Probing Partonic Structure in $\gamma^*\gamma \to \pi\pi$ near Threshold

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Hadron pair production $\gamma^*\gamma \to h\overline{h}$ in the region where the c.m. energy is much smaller than the photon virtuality can be described in a factorized form, as the convolution of a partonic handbag diagram and generalized distribution amplitudes which are new nonperturbative functions describing the exclusive fragmentation of a quark-antiquark pair into two hadrons. Scaling behavior and a selection rule on photon helicity are signatures of this mechanism. The case where h is a pion is emphasized. [S0031-9007(98)06976-2]

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Hadron production at small invariant mass or even near threshold is usually quite inaccessible to a partonic description. We advocate that a notable exception is provided by the process $\gamma^*\gamma \to h\overline{h}$ with a highly virtual photon, more precisely, in the regime where the squared c.m. energy W^2 is much smaller than the photon virtuality Q^2. Here we will focus on pairs of spin zero mesons, e.g., pions or kaons, but our analysis can be extended to other final states such as $p\overline{p}$.

A natural point of reference for this study is the transition form factor $\gamma^*\gamma \to \pi^0$ at large Q^2, which has been extensively studied in QCD [1,2] and is considered to be one of the best environments for studying the pion light cone distribution amplitude. We investigate a similar process, where a *pair* of neutral or charged pions is produced. The hard scattering, $\gamma^*\gamma \to q\overline{q}$ at tree level, is essentially the same, while the hadronic matrix element is a new nonperturbative object describing the transition from partons to hadrons. This mechanism is shown in Fig. 1.

Note that our reaction is related by crossing to deeply virtual Compton scattering, $\gamma^* h \to \gamma h$ which, at large Q^2 and small squared momentum transfer between the hadrons, factorizes into a hard photon-parton scattering and an off-diagonal parton distribution [3,4]. The tree level diagrams are precisely the crossed versions of those in Fig. 1. The $\gamma^*\gamma$ and the Compton process share many features, in particular, their scaling behavior in Q^2 and a helicity selection rule for the virtual photon.

Kinematics.—In addition to the four-momenta shown in Fig. 1, we introduce $P = p + p'$ and $Q^2 = -q^2$, $W^2 = P^2$, $t = (q-p)^2$, $m^2 = p^2$. Let us go to the collision c.m. and define our z axis along $\mathbf{q}$. In terms of the angle $\theta_{\mathrm{c.m.}}$ between $\mathbf{q}$ and $\mathbf{p}$, we may write

$$t = m^2 - \frac{Q^2 + W^2}{2}\left(1 - \sqrt{1 - \frac{4m^2}{W^2}}\cos\theta_{\mathrm{c.m.}}\right). \quad (1)$$

Now we perform a boost along z to the Breit frame and introduce lightlike vectors $v = (1,0,0,1)/\sqrt{2}$ and $v' = (1,0,0,-1)/\sqrt{2}$ which, respectively, set a "plus" and "minus" direction. We then have

$$q = \frac{Q}{\sqrt{2}}(v - v'), \qquad q' = \frac{Q^2 + W^2}{\sqrt{2}\,Q}v',$$
$$P = \frac{Q}{\sqrt{2}}v + \frac{W^2}{\sqrt{2}\,Q}v'. \quad (2)$$

We consider the kinematic region

$$W^2 \ll Q^2, \quad (3)$$

where both p and p' have small minus and transverse components; to describe their plus components, we define the light cone fraction

$$\zeta = \frac{p^+}{P^+} = 1 - \frac{m^2 - t}{Q^2 + W^2}, \quad (4)$$

which controls how they share the momentum P. With (1) it is easy to see that ζ and $1-\zeta$ cannot be smaller than m^2/W^2. This frame is suited to display the factorization of our process into a hard, head-on collision of the two photons in which partons are produced moving fast into the virtual photon direction and a long-distance process of hadronization. From the usual power counting arguments for Feynman graphs [5] it is favorable to have the minimal number of partons, that is, two, between the hard scattering and the soft hadron formation.

Expression of the amplitude.—Let us now derive the tree level expression of the $\gamma^*\gamma$ amplitude as a convolution of a hard scattering amplitude H and an amplitude S for the soft transition from partons to hadrons. Technically,

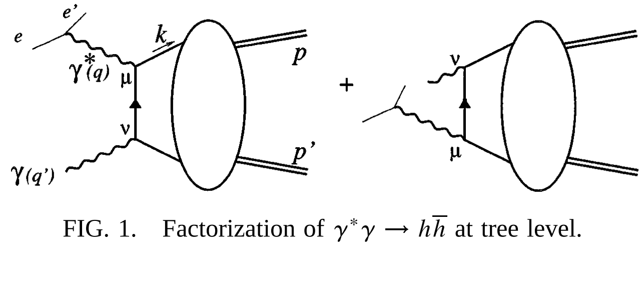

FIG. 1. Factorization of $\gamma^*\gamma \to h\overline{h}$ at tree level.

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factorization means that the behavior of S enforces k^2 and kP to be negligible with respect to the hard scale Q^2. As a result, the minus and transverse components of k can be neglected in H, and the hadronic tensor can be written as an integral over the light cone fraction $z = k^+/P^+$ of the quark with respect to the hadronic system:

$$\begin{aligned} iT^{\mu\nu} &= -\int d^4x\, e^{-i\mathbf{q}\cdot\mathbf{x}} \\ &\quad \times {}_{\text{out}}\langle h(p)\bar{h}(p')\,|TJ^{\mu}_{\text{em}}(x)J^{\nu}_{\text{em}}(0)|0\rangle_{\text{in}} \\ &= \int dz\, H^{\mu\nu}_{\alpha\beta}(z,q,q')S_{\alpha\beta}(z,v',p,p'), \end{aligned} \quad (5)$$

with the soft matrix element

$$\begin{aligned} S_{\alpha\beta} &= \frac{P^+}{2\pi}\int dx^-\, e^{-iz(P^+x^-)} \\ &\quad \times {}_{\text{out}}\langle h(p)\bar{h}(p')\,|\bar{\psi}_\alpha(x^- v')\psi_\beta(0)|0\rangle_{\text{in}}\,. \end{aligned} \quad (6)$$

Here we are working in the light cone gauge $A^+ = 0$; otherwise the usual path-ordered exponential appears between the quark fields. The hard scattering is given by

$$\begin{aligned} H^{\mu\nu}_q &= \frac{ie_q^2}{\sqrt{2}\,Q} \\ &\quad \times \bigg[(g^{\mu\rho}v'^{\nu} + v'^{\mu}g^{\nu\rho} - g^{\mu\nu}v'^{\rho})\gamma_\rho \frac{2z-1}{z(1-z)} \\ &\quad - i\epsilon^{\mu\nu\rho\sigma}\gamma_\rho\gamma_5\bigg(v_\sigma - \frac{v'_\sigma}{z(1-z)}\bigg)\bigg] \end{aligned} \quad (7)$$

for quark charge e_q, and selects the vector and axial vector part of the soft amplitude. To obtain the leading behavior of $T^{\mu\nu}$ in the large-Q^2 limit we replace γ_ρ with γ^+v_ρ and $\gamma_\rho\gamma_5$ with $(\gamma^+\gamma_5)v_\rho$ in (7); note that the v_σ term in the antisymmetric part of $H^{\mu\nu}$ then drops out. From now on, we specialize in the case where h is a spin zero meson, then the axial part of S vanishes due to parity invariance and we are left with the vector part; note that this is just opposite to the case of the $\gamma^*\gamma \to \pi^0$ transition form factor. Introducing for each quark flavor the generalized distribution amplitude

$$S_{q,\alpha\beta}\gamma^+_{\alpha\beta} = \Phi_q(z,\zeta,W^2)P^+, \quad (8)$$

we finally have

$$\begin{aligned} T^{\mu\nu} &= \tfrac{1}{2}(v^\mu v'^\nu + v'^\mu v^\nu - g^{\mu\nu})\sum_q e_q^2 \\ &\quad \times \int_0^1 dz\,\frac{2z-1}{z(1-z)}\,\Phi_q(z,\zeta,W^2)\,. \end{aligned} \quad (9)$$

Contracting with the photon polarization vectors we see that, in order to give a nonzero $\gamma^*\gamma \to h\bar{h}$ amplitude, the virtual photon must have the same helicity as the real one; in particular, it must be transverse. As in the case of virtual Compton scattering, this is a direct consequence of chiral invariance in the collinear hard scattering process [6]. We also find that the $\gamma^*\gamma$ amplitude is independent of Q^2 at fixed ζ and W^2, up to a logarithmic scaling violation to be discussed later.

There will of course be power corrections in $1/Q$ to this leading mechanism. An example is the hadronic component of the real γ, which might be modeled by vector meson dominance. From power counting arguments, one finds that this contribution is suppressed, as in virtual Compton scattering [3,4,7]. Our process can also be treated within the operator product expansion [8], which allows for a systematic analysis of higher twist effects.

Generalized distribution amplitudes.—Although S describes an amplitude, we have not time ordered the fields in its definition (6): The time ordering can be left out because the separation of the field operators is lightlike, as shown for ordinary distribution amplitudes and off-diagonal or diagonal parton distributions in [9]. As a by-product of the proof, one finds that our generalized distributions $\Phi(z,\zeta,W^2)$ are only nonzero in the interval $0 < z < 1$, just the same as ordinary distribution amplitudes.

Whereas time reversal invariance constrains ordinary distribution amplitudes and parton distributions to be real valued functions up to convention dependent phases, our generalized distributions are *complex:* Time reversal interchanges incoming and outgoing states, and $|h\bar{h}\rangle_{\text{out}}$ is different from $|h\bar{h}\rangle_{\text{in}}$ since hadrons interact. We notice in (9) that the hard scattering kernel at Born level is purely real so that the imaginary part of the $\gamma^*\gamma$ amplitude is due to $\text{Im}\,\Phi$; it corresponds to rescattering and resonance formation in the soft transition from the partons to the final state hadron pair.

Charge conjugation invariance provides a symmetry relation

$$\Phi(z,\zeta) = -\Phi(1-z,1-\zeta), \quad (10)$$

where, for ease of writing, we have dropped the argument W^2 so that, in the convolution (9), one may replace

$$\Phi(z,\zeta) \to \tfrac{1}{2}[\Phi(z,\zeta) + \Phi(z,1-\zeta)]. \quad (11)$$

The amplitude is hence symmetric under $\zeta \leftrightarrow 1-\zeta$, i.e., under exchange of the hadron momenta, which reflects the fact that the initial state $\gamma^*\gamma$ of the collision has positive charge conjugation parity C. In the case where $h = \bar{h}$, for instance, if $h = \pi^0$, the corresponding configurations are identical so that $\Phi(z,\zeta)$ is even under $\zeta \leftrightarrow 1-\zeta$ and, by virtue of (10), odd under $z \leftrightarrow 1-z$.

As for parton distributions and ordinary distribution amplitudes, there are sum rules relating moments of Φ to matrix elements of local operators. Appropriately summed over quark flavors, the moment $\int dz\,\Phi(z,\zeta)$ gives the timelike elastic form factor as measured in $\gamma^* \to h\bar{h}$ and thus provides a constraint on our new quantities. It is however inaccessible in our two-photon process, which projects out the part of $\Phi(z,\zeta)$ that is odd under $z \leftrightarrow 1-z$ as we see in (9). This is clear because the elastic form factor is C odd while a $\gamma^*\gamma$ collision produces the C-even projection of $h\bar{h}$. The moment $\int dz(2z-1)\Phi(z,\zeta)$, in contrast, gives a matrix element of the quark part of the energy-momentum tensor, which is C even.

Let us point out some connections to our new distributions with other quantities describing the partonic structure of hadrons. The physics they involve goes beyond that of a $q\overline{q}$ distribution amplitude of a meson: Since two hadrons are formed, the physics of Φ does not select their lowest Fock states; in this respect, it is related to ordinary parton distributions and to fragmentation functions. If, on the other hand, W is at or near the mass of a resonance with appropriate quantum numbers, such as an f_0, it will contain physics of the distribution amplitude for the resonance and of its decay into $h\overline{h}$.

As already mentioned, our distributions can also be viewed as the crossed version of off-diagonal parton distributions [3,4], and we remark that Φ can be obtained from double distributions [4] in the crossed channel. Notice also that matrix elements such as the ones defining Φ have been considered for multihadron production at large invariant mass [10].

Evolution.—QCD radiative corrections to the hard scattering will as usual lead to logarithmic scaling violation. At this point it is useful to remember the analogy of our process with the $\gamma^*\gamma \to \pi^0$ transition form factor. The evolution can be obtained from the hard scattering kernel alone and remains the same if we replace Φ with the distribution amplitude of a meson with the appropriate quantum numbers, say, an f_0. The generalized distribution amplitudes thus follow the usual Efremov-Radyushkin-Brodsky-Lepage (ERBL) evolution [11] for a meson,

$$\mu^2 \frac{d}{d\mu^2} \Phi(z, \zeta, W^2; \mu^2) = \int_0^1 dy\, V_+(z,y) \times \Phi(y, \zeta, W^2; \mu^2), \quad (12)$$

where μ^2 is the factorization scale and $V_+(z,y)$ is the evolution kernel.

The situation here is easier than in virtual Compton scattering, where the evolution [3,4,12] is given by the *extended* ERBL kernel. In that case, the hard subprocess involves the parameter describing the kinematic asymmetry of the two hadrons, whereas in our case the hard scattering and the evolution kernel are independent of ζ.

Up to now, we have discussed generalized $q\overline{q}$ distribution amplitudes. Beyond tree level in the hard scattering the hadron pair can, however, also originate from two gluons. The generalized $q\overline{q}$ and gg distributions mix under evolution, and (12) becomes a matrix equation. We remark that at a one-loop level the calculation of the $q\overline{q}$ diagonal element in the matrix evolution kernel $V_+(z,y)$ is completely analogous to the standard one for pseudoscalar and vector mesons, and results in the same expression given in Ref. [11]. For a discussion of the evolution, including quark-gluon mixing, we refer to [13]. As for the gluons, they are known to be important in fragmentation and in parton distributions, and one can expect the generalized gg distribution to be of the same order as the one for $q\overline{q}$.

Phenomenology.—e^+e^- high energy colliders such as $B\overline{B}$ factories and CERN Large Electron-Positron Collider (LEP) have a rich potential in $\gamma^*\gamma$ physics. Without elaborating on experimental issues we remark only that the $\gamma^*\gamma$ c.m. is not the laboratory frame so that our kinematics, even if W is close to threshold, does not imply slowly moving final state hadrons whose detection would be difficult.

In $e\gamma$ collisions the $\gamma^*\gamma$ process we want to study competes with bremsstrahlung, where the hadron pair originates from a virtual photon radiated off the lepton [14]. This process produces the pair in the C-odd channel and hence does not contribute to $h = \overline{h}$, in particular, not to $h = \pi^0$. Its amplitude can be computed from the value of the timelike elastic form factor measured in $e^+e^- \to h\overline{h}$.

The interference between the $\gamma^*\gamma$ and bremsstrahlung processes provides an opportunity to study the $\gamma^*\gamma$ contribution at *amplitude* level. Thanks to the different charge conjugation properties of the two processes their interference term can be selected by the charge asymmetries

$$d\sigma(h(p)\overline{h}(p')) - d\sigma(\overline{h}(p)h(p')) \quad (13)$$

or

$$d\sigma(e^+\gamma \to h\overline{h}) - d\sigma(e^-\gamma \to h\overline{h}), \quad (14)$$

while it drops out in the corresponding charge averages. We note that bremsstrahlung has an amplitude with both real and imaginary parts, especially at values of W where it is dominated by vector meson resonances, and, in particular, benefits from ρ up to W of order 1 GeV.

In a kinematical regime where bremsstrahlung is negligible, a comparison between charged and neutral pion pair yields will allow one to study the breaking of isospin symmetry: Applying this symmetry to the transition from $q\overline{q}$ or gg to a pair of pions in the C-even channel, one obtains

$$T^{\mu\nu}(\pi^+(p)\pi^-(p')) = -T^{\mu\nu}(\pi^0(p)\pi^0(p')) \quad (15)$$

for the hadronic tensor of the two-photon process.

We finally point out that, by using the methods of [6], the angular correlation between the leptonic and hadronic planes in the $\gamma^*\gamma$ c.m. can be used to test the helicity selection rule of the handbag mechanism and thus the dominance of leading twist at finite value of Q^2.

In conclusion, we have exhibited a new instance of factorization of long- and short-distance dynamics in a process accessible at existing or planned e^+e^- or $e\gamma$ facilities. The investigation of $\gamma^*\gamma \to h\overline{h}$ in the kinematical domain $W^2 \ll Q^2$ provides a complement to the existing tools for the study of hadrons in QCD.

The reaction studies and the generalized distribution amplitudes introduced here are natural extensions of the $\gamma^*\gamma \to \pi^0$ transition form factor and of the pion distribution amplitude, which is of great importance in the QCD description of exclusive reactions at large momentum transfer. Describing a $q\overline{q} \to h\overline{h}$ or $gg \to h\overline{h}$ transition, the new distribution amplitudes contain more variables, specifying the invariant mass of the two hadrons and their sharing of momentum. Note that they do not select a lowest Fock state component of the hadrons.

The generalized distribution amplitudes may also be seen as quantities obtained by crossing from the off-diagonal parton distributions, discussed extensively in the framework of deeply virtual Compton scattering and deep electroproduction of mesons.

Phenomenological aspects of the reaction presented here have been sketched: Charge asymmetries, interference with $h\overline{h}$ production by bremsstrahlung, and angular correlations offer valuable help for its experimental investigation. Once the validity of our leading-twist analysis in a given region of Q^2 has been tested, the extraction of the generalized distribution amplitudes for various mesons will be possible; these new nonperturbative quantities should give us another glimpse of how hadrons are made from partons.

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