Driven Electrons on the Fermi Surface

A. Iomin and S. Fishman
Department of Physics, Technion, Haifa 32000, Israel
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A model for electronic motion on the Fermi surface in the presence of a uniform magnetic field and an alternating electric field is developed. For a sufficiently strong electric field it is found to be effectively kicked. It is demonstrated that for some values of the parameters the model constitutes a realization of a kicked Harper model. A family of models to which it belongs is introduced. A realization of a model where kicking both in position and in momentum takes place is proposed. The required experimental conditions for these realizations are discussed. [S0031-9007(98)06922-1]

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The quantum mechanical behavior of systems that are chaotic in the classical limit has been the subject of many recent studies [1,2] that form the field of quantum chaos. In this field systems modeled by time dependent Hamiltonians, and, in particular, Hamiltonians that are periodically dependent on time, attracted considerable interest. Among these systems maps play a special role, because of their simplicity. In spite of their simplicity maps exhibit many of the physical properties of continuous systems. Maps are generated by Hamiltonians where some part, usually the potential, is proportional to a δ function of time. The standard system that was used for the exploration of classical and quantum dynamics of maps is the kicked rotor leading to the standard map. For sufficiently strong kicks it exhibits classical diffusion that is suppressed by a mechanism similar to Anderson localization in disordered solids [1,3–5]. It was observed experimentally for laser cooled sodium atoms [6,7]. In this Letter it will be shown that in some regimes of parameters, electrons on the Fermi surface of a solid in the presence of a magnetic field and driven by an electric field are effectively kicked. The details will be presented in [8].

The simplest model for the exploration of electronic motion in a periodic potential in the presence of a magnetic field is the Harper model [9] that is defined by the Hamiltonian

$$\mathcal{H}^H = \cos p + \lambda \cos q\,. \tag{1}$$

It models also electronic motion in a one-dimensional potential with two incommensurate periods. The model was studied extensively, and extremely interesting spectral properties were found [10–14]. This model is integrable. A time dependent model that is related to it is the kicked Harper model, which is defined by the Hamiltonian

$$\mathcal{H}^{\mathrm{KH}} = L_H \cos p + K_H \cos q \sum_n \delta(t - n)\,. \tag{2}$$

The model and its variants appear naturally for the kicked harmonic oscillator [15,16]. It is of specific interest since it does not follow the Kolmogorov-Arnold-Moser (KAM) picture. Its dynamics corresponds to kicks combined with rotations of the fourfold symmetry. This system exhibits chaotic motion in a region that increases with L_H and K_H. The model was subject to extensive theoretical studies [14–20]. For some regimes of parameters its spectrum of quasienergies is similar in nature to the energy spectrum of the Harper model [14,18,21]. It exhibits classical and quantum diffusion as well as localization and anomalous diffusion and even ballistic motion [15–20]. The motivation for the explorations of the kicked Harper model in the field of quantum chaos was so far mainly theoretical, because of the variety of interesting phenomena that were found. Since the system can be modeled approximately by the kicked harmonic oscillator it can be realized experimentally [22].

In the present Letter it is demonstrated that a variety of models including the kicked Harper model can be realized also for electrons on a lattice in the presence of a magnetic field driven by a smooth driving electric field. The effective kicking is generated as a result of a resonance between the electronic motion and the driving field. This system also exhibits interesting behavior in other regimes such as bifurcations and phase space acceleration [23]. The materials that are most relevant for experimental realization of the kicked Harper model are the 2D electron gas embedded in lateral superlattices fabricated on GaAs heterostructures [24] and organic metals (where the skin depth is larger than the size of the crystals used) [25,26]. Classical calculations in the chaotic regime were used to explain the microwave photo-conductivity for GaAs heterostructures [27]. The experimental and theoretical explorations of the organic metals focused so far on their Fermi surfaces that are quasi-two-dimensional. The topologies of these surfaces are studied by a variety of strong magnetic field [25] as well as cyclotron resonance [26,28] techniques. Tight-binding models leading to equations like (1) are used for these investigations [29] in the linear response regime. In the present Letter it will be shown that for strong driving fields, where linear response theory does not hold, the model (2) is relevant.

In what follows the dynamics of Bloch electrons, namely, electrons moving in a periodic potential, in the presence of a uniform, constant (in time) magnetic field and a uniform electric field alternating in time is explored. The specific case of electrons on a cubic lattice

is considered. The magnetic field $\vec{H}$ is chosen in the direction of the crystal axis that will be denoted as the z direction in what follows, while the electric field is chosen in the direction of the y axis. In the first stage an effective Hamiltonian for the electronic motion will be derived in the framework of the one band approximation. Then it will be shown that in some limit it is well approximated by a kicked model. For specific values of parameters it reduces to the kicked Harper model (2).

The starting point of our analysis is the tight-binding Hamiltonian $\mathcal{H}^{(\mathrm{t.b.})} = \varepsilon(\hbar\vec{k})$, where $\hbar\vec{k}$ is the lattice momentum. For a uniform magnetic field the Peierls substitution holds that if a magnetic field is applied to a periodic solid, described by $\mathcal{H}^{(\mathrm{t.b.})}$ without the field, the Hamiltonian in presence of the field is [30]

$$\mathcal{H}^A = \varepsilon\left(\vec{P} - \frac{e}{c}\vec{A}\right), \tag{3}$$

where e is the charge of the electron, c is the speed of light, $\vec{P}$ is the canonical momentum, and $\vec{A}$ is the vector potential of the magnetic field. The magnetic field is $\vec{H} = (0, 0, H)$, and the vector potential is chosen to take the form $\vec{A} = \vec{A}^0 = (0, -xH, 0)$. The canonical momentum in (3) is determined in the standard way as $P_x = \hbar k_x; P_y = \hbar k_y + \frac{e}{c}A_y^0$. The components of the velocity $(v_x, v_y) = (\dot{x}, \dot{y})$ and the generalized force $(\dot{P}_x, \dot{P}_y)$ are obtained in the standard way by differentiation of $\mathcal{H}^A$. The corresponding two components of the quasimomentum which are canonical conjugate variables are (see, for example, [31])

$$\hbar\dot{k}_x = -\frac{eH}{c}v_y, \qquad \hbar\dot{k}_y = \frac{eH}{c}v_x, \qquad \vec{v} = \frac{1}{\hbar}\vec{\nabla}_k\varepsilon\,. \tag{4}$$

The effect of the time dependent electric field is introduced by the acceleration theorem [31] that holds if the perturbation is weak and of sufficiently low frequency so that interband transitions can be ignored and is slowly varying in space so that matrix elements between Wannier functions are negligible. The effective approximate Hamiltonian is $\mathcal{H}_{\mathrm{eff}} = \varepsilon(\hbar\vec{k}) + \mathcal{U}(\vec{r}, t)$ [31,32], where $\mathcal{U}$ is the time dependent perturbation. Thus if the alternating electric field $\vec{E} = (0, -E_0\cos\tilde{\nu}t, 0)$ is not too strong its only effect is to add an additional component to the acceleration $\hbar\dot{k}_y$ leading to $\hbar\dot{k}_y = \frac{eH}{c}v_x - eE_0\cos\tilde{\nu}t$, where E_0 is the strength of the alternating field, while $\tilde{\nu}$ is its frequency.

We will consider the orthorhombic lattice. In the tight-binding approximation the Fermi surface is determined by [33]

$$\varepsilon(\vec{\hbar k}) = -\alpha - \gamma_1\cos k_x a - \gamma_2\cos k_y a - \gamma_3\cos k_z a\,, \tag{5}$$

where α is the diagonal part of the energy of lattice electrons, $\gamma_1, \gamma_2, \gamma_3$ are the overlap integrals in tight-binding approximation, and a is the lattice constant. The resulting equations of motion are

$$\dot{P}_x = \frac{eH\gamma_2 a}{\hbar c}\sin\frac{a}{\hbar}\left(P_y - \frac{e}{c}A_y\right), \qquad \dot{P}_y = 0\,,$$
$$\dot{x} = \frac{\gamma_1 a}{\hbar}\sin\frac{a}{\hbar}P_x, \qquad \dot{y} = \frac{\gamma_2 a}{\hbar}\sin\frac{a}{\hbar}\left(P_y - \frac{e}{c}A_y\right). \tag{6}$$

The vector potential $\vec{A}$ is chosen to be time dependent in order to take into account the effect of the electric field, namely, $\vec{A} = \vec{A}^0 - \frac{c}{\tilde{\nu}}\vec{E} = (0, -xH + \frac{cE_0}{\tilde{\nu}}\sin\tilde{\nu}t, 0)$.

We note that P_y is a constant of motion and the time dependence of y is completely determined by x and P_x. Therefore it is convenient to use x and P_x as the conjugate variables. The equations of motion (6) for these are just the Hamilton equations $\dot{P}_x = -\frac{\partial\mathcal{H}_1}{\partial x}$ and $\dot{x} = \frac{\partial\mathcal{H}_1}{\partial P_x}$ for the Hamiltonian

$$\mathcal{H}_1 = -\gamma_1\cos\frac{P_x a}{\hbar} - \gamma_2\cos\frac{a}{\hbar}\left(P_y - \frac{e}{c}A_y\right), \tag{7}$$

where $\vec{A}$ is defined above. For the tight-binding approximation to hold it is required that $\gamma_1, \gamma_2 \gg eE_0 a$ and $\gamma_1, \gamma_2 \gg \hbar\tilde{\nu}$. It is convenient to introduce dimensionless variables, $p = \frac{P_x a}{\hbar}, q = \frac{a}{\hbar}(\frac{eH}{c}x + P_y) = \frac{eHa}{c\hbar}(x - x_0)$, with $x_0 = -\frac{c}{eH}P_y$. The dimensionless electric field is $\kappa = \frac{eE_0 a}{\hbar\tilde{\nu}}$, the time is rescaled by $\Omega t \to t, \frac{\tilde{\nu}}{\Omega} = \nu$, where $\Omega = (\sqrt{\gamma_1\gamma_2}\,a^2 eH)/\hbar^2 c$. The dimensionless Planck's constant is also determined by the Poisson brackets for p and q: $\tilde{h} = \hbar\{q, p\}_P = \hbar\frac{\partial q}{\partial x}\frac{\partial p}{\partial P_x} = \frac{Ha^2}{\hbar c/e} \equiv 2\pi\frac{\Phi}{\Phi_0}$, where Φ is the magnetic field flux through one lattice cell $a \times a$, and $\Phi_0 = \frac{hc}{e}$ is a quantum of the magnetic flux. Rescaling the Hamiltonian $\mathcal{H}_1/\sqrt{\gamma_1\gamma_2} \to \mathcal{H}_1$ and defining $\sqrt{\gamma_1/\gamma_2} = L, \sqrt{\gamma_2/\gamma_1} = K$, we obtain the following Hamiltonian in the form of a driven Harper model:

$$\mathcal{H}_1 = L\cos p + K\cos(q - \kappa\sin\nu t)\,. \tag{8}$$

The driving potential $K\cos(q - \kappa\sin\nu t)$ is well known in the literature, and it has been discussed in the context of the description of dynamical localization in atomic momentum transfer [34,35]. This effect has been observed experimentally following an extension of a theoretical proposal [7]. The Hamilton equations for $\mathcal{H}_1$ of (8) are $\dot{q} = -L\sin p$ and $\dot{p} = K\sin\psi(t)$ with $\psi(t) = q - \kappa\sin\nu t$. For $\kappa \gg 1$ or $eE_0 a \gg \hbar\tilde{\nu}$ the forcing resulting in change of momentum is dominated by the resonant points [7,34–36], where $\dot{\psi} = 0$ or

$$-L\sin p = \dot{q} = \kappa\nu\cos\nu t\,. \tag{9}$$

This condition is consistent with the tight-binding approximation if $\gamma_1, \gamma_2 \gg eE_0 a \gg \hbar\tilde{\nu}$, which can be satisfied for some regimes of the electric field for organic metals and superlattices. Expanding $\psi(t)$ around the resonant point t_r and integrating the Hamilton equation for p taking into account the fact that this integral accumulates most of its contribution from a narrow region (of width $|\kappa\nu^2\sin t_r|^{-1/2}$) around the resonance, one finds that the

momentum transferred at each resonance is

$$\Delta p_r^{\pm} = \sqrt{\frac{2\pi}{\kappa\nu^2|\sin\nu t_r|}}\, K \sin\left(\psi_r^{\pm} \pm \frac{\pi}{4}\right), \qquad (10)$$

where $\psi_r = \psi(t_r)$ and the sign depends on the direction of crossing of the resonance.

The position of the resonance depends on p. For strong driving so that $\kappa\nu \gg L$ (or $eE_0a \gg 2\pi\gamma_1 \frac{\Phi}{\Phi_0} = \gamma_1\tilde{h}$) for the resonance condition it is required that $\cos\nu t_r \approx 0$ or $|\sin\nu t_r| \approx 1$. Consequently the resonant points are $\nu t_r \approx \pm\frac{\pi}{2}$ and therefore are approximately equally spaced in time, and occur at the times $\nu t_r^- = -\frac{\pi}{2} + 2\pi l$ and $\nu t_r^+ = \frac{\pi}{2} + 2\pi l$ (l are integers). The resulting map is $M = M_2M_1$ with

$$M_1: \begin{cases} p_1 = & p + K_1 \sin(q + \kappa_0), \\ q_1 = & q - L_1 \sin p_1, \end{cases}$$
$$M_2: \begin{cases} p_2 = & p_1 + K_1 \sin(q_1 - \kappa_0), \\ q_2 = & q_1 - L_1 \sin p_2, \end{cases} \qquad (11)$$

where $L_1 = \frac{\pi}{\nu}L$ while $K_1 = \sqrt{2\pi/\kappa\nu^2}\,K$ and $\kappa_0 = \kappa - \frac{\pi}{4}$. First we note that the map is periodic in κ with the period 2π, and therefore it is periodic in the magnitude of the electric field.

This map is generated by the Hamiltonian

$$\mathcal{H}_2 = L\cos p + \nu K_1\left[\cos(q+\kappa_0)\sum_{n=-\infty}^{\infty}\delta\left(\nu t + \frac{\pi}{2} - 2\pi n\right) + \cos(q-\kappa_0)\sum_{n=-\infty}^{\infty}\delta\left(\nu t - \frac{\pi}{2} - 2\pi n\right)\right]. \qquad (12)$$

It can be derived also with the help of the asymptotic properties of Bessel functions.

An essential simplification can be made if the electric field is such that $\kappa = 2\pi N + \frac{\pi}{4}$ and $N \gg 1$ is a natural number. The resulting map corresponding to (11) is

$$p_1 = p + \sqrt{\frac{2\pi}{\kappa\nu^2}}\, K \sin q,$$
$$q_1 = q - \frac{\pi}{\nu} L \sin p_1. \qquad (13)$$

It is generated by (2) with $K_H = \frac{K}{\nu}\sqrt{2\pi/\kappa}$ and $L_H = \frac{\pi}{\nu}L$. Classical chaos in large regions is found, for example, for $K_H > 1$. Together with $eE_0a \gg 2\pi\gamma_1\frac{\Phi}{\Phi_0} = \gamma_1\tilde{h}$ this condition yields $eE_0a \gg \hbar\tilde{\nu}(\frac{\gamma_1}{\gamma_2})^2$.

A special role is played by the self-dual point $L_H = K_H$. For (13) this condition is $eE_0a = \frac{2}{\pi}(\frac{\gamma_2}{\gamma_1})^2\hbar\tilde{\nu}$. Consistency with $eE_0a \gg \hbar\tilde{\nu}$ requires $\frac{2}{\pi}(\frac{\gamma_2}{\gamma_1})^2 \gg 1$. It is consistent also with the inequality $eE_0a \gg \hbar\tilde{\nu}(\frac{\gamma_1}{\gamma_2})^2$. Therefore the interesting regimes of the kicked Harper model can in principle be realized experimentally.

Choosing $\kappa = \pi(2N+1) + \frac{\pi}{4}$ one finds a model similar to (2) but with q shifted by π. For $\kappa = \pi(N + \frac{1}{2}) + \frac{\pi}{4}$ one obtains the two-sided kicked Harper model defined by the Hamiltonian

$$\mathcal{H}^{\mathrm{TKH}} = L_H\cos p + K_H(-1)^N\cos(q+\pi/2) \times \sum_n (-1)^n\delta(t-n). \qquad (14)$$

So far the electric field was chosen to be parallel to the y axis. Now turn to the case where the electric field is perpendicular to the magnetic field but is not confined to an axis, namely, $\vec{E} = (-E_x\cos\tilde{\nu}t, -E_y\cos\tilde{\nu}t, 0)$. Following the argumentation used for the electric field parallel to the y axis one finds that the motion is described by the effective Hamiltonian

$$\mathcal{H}_3 = L\cos(p - \kappa_x\sin\nu t) + K\cos(q - \kappa_y\sin\nu t), \qquad (15)$$

where the canonical variables are p and q, the parameters L, K, and ν are defined as before, while $\kappa_x = \frac{eE_xa}{\hbar\tilde{\nu}}$ and $\kappa_y = \frac{eE_ya}{\hbar\tilde{\nu}}$.

In the asymptotic limit of large κ_x and κ_y, namely, $\kappa_x\nu \gg K$ and $\kappa_y\nu \gg L$, one obtains an approximate Hamiltonian corresponding to (12)

$$\mathcal{H}_3 = \mathcal{H}_3^+\sum_{n=-\infty}^{\infty}\delta\left(\nu t + \frac{\pi}{2} - 2\pi n\right) + \mathcal{H}_3^-\sum_{n=-\infty}^{\infty}\delta\left(\nu t - \frac{\pi}{2} - 2\pi n\right) \qquad (16)$$

with $\mathcal{H}_3^{\pm} = \sqrt{2\pi/\kappa_x}\,L\cos(p \pm \kappa_x \mp \frac{\pi}{4}) + \sqrt{2\pi/\kappa_y}\times K\cos(q \pm \kappa_y \mp \frac{\pi}{4})$. The Poisson brackets $\{\mathcal{H}_3^+, \mathcal{H}_3^-\}$ do not vanish in general, and therefore the motion is expected to be chaotic. For the system to be considered kicked it is required $eE_xa, eE_ya \gg \hbar\tilde{\nu}$. For the kicks to be equally spaced in time the conditions $eE_ya \gg \gamma_1\tilde{h}$ and $eE_xa \gg \gamma_2\tilde{h}$ should hold. The self-dual case is $\sqrt{2\pi/\kappa_x}\,L = \sqrt{2\pi/\kappa_y}\,K$ or $(\frac{\gamma_1}{\gamma_2})^2 = \frac{E_x}{E_y}$. The situation is simpler when the Poisson bracket $\{\mathcal{H}_3^+, \mathcal{H}_3^-\}$ vanishes. This requires

$$\kappa_x - \kappa_y = N_1\pi, \qquad \kappa_x + \kappa_y = N_2\pi + \frac{\pi}{2}, \qquad (17)$$

where N_1 and N_2 are integers. The situation is particularly simple when $\kappa_x = \kappa_y = 2\pi N + \frac{\pi}{4}$. In this case $\mathcal{H}_3^+ = \mathcal{H}_3^- = \overline{\mathcal{H}}$ and $\mathcal{H}_3$ reduces to

$$\mathcal{H}_3 = \overline{\mathcal{H}}\sum_n \delta(t - nT) \qquad (18)$$

with $\overline{\mathcal{H}} = L_3\cos p + K_3\cos q$, where $L_3 = \frac{L}{\nu}\sqrt{2\pi/\kappa_x}$ and $K_3 = \frac{K}{\nu}\sqrt{2\pi/\kappa_y}$, while $T = \frac{\pi}{\nu}$ is the period. This model is integrable since $\overline{\mathcal{H}}$ is such, the classical Hamilton equations can be integrated and the solution can be expressed explicitly in terms of elliptic functions. The final answer takes the form $\cos p(t) = f(B[\frac{t-t_0}{T}]_{\mathrm{int}})$, where f is a periodic function of period B_0, while $[X]_{\mathrm{int}}$ is the integer part of X and B is a constant. If B is

commensurate with B_0 then p is periodic and takes a finite number of values, while if it is incommensurate it covers ergodically the interval $[0, 2\pi]$. There are different other integrable possibilities for the various combinations (17).

The one-step evolution operator for the Hamiltonian $\overline{H}_3$ and $\overline{\mathcal{H}}$ have the same eigenstates. The quasienergies in this case are just $\eta_m = E_m/\tilde{h}|_{\mathrm{mod}\,2\pi}$, where E_n are the eigenenergies of $\mathcal{H}$. Since $\mathcal{H}$ is the Harper Hamiltonian, $\{E_m\}$ are known [11–14] and the η_m can be obtained from them. In particular, for $L_3 = K_3$ the quasienergy spectrum will have a self-similar structure, but folded on the interval $[0, 2\pi]$.

In this Letter a model for electronic motion in solids, where as a result of variation of the electric field a transition takes place from smooth driving for weak fields to effective kicking for strong fields, was presented. A variety of models of which the well known kicked Harper model is an example was developed. It is expected that at least some of them will be studied experimentally and theoretically. Preliminary results [8] indicate that interesting behavior is expected. In particular, it is found that the behavior of single trajectories for the models characterized by various values of κ, corresponding to values of the electric field may be very different [8]. Most physical properties depend, however, on the evolution of phase space densities (and related averages and variances). In the chaotic regime these are not very sensitive to small errors in the value of κ. The behavior predicted by the model presented in this Letter can be observed in conductivity and absorption experiments. Deviations from standard resonance phenomena predicted by linear response theory are expected, and their detailed nature should be analyzed in the future.

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