Mesoscopic Superconducting–Normal Metal–Superconducting Transistor

Frank K. Wilhelm,[1] Gerd Schön,[1] and Andrei D. Zaikin[1,2]
[1] *Institut für Theoretische Festkörperphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany*
[2] *P. N. Lebedev Physics Institute, Leninskii Prospect 53, 117924 Moscow, Russia*
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In a mesoscopic superconductor–normal metal–superconductor (*SNS*) heterostructure the quasiparticle distribution can be driven far from equilibrium by a voltage applied across the normal metal. This reduces the supercurrent between the superconducting electrodes, which creates the possibility of using these *SNS* junctions as fast switches and transistors. We describe the system in the framework of the quasiclassical theory and find good agreement with recent experiments. We propose further experimental tests, for instance, the voltage dependence of the current-phase relation, which includes a transition to a π junction. [S0031-9007(98)06894-X]

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Nonequilibrium effects in superconducting systems [1] have been gaining new attention due to the increased activities in the field of mesoscopic electron transport [2]. In contrast to earlier work on nonequilibrium superconductivity the new experiments show nonlocal and size dependent (d) effects by reaching temperatures below the characteristic Thouless energy $E_{\rm Th} = \mathcal{D}/d^2$. The understanding of this regime is not only of fundamental interest, but also important for nanoelectronic applications. The size reduction of electronic devices is accompanied by an increase in operation frequency. For instance, in the system considered below the latter is limited by $E_{\rm Th}$.

Recently Pothier *et al.* [3] probed the quasiparticle properties in short diffusive wires by coupling tunneling contacts to it. They found that in mesoscopic wires the distribution function has a nonequilibrium energy dependence, with a double-step structure at the electrochemical potentials of both reservoirs. In different setups, sketched in Fig. 1, Morpurgo *et al.* [4] demonstrated that in a superconductor–normal metal heterostructure the nonequilibrium quasiparticle distribution due to a normal current flow in N can be used to tune the supercurrent between the superconducting electrodes. This opens the perspective to use such devices as ultrafast transistors.

To account for their experimental findings, Morpurgo *et al.* [4] proposed a qualitative model based on a quasiequilibrium distribution function with locally enhanced effective electron temperature. This picture is appropriate in the limit of strong electron-electron interactions. However, inelastic processes have only a weak effect in the mesoscopic sample considered here. In fact, we find the best performance in the opposite limit.

In this article we will describe mesoscopic superconductor–normal metal heterostructures as shown in Fig. 1. For a quantitative analysis we use the quasiclassical theory. It accounts well for the relevant physics: (i) The spectral properties in the normal metal are modified by the proximity effect due to the presence of the superconducting electrodes. (ii) The quasiparticle distribution function is found as the solution of a kinetic equation. If the normal wire between the normal reservoirs is shorter than the inelastic scattering length, the distribution function f has the nonequilibrium (two-step) form observed in the experiments of Pothier *et al.* [3]. (iii) This nonequilibrium distribution reduces the supercurrent between the two superconducting contacts through the normal metal, which allows the tuning of the supercurrent by a perpendicular voltage. For best performance as a transistor, in the setup of Fig. 1a, the width of the superconducting contacts d_S should be chosen narrow compared to the width and length of the normal wire, d and L, respectively. The first condition ensures that only a small fraction of the control normal current is diverted through the superconductors, and accordingly the distribution function in N is little disturbed by the presence of the superconducting electrodes. The second condition ensures that the voltage is nearly constant along the superconducting leads, while the total voltage drop responsible for the nonequilibrium and reduction of the supercurrent may be large. Since the effect relies on a deviation of the distribution function from local equilibrium with shifted electrochemical potential, the normal wire should have mesoscopic dimensions, i.e., the length L should not exceed the inelastic relaxation length $l_{\rm in}$.

The presence of the superconducting electrodes induces correlations in the normal metal (proximity effect), which are responsible for a supercurrent. Their decay length depends on energy: Correlations with energy $\epsilon \gg E_{\rm Th}$

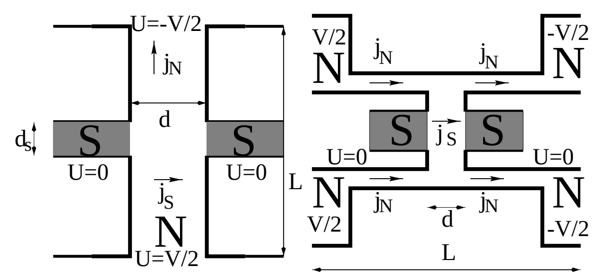

FIG. 1. Realizations of the *SNS* transistors. The supercurrent is tuned by (a) a perpendicular or (b) a parallel normal current.

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decay exponentially, while those within the range of order E_{Th} carry the supercurrent. If, in a nonequilibrium situation, these states are occupied and in this way blocked for superconducting correlations, the superconductivity is weakened and the supercurrent reduced. This suppression mechanism will be described in this paper, based on the real-time formalism of quasiclassical Green-Keldysh functions in the diffusive limit [1,5].

In the first step, we describe the proximity effect in the normal metal by analyzing Usadel's equations. The standard parametrization of normal and anomalous retarded Green functions $G^{\mathrm{R}} = \cosh\alpha$ and $F^{\mathrm{R}} = \sinh\alpha\, e^{i\chi}$ allows us to write these equations, for the normal metal region between the superconducting electrodes, in the form

$$\begin{aligned} \mathcal{D}\,\partial_x^2\alpha &= -2i\epsilon\sinh\alpha - (\mathcal{D}/2)\,(\partial_x\chi)^2\sinh 2\alpha\,, \\ \partial_x j_\epsilon &= 0, \qquad j_\epsilon = (\partial_x\chi)\sinh^2\alpha\,. \end{aligned} \tag{1}$$

Here, $\mathcal{D}$ is the diffusion coefficient and x is the coordinate normal to the NS interfaces. We introduced the energy-dependent "spectral current" j_ϵ. For simplicity we ignored the dependence of the spectral quantities on the coordinate y parallel to the interfaces, which merely leads to a quantitative modification of our results. In the realistic limit $\Delta \gg E_{\mathrm{Th}}$, for transparent interfaces, the boundary conditions at these interfaces, $x = \pm d/2$ read [6]

$$\alpha(\pm d/2) = -i\pi/2, \qquad \chi(\pm d/2) = \pm\phi/2\,. \tag{2}$$

In two limits we find analytic solutions of Eqs. (1) and (2). For low energies $\epsilon \ll E_{\mathrm{Th}}$ we obtain

$$\begin{aligned} \alpha &\simeq -i\pi/2 + (\epsilon/E_{\mathrm{Th}})a(\phi) + O(\epsilon^3)\,, \\ \chi &\simeq \phi x/d - (\epsilon/E_{\mathrm{Th}})^2 b(\phi) + O(\epsilon^3)\,, \end{aligned}$$

where $a(\phi)$ and $b(\phi)$ are real-valued functions (omitted for brevity). For higher energies $\epsilon \gg E_{\mathrm{Th}}$ we obtain [6]

$$F^{\mathrm{R}} = F_0(x - d/2)e^{i\phi/2} + F_0(d/2 - x)e^{-i\phi/2}, \tag{3}$$

where

$$F_0(x) = 4q\,\frac{1+q^2}{(1-q^2)^2}\,, \qquad q = i(\sqrt{2} - 1)e^{-x\sqrt{-2i\epsilon/\mathcal{D}}}$$

and

$$\mathrm{Im}(j_\epsilon) \approx 64 q\,\partial_x q|_{x=L/2}\sin\phi\,. \tag{4}$$

In addition, we have studied the problem numerically. The results for the spectral current $\mathrm{Im}(j_\epsilon)$ are combined in Fig. 2. It is an odd function of ϵ and shows a proximity induced minigap [7], $\epsilon_{\mathrm{g}} \simeq 3.2E_{\mathrm{Th}}$ at $\phi = 0$, below which $\mathrm{Im}(j_\epsilon) = 0$. This gap decreases [8] with increasing ϕ and vanishes at $\phi = \pi$. At energies directly above the gap, $\mathrm{Im}(j_\epsilon)$ increases sharply, but rapidly decreases at higher ϵ. At large energies, it changes sign and oscillates around zero with exponentially decaying amplitude.

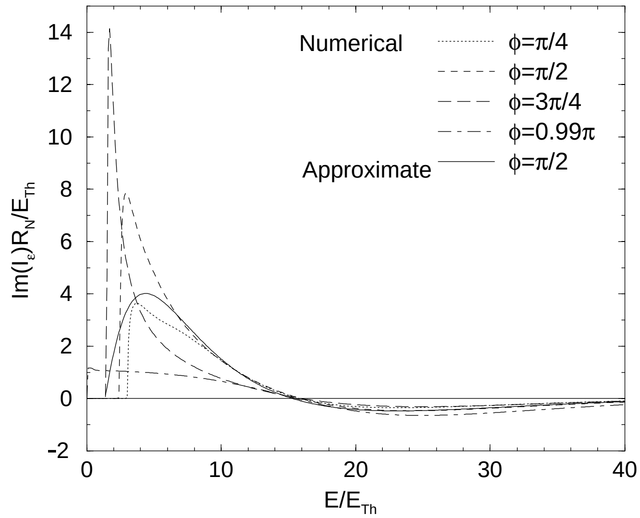

FIG. 2. The spectral current $\mathrm{Im}(j_\epsilon)$ as a function of energy for different values of the phase difference ϕ. At higher energies, we find oscillations according to Eq. (4).

Next we determine the nonequilibrium quasiparticle distribution in the normal metal between the reservoirs, which are at different electrochemical potentials $\pm eV/2$. In a wire of mesoscopic length, $L \ll l_{\mathrm{in}}$, in the diffusive limit the distribution function obeys the kinetic equation

$$\mathcal{D}\,\partial_y^2 f = 0\,. \tag{5}$$

In the absence of superconducting contacts, its solution

$$\begin{aligned} f(\epsilon, y) &= (1/2 - y/L)f^{\mathrm{eq}}(\epsilon + eV/2) \\ &\quad + (1/2 + y/L)f^{\mathrm{eq}}(\epsilon - eV/2) \end{aligned} \tag{6}$$

has two temperature-rounded steps at the electrochemical potentials of both reservoirs. The step heights depend on the position along the wire; in this way the distribution function interpolates linearly between the boundary conditions at $y = \pm L/2$. This functional dependence was detected in the experiments of Pothier *et al.* [3].

Although the distribution function definitely does not have a thermal form, a local electrochemical potential of the normal metal, $\mu(y)$, and an effective electron temperature can be defined by its moments. In the following we will consider the situation where the electrochemical potential of the superconductors coincides with the local value of the normal metal, which guarantees that there is no net current out of the normal metal into the superconductors. Since we further have chosen the size of the superconducting contacts, d_S, small compared to the width and length, d and L, of the normal wire, the distribution function in N is little disturbed by the contacts.

On the other hand, the quasiparticle distribution function influences the induced superconducting correlations. The supercurrent through the structure is given by

$$I_{\mathrm{S}} = \frac{d}{2R_d}\int_{-\infty}^{\infty} d\epsilon[1 - 2f(\epsilon, y_S)]\,\mathrm{Im}(j_\epsilon)\,, \tag{7}$$

where $1/R_d = 2e^2 N_0 \mathcal{D} S/d$, S is the junction area, and y_S denotes the position of the superconducting electrodes. It is important to note that the energy is measured relative to the electrochemical potential of the superconductors, which we have chosen to coincide with the local value of the normal metal, $\mu(y)$. Because of the odd symmetry of $\mathrm{Im}(j_\epsilon)$, only the odd component of a nonequilibrium quasiparticle distribution modifies I_S. Accordingly the first term in the integral (7) can be written as [1,6] $-f(\epsilon, y_S) + f(-\epsilon, y_S)$, which displays that an excess number of electronlike or of holelike excitations have the same effect on the supercurrent.

The largest effect is found when the superconducting electrodes are placed symmetrically between the two normal reservoirs [i.e. at $y = 0$; see Eq. (6)]. The resulting supercurrent across the superconducting–normal metal–superconductor (SNS) junction is presented in Fig. 3 as a function of the voltage V across the normal metal.

At low temperatures $T \ll eV$, $f(\epsilon, 0)$ deviates from the equilibrium value only in the window $-eV/2 < \epsilon < eV/2$. Since the spectral current $\mathrm{Im} j_\epsilon$ vanishes for $\epsilon < \epsilon_g$, there is no modification of of the supercurrent for small voltages $eV < 2\epsilon_g$. On the other hand, for $eV > 2\epsilon_g$, extra quasiparticle and hole states with energies $|\epsilon|$ below $eV/2$ are occupied and the supercurrent is diminished. Since this energy window increases with increasing V, the supercurrent decays rapidly with voltage (cf. Fig. 3). This is exactly what has been observed in the experiments [4]. Furthermore, at still larger voltage $eV \gtrsim 10 E_{\mathrm{Th}}$, the supercurrent changes sign since the integral in (7) is dominated by the energy interval where $\mathrm{Im}\, j_\epsilon < 0$ (Fig. 2). We thus find a transition to a so-called π junction [9], controlled by nonequilibrium effects. This effect is rather pronounced; the critical current of the π junction is approximately 30% of I_c at $T = eV = 0$.

For high voltages or temperatures $eV, T \gg \epsilon_g$, we find $I_S = I_c \sin\phi$ with critical current

$$I_c R_d = \frac{64\pi}{3 + 2\sqrt{2}}\, e^{-\sqrt{\Omega_V/E_{\mathrm{Th}}}} \cos\left(\frac{eV}{2\sqrt{\Omega_V E_{\mathrm{Th}}}} + \varphi_0\right) \times \begin{cases} eV & \text{for } eV \gg \epsilon_g, T \ll \epsilon_g \\ T\left(\frac{(2\pi T)^2 + e^2V^2}{E_{\mathrm{Th}}^2}\right)^{1/4} & \text{for } T \gg \epsilon_{\mathrm{g}} \end{cases}. \tag{8}$$

Here $\Omega_V = \pi T + \sqrt{(\pi T)^2 + (eV/2)^2}$ and

$$\varphi_0 = \begin{cases} \pi/2 & \text{for } eV \gg \epsilon_g, T \ll \epsilon_g \\ \frac{1}{2}\tan^{-1}\left(\frac{eV}{2\pi T}\right) & \text{for } T \gg \epsilon_{\mathrm{g}} \end{cases}.$$

Nonequilibrium effects also influence the current-phase relation $I_S(\phi)$. The rich variety of different curves at different voltages is displayed in Fig. 4. In contrast, a quasiequilibrium theory would always predict a functional dependence as shown in the inset of the figure.

Since the supercurrent decays exponentially as a function of both T and V, one might try to describe the system properties by a quasi equilibrium theory with effective V-dependent temperature T^*. Exactly this strategy was adopted in Ref. [4] with $T^* = \sqrt{T^2 + \gamma^2 (eV)^2}$. The the best fit to the experimental data was obtained for $\gamma \simeq 6\ \mathrm{K}(mV)^{-1}$, whereas the theoretical prediction from a simple model was $\gamma = 3.2\ \mathrm{K}(mV)^{-1}$. In our approach, the distribution function in the normal metal, in general, *cannot* be described by the Fermi function with effective temperature T^*, and the analogy between temperature and voltage is misleading. One of the most striking consequences, the transition to a π junction, is not obtained in the quasiequilibrium description.

Nevertheless, our theoretical results agree fairly well with the experiment in the relevant range of temperatures and voltages. The transition to a π junction is predicted for higher voltages than studied in Ref. [4], and was recently observed [10].

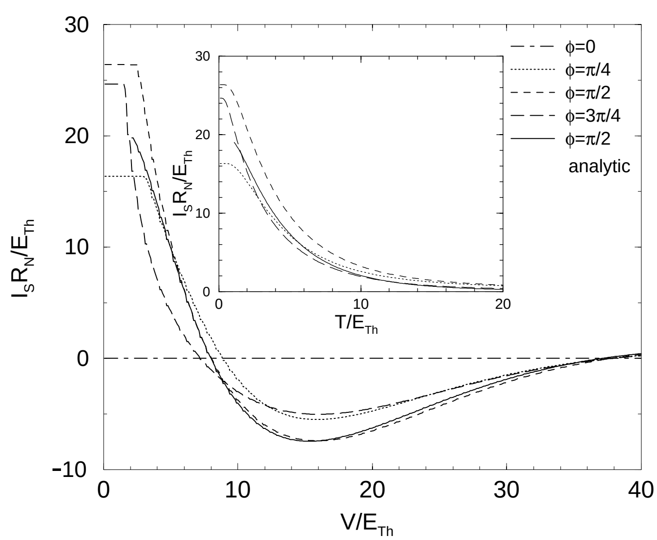

FIG. 3. The supercurrent as a function of control voltage (at $T = 0$) and temperature (at $V = 0$) for various values of ϕ.

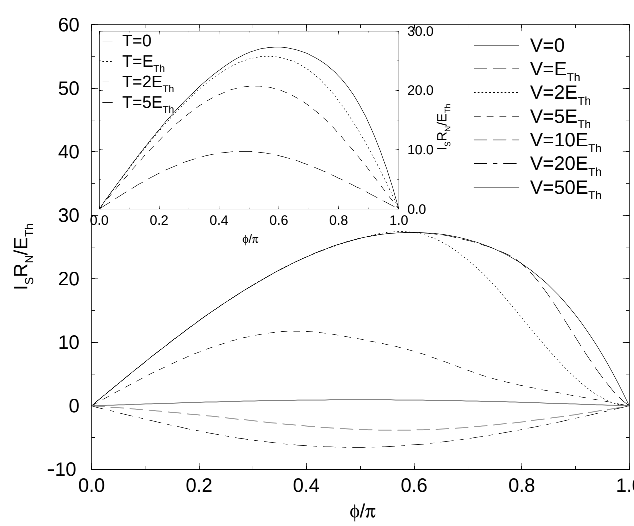

FIG. 4. The supercurrent-phase relation at different temperatures (inset) and control voltages.

The configuration of Fig. 1a is but one realization of a mesoscopic system where the supercurrent can be controlled by an externally applied voltage. Another realization, also studied in Ref. [4], is depicted in Fig. 1b. In this case the distribution function in the N layer between two superconductors is driven out of equilibrium by the normal current flowing parallel to the supercurrent. Provided $d \ll L$ there is practically no voltage drop across the *SNS* junction and only dc Josephson effect can be considered. Since the distribution function has the same form (6) as before, the previous results for $I_S(V)$ apply also to the structure of Fig. 1b. Again, good agreement with the experimental findings [4] is observed.

A similar system, namely, a diffusive *SINIS* system (S: superconducting; I: insulating; N: normal metal) with a thin ($d \ll \xi_0$) N-layer separated by low transparency barriers from the superconductors was already studied by Volkov [11]. In this case, the superconductors and the normal metal are in equilibrium, except that their electrostatic potentials are shifted relative to each other, i.e., the total voltage drop occurs across the barriers. Also, in this case, a voltage-dependent supercurrent reduction as well as a transition to a π junction was predicted. However, there are important qualitative differences in (i) the form of the nonequilibrium distribution function in the N metal (which has a *local* equilibrium form in Ref. [11], while it depends in a *nonlocal* way on the applied voltages in our case), (ii) the question whether the chemical potentials of S and N metals are shifted, and (iii) the resulting voltage dependence of the supercurrent. Various realizations of related effects in ballistic structures have also been studied theoretically [2,12]. In some cases discrete Andreev levels play a prominent role and the nonequilibrium effects lead to their depopulation. The observation of these effects in experiments is still lacking.

If L is not large compared to d_S and d, the space dependence is more complicated, and a conversion between supercurrents and normal currents occurs in the junction area. In this case, the theoretical analysis has to be performed along the lines of Refs. [5] and [1]. The expression for the current (7) has to be modified, and the odd and even components (in energy) of the distribution function, f_L and f_T, obey two coupled diffusion equations with different, position-dependent diffusion coefficients. While qualitatively the physical situation remains unchanged, this type of analysis also describes the limitations for the operation of the device as a transistor.

We now turn to an important practical question: How efficient is this device as a transistor? The control and signal voltages, V and $V_{\mathrm{S}} = I_{\mathrm{c}} R_d$, are both of the order of the Thouless energy E_{Th}. Thus, no voltage gain is obtained. However, the power amplification is proportional to the ratio of the relevant resistors R_L/R_d. Here R_L is the resistance of the normal metal of length L, while R_d, introduced in (7), is the resistance between the superconducting electrodes in a situation where I_{c} is low and this transport is dissipative as well. Since both are governed by the same material-dependent conductivity the ratio depends on the relevant lengths, $R_L/R_d \propto L d_S/d^2$. Hence, by choosing a sufficiently long control line, $L \gg d, d_S$, a power amplification can be achieved.

In summary, we have presented a microscopic description of nonequilibrium electronic properties of mesoscopic *SNS* heterostructures. The distribution function in the normal metal can be driven far from equilibrium by a voltage applied at a distance $\sim L$ from the junction. This distance is limited only by the inelastic relaxation length l_{in}. We analyzed how the supercurrent across the sample is reduced by this control voltage. The strongest reduction and, hence, best performance of the device are found in a mesoscopic situation, when the distribution function deviates significantly from a local equilibrium form. We established the connection to experiments and suggested further tests. The possibility of controlling the supercurrent by an external voltage allows several technical applications, for instance, the use as a high-frequency transistor with power gain proportional to the ratio between length and width of the normal wire.

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