Manipulation, Stabilization, and Control of Pattern Formation Using Fourier Space Filtering

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We present an experimental realization of an almost noninvasive stabilization and manipulation method of coexisting and underlying states of pattern forming systems. In a photorefractive single feedback system, a ring control path is used to realize amplitude and phase-sensitive Fourier-plane filtering, utilizing only a few percent of the system's intensity. We were able to stabilize desired but not predominantly excited patterns in parameter space regions where several patterns are present as underlying solutions. By positive (in-phase) and negative (out-of-phase) control, rolls could be excited in parameter regions where hexagonal structures are preferably stable. [S0031-9007(98)06875-6]

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Transverse modulational instabilities of counterpropagating beams [1] are well known to lead to the spontaneous formation of a variety of complex spatial structures, among them conical rings, pairs of spots (rolls), or arrays of spots arranged in hexagonal or square symmetry. Many systems have revealed to show similar phenomena as, e.g., atomic vapors [2], liquid crystals [3], organic films [4], or recently also photorefractives [5]. One characteristic of all these systems is that they possess a large number of underlying pattern states even in the presence of a single stable output.

From an application point of view of pattern formation in these systems, it is of interest to access the whole range of solutions and to control the structures inherent in the system. Controlling such a system by suppressing predominantly excited solutions or by encouraging underlying solutions to become stable therefore offers the opportunity to stabilize, select, and manipulate these patterns in a defined way for a wide range of technological applications, e.g., in optical information processing.

A common approach to the control of spatially extended systems is providing a control signal that locks the system to one member of a possibly infinite family of solutions that are inherent to the system. The technological aim is to produce a desirable behavior by applying carefully the control signal that directs the system to the goal state and keeps it there, and at the same time not changing or influencing the system dramatically. The intervention into the system should be as small as possible. Moreover, it is desirable to design the control signal in such a way that its magnitude decreases as the system approaches the desired state and—in the absence of noise—vanishes when the system is locked to a certain solution. It is also highly desirable to control a state that is only approximately a true stable state of the system, e.g., if a real experimental system under stress produces states that are distorted compared to the solutions of the infinite, idealized system. In that case, one might expect the feedback to become small, but not to vanish completely.

In experimental optical systems, propagational effects associated with pattern formation are easily observed in the far field, which is nothing else than the representation of the power spectrum of the pattern—its Fourier transform. The big advantage of optics is that the Fourier transform is easily obtained by use of a simple lens. Manipulation in the Fourier domain, i.e., spatial filtering, is one of the most important concepts in modern optics. The combination of feedback control with manipulation of the Fourier space therefore allows one to control the participation of certain spectral components on the nonlinear spatiotemporal pattern formation. These features have led Martin *et al.* [6] to suggest the possibility of using control techniques which operate in the spatial Fourier domain of the control arm to stabilize unstable patterns and to choose between alternative stable states. This technique has been applied theoretically to a variety of nonlinear optical systems, e.g., a feedback system including a liquid crystal light valve as the nonlinear element. The potential of that method has also been shown experimentally for the case of a strongly invasive method in photorefractive feedback mirror experiments [7,8]. Here, we present an experimental realization of an almost noninvasive, phase-sensitive Fourier filtering method in a spontaneous pattern formation system, implemented with a supplementary ring feedback system acting as a control signal in a photorefractive single feedback mirror experiment. The control signal contains only a small percentage of the total energy of the system securing minimum invasion. To the best of our knowledge, this is the first experimental realization of an almost noninvasive stabilization and control system for patterns by Fourier space techniques.

The photorefractive single feedback mirror system allows one to realize and observe a rich variety of

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spontaneously formed patterns. A recent realization of such a system led to the first observation of square patterns and squeezed hexagons in an optical pattern formation system [9]. Pattern formation in this system occurs through modulational instabilities that arise due to the formation of reflection gratings [10]. Above a certain threshold for the photorefractive coupling strength γl, satellite beams are generated with a particular angle θ relative to the central beam. Because the nonlinearity of these materials is proportional to the intensity ratio of the interacting beams, this configuration allows for pattern formation with moderate laser power. Moreover, photorefractives are well suited for experimental pattern control since their intrinsically slow dynamics simplifies time-resolved measurements.

In our contribution, we demonstrate that our system effectively stabilizes certain periodic patterns in regions where a single pattern is predominantly excited. Moreover, we are able to manipulate stable patterns, e.g., change to a desired orientation and to select different patterns out of coexisting stable solutions. We present evidence that enhancement of a desired pattern is possible by positive (in-phase) control and show that the suppression of a predominant pattern in order to allow other patterns to stabilize is possible by negative (out-of-phase) control. In the latter case, the feedback signal tends to decrease down to zero thus allowing one to realize a sensitive noninvasive control.

Our experimental configuration is shown in Fig. 1. A laser beam with a power of 23 mW, obtained from a frequency-doubled Nd:YAG laser, is focused onto the exit face of an iron-doped $KNbO_3$ crystal measuring 5.6 mm along its c axis. In order to avoid undesired backreflections from the surfaces, the crystal was inclined about 6° relative to the direction of propagation. In this geometry, it is well known that energy coupling takes place. The crystal's c axis lies in the direction of the input beam leading to depletion of the incoming beam. The direction of polarization is parallel to the crystal's a axis, thus allowing one to exploit the largest electro-optic coefficient r_{13} of $KNbO_3$. The counterpropagating, backward pump beam is generated by a dielectric mirror, variable in reflection by lateral movement. This mirror is positioned at the end of a confocal ($2f$-$2f$) feedback system with a $f = 120$ mm focal lens in its middle. The relative distance of the mirror to the $2f$-$2f$ position (dotted line) represents a propagation length and is denoted as L in Fig. 1. This specific configuration allows one to adjust positive as well as negative propagation lengths. Moreover, it allows more exact positioning compared to conventional feedback configurations. Thus, reflectivities and propagation lengths that guarantee spontaneous roll, hexagon, or square pattern formation can be adjusted as well as regions of instability that allow for observation of competition between different pattern types [8,9].

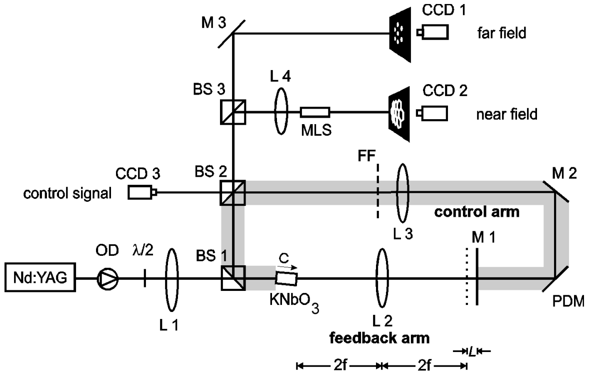

FIG. 1. Experimental setup. OD = optical diode; L = lens; BS = beam splitter; M = mirror; PDM = piezodriven mirror; MLS = microscope lens system; FF = Fourier filter; dotted line: exact $2f$-$2f$ position; L = propagation length.

In order to realize an almost noninvasive feedback control unit, the small transmission signal of mirror M_1 at the end of the feedback path was utilized as the input to the feedback control ring. The feedback control ring redirects the Fourier filtered beam into the input face of the $KNbO_3$ crystal. The intensity that is fed back into the crystal was in all our experiments about 1%–2% of the total intensity incident on the crystal (corresponding to 10%–14% of the field amplitude). Therefore, the control signal is a Fourier filtered and low intensity part of the original pattern reinjected into the feedback system. In order to have an imaging control feedback arm, a lens with a focal length of $f_3 = 300$ mm is introduced into the control arm constituting a new confocal imaging system. Mirror PDM is mounted on a piezotranslation stage in order to vary the phase of the control arm relative to the feedback signal arm. Near the lens L_3, the far field of the pattern is observable, thus representing the Fourier plane. Our filters consisted mainly of slits, being adjustable in any angle in the filter plane. To avoid undesired resonance effects in the control arm, the central spot was always blocked in our experiments. Note that the ring geometry of the control loop is bidirectional and that the control signal therefore in practice approaches the feedback system from both sides.

We applied the Fourier-filtering technique in different parameter regions of the feedback reflectivity R, which represents one of the control parameters of the system. We distinguish between a region of pure and stable hexagonal patterns (R large), an intermediate region where roll and hexagonal patterns are coexisting, and a region where the only stable solutions are rolls (R small).

Figure 2a shows the stationary pattern that is obtained for the intermediate reflectivity region. The stationary pattern shows coexisting rolls and hexagons. Using a slit filter in the Fourier plane, positive feedback stabilized the "nearest" stable pattern, which is a roll pattern in the direction of the filter as shown in Figs. 2b–2e. By changing the orientation of the Fourier filter (see insets in Fig. 2), rolls can be

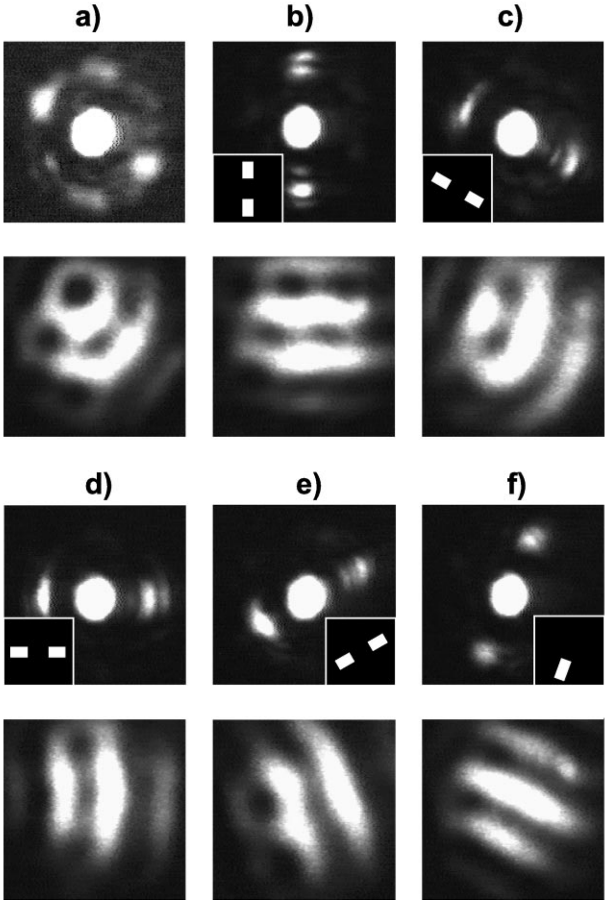

FIG. 2. Manipulation of transverse patterns due to Fourier-plane filtering. Upper rows: far field; lower rows: near field. Insets show the orientation of the Fourier filter. (a) Stationary pattern in the transition region without control. (b)–(e) Roll patterns excited in the direction of the control filter. (f) Roll pattern excited by filtering a single sideband spot.

excited in different desired directions. In the lower rows of Fig. 2, the corresponding near field is shown. By emphasizing its contribution through positive control, the roll pattern is excited and stabilized, while some of the Fourier components that are necessary to produce stable symmetric hexagonal patterns are suppressed. In the stationary state, the control signal beam becomes maximum, allowing the whole system energy to be concentrated in the roll pattern. If only a single spot is allowed to pass the Fourier filter and is reinjected as a positive control, the corresponding roll solution is again excited (Fig. 2f). These results indicate that the roll pattern is a coexisting and stable solution of the system that can be excited by a positive feedback by a small control signal. When the loop was first closed and then opened (situation without and with control), the new (controlled) state was reached within 1 s corresponding to the buildup time of the hexagonal pattern. Figure 3 shows the behavior of positive and negative control in a region where the reflectivity is chosen to be high enough to guarantee stable hexagonal pattern formation.

However, in this region, two different hexagon orientations can be excited. These two hexagons are orthogonal in space and fill the whole scattering ring in the Fourier plane together. Our control is able to stabilize these differently oriented patterns, using again a slit filter in the control arm. By positive control, the direction of the hexagon adjusts to the position of the filter. When the control signal is blocked, the hexagonal pattern stays in the position last chosen, independent of its original position. In order to realize negative feedback, we change the phase of the control arm by adjusting its length via the piezodriven mirror. While a positive control signal means addressing a wanted but stable solution of the system, such a negative control signal suppresses the stable solution by destructive interference and allows the system to choose another solution. For the case that all stable solutions are prevented by the negative control, the system goes into otherwise unstable solutions. Here, no attempt on exploring the unstable solutions is made. For positive control, the hexagonal pattern reaches a particular state with a certain orientation. When the mirror is switched to a position that gives a π phase shift of the control arm, the orthogonal hexagonal pattern appears. Thus, by changing the phase of the control signal, we were able to switch between two equally stable stationary hexagons which have orthogonal orientations in the Fourier plane.

For a reflectivity region where the system is in the stable roll region, we were able to realize a similar switching behavior of different roll patterns using positive as well as negative control. The positive control induces the roll pattern that is appropriate to the slit filter direction. When this pattern is suppressed by negative control, a stable roll

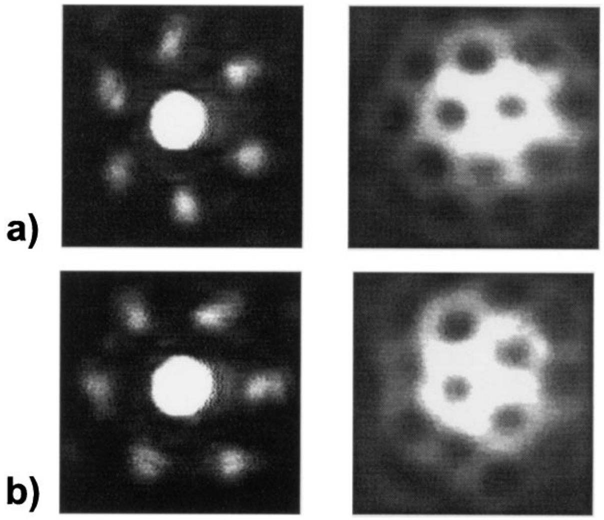

FIG. 3. Positive and negative control of the hexagon direction. Upper row: far field; lower row: near field. The control signal consists of two horizontally arranged spots (compare to Fig. 2d). (a) Hexagon excited by positive control. (b) Hexagon excited by negative control.

pattern appears in the orthogonal direction. Thus it is again possible to switch between two discrete solutions.

For a reflectivity region where the system is in the stable hexagon region, no other pattern could be induced. Obviously, the control signal was too weak ($<0.5\%$ of the original intensity) to induce any pattern changes. These features are currently under investigation. In conclusion, the technique of Fourier filtering and reinjecting a small amount of a pattern forming system's intensity is a powerful and flexible method of changing between the variety of patterns that can be excited in such a system. Wave-front alignment across the whole aperture was not necessary in all our experiments. Therefore, our method can be transferred easily to other pattern forming systems. In parameter regions where different patterns coexist and otherwise unstable solutions are underlying, this method should allow one to excite and explore these patterns.

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