Does Inflationary Particle Production Suggest $\Omega_m < 1$?

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We study a class of Friedmann-Robertson-Walker spacetimes with a nonminimally coupled light massive scalar field. Values of the coupling parameter $\xi < 0$ enhance long range power in the vacuum expectation value of the energy-momentum tensor $\langle T_{\mu\nu}\rangle$ and fundamentally alter the nature of inflationary particle production: the energy density of created particles behaves like an effective cosmological constant, leading generically to $\Omega_m < 1$ in clustered matter and providing a possible resolution of the "Ω problem" for low density cosmological models. [S0031-9007(98)06950-6]

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In this Letter we discuss particle creation of light nonminimally coupled scalar fields due to the changing geometry of a spacetime which underwent an early inflationary phase. Nonminimally coupled fields have arisen in diverse cosmological contexts, e.g., density perturbations from strongly coupled scalars [1], the possibility that a very light scalar particle such as the axion may couple nonminimally to gravity [2] and production of primordial magnetic fields from nonminimal electromagnetism [3]. Ultralight scalars (with $m \sim H$) have been previously discussed in the context of pseudo-Nambu-Goldstone bosons [4].

As shown below, for negative values of the coupling the dominant contribution to both $\langle\Phi^2\rangle$ and $\langle T_{\mu\nu}\rangle$ comes from modes having wavelengths larger than the Hubble radius which makes both quantities formally infrared (IR) divergent. IR divergences cannot be renormalized away, instead a physically motivated IR cutoff has to be invoked [5]. The IR-regularized $\langle T_{\mu\nu}\rangle$ describing particle creation is finite, and behaves like an effective cosmological constant as the Universe expands; consequently, the energy density of created particles can dominate the matter density leading to $\Omega_m < 1$ in a flat Universe.

We consider a spatially flat Friedmann-Robertson-Walker (FRW) model with expansion factor either de Sitter-like $a \propto e^{Ht}$, or power law $a \propto t^p$. Massive free scalar fields satisfy the wave equation

$$[\Box + \xi R + m^2]\Phi = 0, \tag{1}$$

where R is the Ricci scalar and ξ parametrizes the coupling to gravity, $\xi = 0, 1/6$ corresponding to minimal and conformal coupling, respectively. In a spatially flat FRW Universe the field variables separate and $\Phi_k = (2\pi)^{-3/2}\phi_k(\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}$. The comoving wave number $k = 2\pi a/\lambda$ where λ is the physical wavelength of scalar field quanta. Defining the conformal field $\chi_k = a\phi_k$ and using $R = 6\ddot{a}/a^3$ (differentiation being with respect to the conformal time $\eta = \int dt/a$), Eq. (1) leads to

$$\ddot{\chi}_k + [k^2 + m^2a^2 - (1 - 6\xi)\ddot{a}/a]\chi_k = 0. \tag{2}$$

For de Sitter space (2) has the exact solution

$$\chi_k(\eta) = c_1\sqrt{\eta}\,H_\mu^{(2)}(k\eta) + c_2\sqrt{\eta}\,H_\mu^{(1)}(k\eta), \tag{3}$$

where $\mu^2 = 9/4 - 12\xi - m^2/H^2$, and $c_1 = \sqrt{\pi}/2$, $c_2 = 0$ gives the state associated with the Bunch-Davies vacuum [6].

For ultralight fields $m/H \sim 1$ today, corresponds to $m/H \sim 10^{-60}$ during inflation, hence the field modes can be treated as being effectively massless. Setting $m = 0$ in (2) and specializing to a power law expansion

$$a = (t/t_0)^p \equiv (\eta/\eta_0)^{(1-2\nu)/2}, \tag{4}$$

where $2\nu = (1 - 3p)/(1 - p)$ (the inflationary range $p > 1$ corresponds to $\nu \geq 3/2$ and for $a \propto e^{Ht}$, $\nu = 3/2$), we find exact solutions of (2) having the form (3) with $\mu^2 - 1/4 = (\nu^2 - 1/4)(1 - 6\xi)$. For $\xi = 1/6$, $\mu = 1/2$ while for $\xi = 0$, $\mu = \nu$. The choices $c_1 = \sqrt{\pi}/2$, $c_2 = 0$ correspond to the adiabatic vacuum state.

To study quantum fluctuations we define the operator

$$\Phi(x) = \int d^3k[a_k\Phi_k(\mathbf{x},\eta) + a_k^\dagger\Phi_k^*(\mathbf{x},\eta)], \tag{5}$$

where a_k, $a_k^\dagger$ are annihilation and creation operators $[a_k, a_{k'}^\dagger] = \delta_{kk'}$, defining the vacuum state $a_k|0\rangle = 0\ \forall k$. The two-point function

$$\langle\Phi(x)\Phi(x')\rangle_{\rm vac} = \frac{1}{(2\pi)^3}\int d^3k e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\phi_k(\eta)\phi_k^*(\eta') \tag{6}$$

is IR divergent over a certain range of μ values. To see this, substitute Eq. (3) in Eq. (6), using $\phi_k = \chi_k/a$ and the small-argument limit of the Hankel functions

$$H_\mu^{(2,1)}(k\eta) \underset{k\eta<1}{\simeq} \frac{(k\eta/2)^\mu}{\Gamma(1+\mu)} \pm \frac{i}{\pi}\Gamma(\mu)\left(\frac{k\eta}{2}\right)^{-\mu}. \tag{7}$$

The integral controlling the presence of IR divergences is

$$\int dk k^2 k^{-2\mu}|c_1 - c_2|^2. \tag{8}$$

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For the adiabatic vacuum state (3), IR divergences arise when $\mu^2 \geq 9/4$. For $\xi = 0$, IR divergences are encountered for $p \geq 2/3$ $(p \neq 1)$ [5]. The situation is substantially different for $\xi \neq 0$ (Fig. 1), IR divergences being present over a wide range of expansion rates: $p < 0$; $p > 1/2$ $(p \neq 1)$; $0 < p < 1/2$. Since there is no particle production when $p = 0,\ 1/2,\ \xi = 1/6$, these special cases are free of IR divergences.

Significantly $\langle T_{\mu\nu} \rangle$ can also be IR divergent: The (bare) energy density in a spatially flat Universe is

$$\rho = \langle T_{00} \rangle = \frac{1}{4\pi^2 a^2} \int_0^\infty dk k^2 (|\dot{\phi}_k|^2 + k^2 |\phi_k|^2) + \frac{3}{2\pi^2} \xi H \int_0^\infty dk k^2 \left[H |\phi_k^2| + \frac{\phi_k \dot{\phi}_k^* + \dot{\phi}_k \phi_k^*}{a} \right] + \frac{m^2}{4\pi^2} \int_0^\infty dk k^2 |\phi_k|^2 . \tag{9}$$

Restricting attention momentarily to $\xi = 0$, $m = 0$, we conclude that $\langle T_{00} \rangle$ is IR divergent for $|\nu| \geq 5/2$, i.e., $3/4 \leq p \leq 2$ $(p \neq 1)$. [We correct a minor error in Ref. [5] who quote $2/3 \leq p \leq 2$ $(p \neq 1)$.] This is a much smaller range than for the two-point function which is divergent for $p \geq 2/3$ $(p \neq 1)$. In contrast, the key aspect of nonminimal coupling is that $\langle T_{00} \rangle$ contains terms proportional to $\xi \langle \Phi^2 \rangle$, and consequently (for ultralight fields) $\langle T_{00} \rangle$ is IR divergent *over the same range of parameters* as $\langle \Phi^2 \rangle$.

The curing of IR divergences requires that mode functions be modified in the IR limit. The only freedom to accomplish this in Eq. (8) is to change the behavior of $|c_1 - c_2|$ as $k \to 0$: There are no IR divergences if $\lim_{k \to 0} |c_1 - c_2|^2 \propto O(k^{2|\mu| - 2}) \to 0$ (maintaining $c_2 = 0$, $c_1 = 1$ at large k). One way to determine c_1 and c_2 is to assume the existence of a preinflationary radiation-dominated phase [5,7], as a result [8], $|c_1 - c_2|^2 \simeq [1 + 4\pi^2 C (k \tilde{\eta}_0)^{1 - 2|\mu|}]^{-1}$, where $\tilde{\eta}_0$ marks the onset of inflation. Finiteness in the ultraviolet is achieved by imposing a cutoff at the horizon scale. It then follows that

$$\langle \Phi^2 \rangle = \overline{C} \eta^{2(\nu - |\mu|)} \int_{\tilde{\eta}_0^{-1}}^{\eta^{-1}} dk k^2 k^{-2|\mu|}, \tag{10}$$

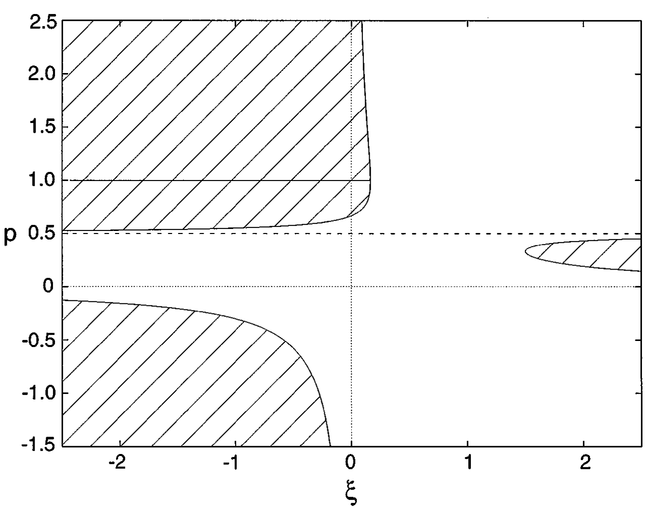

FIG. 1. IR divergent regions (shaded) of $\langle \Phi^2 \rangle$: The special cases $p = 0$, $1/2$, 1, and $1/6 \leq \xi \leq 3/2$ have no IR divergences.

where $\overline{C} = C \tilde{\eta}_0^{1 - 2\nu}$. The behavior of $\langle \Phi^2 \rangle$ depends crucially upon the value of $\nu - |\mu|$. For $\xi = 0$, $\langle \Phi^2 \rangle \simeq \overline{C} \int_{1/\tilde{\eta}_0}^{1/\eta} dk k^{2(1 - \nu)}$, which gives for exponential inflation $(\nu = 3/2)$ the standard result $\langle \Phi^2 \rangle \simeq H^3 \Delta t / (4\pi^2)$ [9]. In the case of power law inflation $\nu > 3/2$, and $\langle \Phi^2 \rangle$ freezes to a large value at late times [8,10]. With negative values of ξ, $\mu > \nu$, $\langle \Phi^2 \rangle$ grows with time approaching the asymptotic form

$$\langle \Phi^2 \rangle \underset{\eta \to 0}{=} \frac{C}{2|\mu| - 3} \left(\frac{\tilde{\eta}_0}{\eta} \right)^{2(|\mu| - \nu)} \tilde{\eta}_0^{-2} \tag{11}$$

at late times. For $\xi \ll -1/6$ and $\nu^2 \gg 1/4$ we have $|\mu| \approx \nu \sqrt{6|\xi|}$, which substituted in (11) gives

$$\langle \Phi^2 \rangle \propto a^c, \quad c = \frac{4\nu}{2\nu - 1} (\sqrt{6|\xi|} - 1) > 1 . \tag{12}$$

Thus $\xi < 0$ can greatly accelerate the growth of fluctuations in inflationary models. Even for minimal coupling, $\xi = 0$, IR finite physical quantities, e.g., $\langle \Phi(x) \Phi(x') \rangle$ and $\langle T_{00} \rangle$, remain sensitive to the presence of long range power in the field modes. This is reflected in the growth of scalar fluctuations, generation of density fluctuations, and the quantum creation of gravitational waves.

We now examine particle production in a Universe that inflates and then transits to a matter dominated regime of expansion. To do this, we return to Eq. (2): this equation closely resembles the one-dimensional Schrödinger equation, the role of the "potential barrier" $V(x)$ being played by $V(\eta) = -m^2 a^2 + (1 - 6\xi) \ddot{a}/a$ [11]. The form of the barrier is shown in Fig. 2 assuming $\xi < 1/6$, $m \simeq 0$. The process of superadiabatic amplification of zero-point fluctuations (particle production) can be qualitatively described as follows: the amplitude of modes having wavelengths smaller than the Hubble radius decreases conformally with the expansion of the Universe, whereas that of larger-than Hubble radius modes freezes (if $\xi = 0$) or grows with time $(\xi < 0)$. Consequently, modes with $\xi \leq 0$ have their amplitude superadiabatically amplified on reentering the Hubble radius after inflation (Fig. 2).

The nonvacuum state of the scalar field at late times $(\eta > |\eta_0|)$ is described by a linear superposition of positive and negative frequency solutions

$$\phi_{\rm out}(k, \eta) = \alpha \phi_k^+ + \beta \phi_k^- , \tag{13}$$

where $\phi_k^{+,-} = (\sqrt{\pi \eta_0}/2)(\eta/\eta_0)^{\overline{\nu}} H_{|\overline{\mu}|}^{(2,1)}(k\eta)$. $[a \propto t^p$, $p < 1$; $2\overline{\nu} = (1 - 3p)/(1 - p)$, $-3/2 \leq \overline{\nu} \leq$

$-1/2 \Leftrightarrow 2/3 \geq p \geq 1/2$ corresponding to matter with an equation of state $0 \leq w \leq 1/3$, $\overline{\mu}^2 - 1/4 = (\overline{\nu}^2 - 1/4)(1 - 6\xi)$.] The transition from inflation to a radiation/matter dominated epoch is marked by η_0, $H_{\rm rh}^2 \equiv 1/\eta_0^2$ being the Hubble parameter at reheating. [We assume $m^2 < |\xi| R$ or $m^2/H^2 < 6|\xi|(\overline{\nu}^2 - 1/4)/(\overline{\nu} - 1/2)^2$ allowing us to treat field modes as being effectively massless.]

The Bogoliubov coefficients α and β are determined by matching $\phi_{\rm in}^+(k,\eta)$, $\dot{\phi}_{\rm in}^+(k,\eta)$ given in (3) and $\phi_{\rm out}(k,\eta)$, $\dot{\phi}_{\rm out}(k,\eta)$ at $\eta = \eta_0$. For modes with $k\eta_0 < 1$ we obtain

$$\begin{aligned} \alpha + \beta &= A\,\frac{i}{\pi}\,\Gamma(\mu)\Gamma(1+\overline{\mu})\left(\frac{k\eta_0}{2}\right)^{-(\mu+\overline{\mu})} + B\,\frac{\Gamma(1+\overline{\mu})}{\Gamma(1+\mu)}\left(\frac{k\eta_0}{2}\right)^{\mu-\overline{\mu}}, \\ \alpha - \beta &= C\,\frac{i\pi}{\Gamma(\overline{\mu})\Gamma(1+\mu)}\left(\frac{k\eta_0}{2}\right)^{\mu+\overline{\mu}} + D\,\frac{\Gamma(\mu)}{\Gamma(\overline{\mu})}\left(\frac{k\eta_0}{2}\right)^{\overline{\mu}-\mu}, \\ |\alpha|^2 - |\beta|^2 &= 1, \end{aligned} \tag{14}$$

where $A = (\overline{\mu} + |\overline{\nu}| + \nu - \mu)/2\overline{\mu}$, $B = (\overline{\mu} + |\overline{\nu}| + \nu + \mu)/2\overline{\mu}$, $C = (\nu + \mu + |\overline{\nu}| - \overline{\mu})/2\overline{\mu}$, $D = (\overline{\mu} - |\overline{\nu}| + \mu - \nu)/2\overline{\mu}$. (Indices ν, μ refer to the inflationary epoch; $\overline{\nu}$, $\overline{\mu}$ to the matter-radiation dominated epoch.)

Since $|\beta|^2 \propto (k\eta_0)^{-2(\mu+\overline{\mu})}$ and $\mu = \mu(\nu,\xi)$, the number density of created particles is sensitive to (1) the inflationary expansion rate parametrized by ν, (2) the equation of state after inflation, parametrized by $\overline{\nu}$, and (3) the coupling to gravity ξ. More particles are created for $\xi < 0$ than for $\xi = 0$ (see Fig. 2 and Ref. [2]).

From (13) and (14) we find that on larger than horizon scales ($k\eta < 1$),

$$\phi_{\rm out} \simeq iA\sqrt{\eta_0/4\pi}\,\Gamma(\mu)\,(k\eta_0/2)^{-\mu}(\eta/\eta_0)^{\overline{\mu}-|\overline{\nu}|}, \tag{15}$$

from which follows the important observation that $\langle\Phi^2\rangle_{\rm out} = 1/2\pi^2 \int dk |\phi_{\rm out}|^2 k^2$ has *exactly the same* infrared properties as $\langle\Phi^2\rangle_{\rm in}$ and that on scales *larger* than the horizon $\langle\Phi^2\rangle_{\rm out}$ grows with time at a rate determined by $\overline{\mu} - |\overline{\nu}| > 0$ ($|\overline{\nu}| = 3/2$ in a matter dominated Universe). This result bears directly on the energy density of created particles $\langle T_{00}\rangle_{\rm out}$, which can be determined from (9) after substituting $\phi \to \phi_{\rm out}$. Equation (9) informs us that the main contribution to $\langle T_{00}\rangle_{\rm out}$ comes from long wavelength modes and, for $\xi < 0$, $\langle T_{00}\rangle_{\rm out}$ is IR divergent if the field is effectively massless. A radiation dominated phase prior to inflation leads to an effective IR cutoff $k_{\rm min} \simeq \tilde{\eta}_0^{-1}$ in $\langle T_{00}\rangle$, since IR divergent states cannot arise from IR finite initial conditions [12]. In addition, a high frequency cutoff appears due to suppression of particle creation at large k [13]. For particles created during inflation, this cutoff is set by the Hubble parameter at the end of reheating and the commencement of radiation domination: $k_{\rm max} \simeq H_{\rm rh} \simeq \eta_0^{-1}$ [13,14].

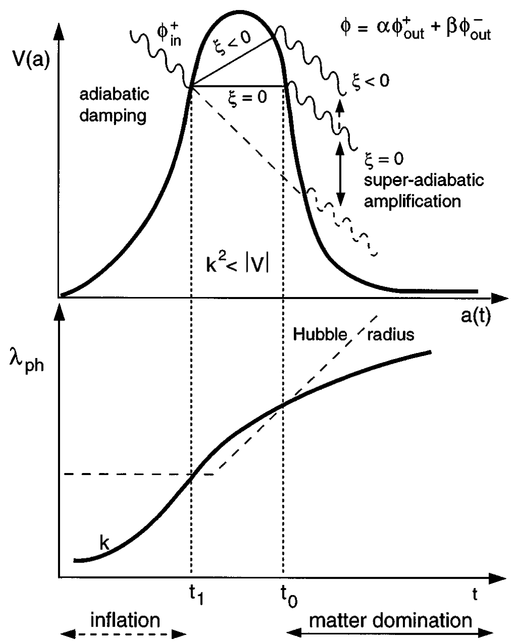

FIG. 2. The timelike "potential barrier" $V(\eta)$ for inflation followed by a matter dominated epoch.

The integration limits in (9) are therefore $\int_{k_{\rm min}}^{k_{\rm max}}$. For $m/H \leq 1$, $\langle T_{00}\rangle$ is dominated by modes larger than the Hubble radius, thus the integration limits are effectively $\int_{\tilde{\eta}_0^{-1}}^{\eta^{-1}}$ leading, for a matter dominated Universe, to

$$\langle\Phi^2\rangle \simeq \frac{N}{\eta_0^2}\left(\frac{\eta}{\eta_{\rm MD}}\right)^{2\overline{\mu}-3}\left(\frac{\tilde{\eta}_0}{\eta_0}\right)^{2\mu-3}\left[1 - \left(\frac{\eta}{\tilde{\eta}_0}\right)^{2\mu-3}\right], \tag{16}$$

where $N = (A^2/8\pi^3)2^{2\mu}\Gamma^2(\mu)/(2\mu - 3)$.

Although a full treatment of the semiclassical Einstein equations $G_{\mu\nu} = -8\pi G(T_{\mu\nu} + \langle T_{\mu\nu}\rangle)$ lies beyond the scope of this work, it is easy to perform a qualitative analysis. For small values $|\xi| < 1$, the term $1/(4\pi a^2)\int dk k^4 |\phi_k|^2$ in Eq. (9) is dominated by horizon-size modes, and is small compared to the remaining terms in $\langle T_{00}\rangle$, which are dominated by the larger infrared cutoff scale $\tilde{\eta}_0$ ($\tilde{\eta}_0 \gg \eta$). These terms, excluding $m^2\langle\Phi^2\rangle$, are of the form $H^2\langle\Phi^2\rangle$ and can be absorbed into the left hand side of the (00) Einstein equation leading to

$$3H^2 \simeq 8\pi\overline{G}[\rho_m + \tfrac{1}{2}m^2\langle\Phi^2\rangle], \tag{17}$$

where $\overline{G} \simeq G/(1 + 8\pi G|\xi|\langle\Phi^2\rangle)$. For ultralight fields $|\xi| < 0.1$ ensures that $\langle\Phi^2\rangle$ grows at a slow rate, satisfying constraints on the time variation of $\overline{G}$ [15]. Consequently,

$$8\pi\overline{G}\langle\Phi^2\rangle \simeq N\left(\frac{H_{\rm rh}}{m_{\rm pl}}\right)^2(1 + z_{\rm MD})^{\mu+\overline{\mu}-3}\left(\zeta\,\frac{T_{\rm rh}}{T_{\rm MD}}\right)^{2\mu-3}, \tag{18}$$

where $\zeta = \tilde{\eta}_0/\eta$. (Note that $\zeta \equiv k_*/k_{\min} = \exp \int_{t_{\rm in}}^{t_*} H dt$, where $t_{\rm in}$ is the beginning of inflation, and t_* the time when a mode entering the horizon today, left the Hubble radius during inflation. In general ζ can be very large, $\log \zeta \gg 1$.) Substituting $H_{\rm rh}/m_{\rm pl} \simeq 10^{-5}$, $T_{\rm rh} \simeq 10^{14}$ GeV, $T_{\rm MD} \simeq (1 + z_{\rm MD}) \times 2.3 \times 10^{-13}$ GeV, $1 + z_{\rm MD} \simeq 23\,219 \Omega_m h^2$, we find that $8\pi \overline{G} \langle \Phi^2 \rangle$ can also be very large (since $\overline{G} < G$ this holds for $8\pi G \langle \Phi^2 \rangle$ as well). For large $8\pi G |\xi| \langle \Phi^2 \rangle \gg 1$, $\overline{G} \simeq 1/(8\pi |\xi| \langle \Phi^2 \rangle)$ and

$$\Lambda_{\rm eff} \equiv 8\pi \overline{G} \langle T_{00} \rangle \simeq m^2/2|\xi|\,,$$

$$\Omega_\phi \equiv \Lambda_{\rm eff}/3H^2 \simeq \frac{1}{6|\xi|}(m/H)^2. \tag{19}$$

We thus find that the energy density of created particles behaves like an *effective cosmological constant*. Furthermore, for ultralight fields $m/H \sim 1$, Ω_ϕ can be significant fraction of the total density of the Universe at late times. (Recent observations of high redshift supernovae suggest $\Omega_\Lambda > 0$, which could be quite large if the Universe is flat [16].)

At first glance it may appear strange that the equation of state of created nonminimal scalars is $p \simeq -\rho$ and not the familiar $p = \rho/3$ expected for relativistic particles. This is because the dominant contribution to $\langle T_{00} \rangle$ comes from wavelengths larger than the horizon size; as a result $\langle T_{\mu\nu} \rangle \simeq \frac{1}{2} g_{\mu\nu} m^2 \langle \Phi^2 \rangle$ at late times. In a similar context it is well known that $\langle T_{00} \rangle$ for gravitational waves (and minimally coupled massless scalars) created during inflation is dominated by wavelengths of order the horizon size, because of which the equation of state of relic gravitational waves is identical to that of classical matter driving the expansion, and in a matter-dominated Universe, is $p = 0$ [16]. (For wavelengths larger than the Hubble radius the distinction between "real" particles and vacuum polarization becomes ambiguous.)

Our analysis indicates that, at a local level, the Universe may be filled with a quasiclassical, stochastic, nearly homogeneous nonminimal scalar field having a Gaussian probability distribution with dispersion $\langle \Phi^2 \rangle$. (This description agrees with the general stochastic approach to inflationary cosmology [9,17].) Because of cosmic variance, the local value of Φ^2 will generally not equal $\langle \Phi^2 \rangle$; remarkably, this does not affect the value of Ω_ϕ since (19) does not depend upon Φ (provided Φ is large).

To summarize, the inflationary production of light nonminimal particles ($\xi < 0$) leads to an energy density which mimics an effective cosmological constant implying $\Omega_m < 1$ in clustered matter in a critical density Universe. Hence, a low density Universe can exist without an "Ω problem" making inflation compatible with observations indicating a low value for Ω_m [18,19]. Since nonminimal scalars naturally arise in the low energy limit of string theory [20], our results may apply to a wider class of models than the ones considered here.

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