) and using Eq. (3). The solution to Eq. (7) is

$$\phi_\omega(x) = V_{3,\omega} + A_\omega^+ e^{iqx} + A_\omega^- e^{-iqx}, \tag{8}$$

where $q = gq_F$, and

$$A_\omega^\pm = \pm \frac{(1 \pm g)(1 - g^2) e^{\mp iqL}}{(1 + g)^2 e^{-iqL} - (1 - g)^2 e^{iqL}} \times \left[V_{1,\omega} - V_{3,\omega} \left(1 - \frac{1 \mp g}{1 \pm g} e^{\pm iqL} \right) \right]. \tag{9}$$

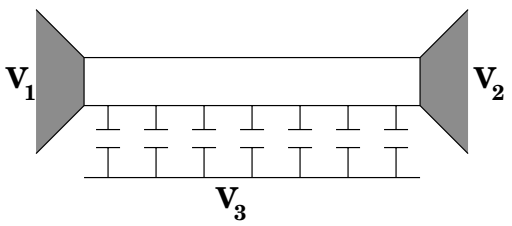

FIG. 1. The 1D wire, connected to two reservoirs and coupled capacitively to a gate.

Conductance matrix.—Now we are in a position to find the full conductance matrix for the capacitively coupled wire. The particle current in the wire is expressed through the density difference ζ of right- and left-moving electrons [15] as $I_p(x,\omega) = e v_F \zeta(x,\omega)$,

$$\zeta = \rho_{\omega,0}(x) - \frac{i q_F \nu_F}{2} \times \int_0^L dx' \, \mathrm{sgn}(x - x') e^{i q_F |x - x'|} \phi_\omega(x') ,$$

where $\phi_\omega(x)$ is the self-consistent potential (8). The displacement current, directed from the gate to the wire, has the density $j_d(x) = i\omega e \rho_\omega(x)$.

The conductance matrix $\mathcal{G}_{\alpha\beta}(\omega)$ relates the current $I_{\alpha,\omega}$ at contact α to the voltage $V_{\beta,\omega}$ applied at contact β ($\alpha, \beta = 1,2,3$): $I_{\alpha,\omega} = \mathcal{G}_{\alpha\beta}(\omega) V_{\beta,\omega}$. With the definitions

$$G_\omega = 2g \frac{e^2}{h} \frac{(1+g)e^{-iqL} + (1-g)e^{iqL}}{(1+g)^2 e^{-iqL} - (1-g)^2 e^{iqL}} , \qquad \bar{G}_\omega = -\frac{e^2}{h} \frac{4g}{(1+g)^2 e^{-iqL} - (1-g)^2 e^{iqL}} , \tag{10}$$

the conductance matrix takes a form

$$\mathcal{G}(\omega) = \begin{pmatrix} G_\omega & \bar{G}_\omega & -G_\omega - \bar{G}_\omega \\ \bar{G}_\omega & G_\omega & -G_\omega - \bar{G}_\omega \\ -G_\omega - \bar{G}_\omega & -G_\omega - \bar{G}_\omega & 2G_\omega + 2\bar{G}_\omega \end{pmatrix} . \tag{11}$$

The matrix $\mathcal{G}$ has the following properties [3]: First, it is symmetric, which reflects the fact that the geometry considered here is symmetric under the exchange of the left and right reservoirs, and no magnetic field is present. Then, $\sum_\alpha \mathcal{G}_{\alpha\beta} = 0$, which restates current conservation. Finally, the property $\sum_\beta \mathcal{G}_{\alpha\beta} = 0$ manifests the fact that a simultaneous shift of all potentials V_β by the same amount does not produce any current (gauge invariance). Furthermore, dissipation of power requires that the matrix $\mathrm{Re}\,\mathcal{G}$ is positive definite. Equation (11) can, after some algebra, also be obtained from formulas of Ref. [10].

In the static limit $\omega = 0$ one reproduces the known result [5,10]: $G = -\bar{G} = e^2/h$, with no current flowing through the gate. Another limiting case is $g = 0$ [15], where one finds $G = -\bar{G} = (e^2/h)(1 - i q_F L/2)^{-1}$. Generally, all the components of the conductance matrix are oscillating functions of frequency (for $g \neq 0$) with the period $2\pi v_F (gL)^{-1}$. In particular, the real part of the conductance reaches zero with a period $2\pi v_F (gL)^{-1}$. It has been suggested that measurement of this period should be used to determine the interaction constant [12,13]. However, this period is a consequence of the linearization of the spectrum near the Fermi energy and not really a signature of an interacting system. Furthermore, this frequency is already in the absence of interactions of the order of an electron transit frequency and therefore rather high. A better strategy consists in analyzing one of the purely capacitive conductances. In particular, we consider $\mathcal{G}_{33}(\omega) = 2(G_\omega + \bar{G}_\omega)$, which we call the gate conductance. The real and imaginary parts of the frequency dependence of the gate conductance $\mathcal{G}_{33}(\omega)$ are displayed in Fig. 2. The real part shows peaks around $\omega = (\pi v_F / gL)(2n+1)$, $n \in \mathcal{Z}$. The height of each peak is equal to 4 times the conductance quantum e^2/h (and thus independent of g), while the width decreases with decreasing g. In contrast, the imaginary part of $\mathcal{G}_{33}(\omega)$ changes sign at these points, and exhibits extrema of height $2e^2/h$ (sharp ones for small g) at the points

$$\Omega_n = \frac{v_F}{gL} \left[\pi(2n+1) \pm \arccos \frac{1-g^2}{1+g^2} \right] .$$

All elements of the conductance matrix $\mathcal{G}_{\alpha\beta}$ are characterized by the common denominator $(1+g)^2 e^{-iqL} - (1-g)^2 e^{iqL}$, which has zeros at frequencies

$$\omega_n = \frac{v_F}{gL} \left[n\pi - i \ln \frac{1+g}{1-g} \right] . \tag{12}$$

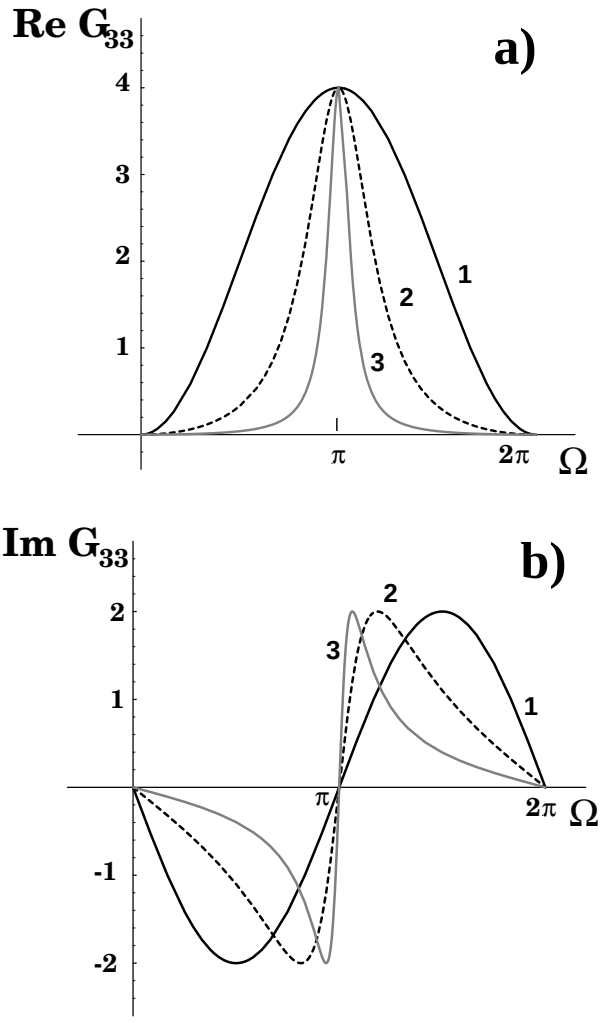

FIG. 2. One period of the frequency dependence of the real (a) and imaginary (b) parts of the gate conductance $\mathcal{G}_{33}(\omega)$ (in units e^2/h). The parameter g is equal to 1 (curve 1), 0.3 (2), and 0.1 (3); the argument is $\Omega = \omega L g / v_F$.

Equation (12) defines the (collective) excitation spectrum of the quantum wire. For $g = 0$ (local charge neutrality) only the $n = 0$ mode survives, and all other frequencies are pushed up to infinity. This mode is purely imaginary, and does not correspond to any type of quasiparticles [15]. On the other hand, for $g = 1$ (noninteracting system) all modes are infinitely damped: Thus, charge relaxation can be caused only by electron-electron interactions. We mention that the same modes are obtained in Ref. [12]; the modes with even n are also obtained in Ref. [13]. Any treatment, whether self-consistent or not, which at some stage invokes the effective interaction, will exhibit this frequency spectrum.

We have now characterized the dynamic conductance and its properties over a wide frequency range. But it is the low-frequency regime that is experimentally most easily accessed. A low-frequency measurement works only if we consider the gate current since at small frequencies the ac component of the conductance $\mathcal{G}_{11}$ represents only a small deviation from the quantized dc conductance and is hard to identify [17]. The gate conductance has the following low-frequency expansion:

$$\mathcal{G}_{33} = -iC_\mu\omega + R_q C_\mu^2 \omega^2 - i\,\frac{1 - 3g^2}{3g^2}\,R_q^2 C_\mu^3 \omega^3 + \ldots .$$

Here $C_\mu = c_\mu L$ is the total electrochemical capacitance of the wire *vis-à-vis* the gate, c_μ is given by Eq. (1). The second order term is determined by the charge relaxation resistance [18] $R_q = h/4e^2$ which is the parallel resistance of two Sharvin-Imry contact resistances of *half* a resistance quantum per contact. It is independent of the interaction constant. The third order term is proportional to the third power of the electrochemical capacitance C_μ, but most importantly it is proportional to a factor of $1/3g^2 - 1$, which is a sensitive function of the interaction strength. Thus, a measurement which determines the out-of-phase (nondissipative) part of the gate conductance up to the third order in frequency is sufficient to determine the interaction parameter g.

In conclusion, we have investigated the ac response of a quantum wire with short-range interactions. We formulated a self-consistent, charge and current conserving, approach using RPA. The boundary condition which couples the density of the wire to the electron reservoirs is of electrochemical nature. Because of the coupling with the reservoir all the collective modes of the system acquire a damping constant. In the present Letter only the one-channel case is considered. The case of two channels with the same velocity v_F (corresponding to one spin-degenerate channel) can be obtained from the above results simply by replacing the density of states of the one-channel problem ν by that appropriate for the spin-degenerate channel 2ν: Spin-charge separation cannot be probed by the ac response.

We find that the measurement of the low-frequency, nondissipative component of the gate conductance including only its first two leading coefficients is sufficient to determine the interaction strength. Such measurements are very desirable and will provide a strong stimulation for further research on the role of electron-electron interactions.

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