Bethe-Brueckner-Goldstone Expansion in Nuclear Matter

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(Received 30 December 1997)

The equation of state of symmetric nuclear matter at zero temperature is calculated up to the three hole-line level of approximation in the Bethe-Brueckner-Goldstone expansion. Both the standard and the continuous choices for the single particle auxiliary potential are considered. The resulting equation of state shows independence from the choice of the auxiliary potential to a high degree of accuracy. This result gives strong evidence for the convergence of the expansion and establishes the nuclear matter saturation curve for the adopted nucleon-nucleon interaction, the Argonne v_{14} potential. [S0031-9007(98)06905-1]

PACS numbers: 21.65.+f, 21.30.Fe, 21.60.–n

In the long history of the many-body theory of nuclear matter equation of state (EOS) at zero temperature, the Bethe-Brueckner-Goldstone (BBG) expansion [1] has played a major role. The BBG expansion for the EOS can be ordered according to the number of independent hole lines appearing in the diagrams representing the different terms of the expansion. This grouping of diagrams generates the so-called hole-line expansion [1,2]. The smallness parameter of the expansion is assumed to be the "wound parameter" [2], roughly determined by the ratio between the core volume and the volume per particle in the system. The parameter turns out to be small enough up to 2–3 times nuclear matter saturation density. The diagrams with a given number n of hole lines describe the n-particle correlations in the system. At the two hole-line level of approximation the corresponding summation of diagrams produces the Brueckner-Hartree-Fock (BHF) approximation, which incorporates in an exact way the two particle correlations. The BHF approximation includes the self-consistent procedure of determining the single particle auxiliary potential, which is an essential ingredient of the method. The self-consistent procedure, first devised by Brueckner [3], was the real breakthrough towards microscopic calculations of nuclear matter EOS, and the BHF results indicate that already the two hole-line approximation is able to produce reasonable values for the saturation point. The remaining discrepancies can be summarized in the celebrated Coester band [4,5], the line along which the results for different "realistic" nucleon-nucleon (*NN*) forces appear to be approximately concentrated and which misses the phenomenological saturation point. According to the force used, either the saturation density is too high or the binding energy is too small. As it is well known, the BHF contains an arbitrariness; i.e., the definition of the auxiliary potential is not unique. The final result of a hypothetically exact BBG calculation is independent of the auxiliary potential, but the rate of convergence can, of course, depend on the particular choice adopted. The "standard" choice assumes that the auxiliary potential U is zero above the Fermi momentum k_F. The Liège group has advocated another choice [6], the "continuous choice," for the potential U, in which the definition of the potential is extended to momenta k larger than k_F, thus making U a continuous function through the Fermi surface. There are many physical arguments that favor this choice [6]. In several works [7–9] a comparison was made between the two possible choices adopted to calculate the EOS. The BHF calculations with the continuous choice are numerically more delicate. Recently the analysis of the BHF calculations with the continuous choice was presented in Ref. [7] for the case of the *NN* Paris potential. In general, the results indicate that the saturation point in the continuous choice is slightly closer to the phenomenological one, but still the disagreement remains. Other choices of the auxiliary potential have been proposed, in particular, by Brandow [10]. We will restrict the analysis to the standard and continuous choices, since they appear to be two extreme opposite cases.

The three hole-line diagrams can be summed up by solving the Bethe-Fadeev equations for the three-body scattering matrix $T^{(3)}$ inside nuclear matter. As shown by Rajaraman and Bethe [11], the summation is essential, since individual three hole-line diagrams can be quite large, but substantial cancellation occurs once the whole set of three-hole diagrams is considered. Unfortunately, at this level of approximation the theoretical analysis is quite scarce. Indeed, only recently [12] the early extensive study of the three hole-line contribution, presented by Day in Ref. [13], was reexamined and checked. Satisfactory agreement was found between the two sets of calculations, where the standard choice for the single particle potential was adopted. The saturation point turns out to be still away from the phenomenological one and actually very

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close to the one obtained in the BHF approximation within the continuous choice [12]. The conclusion that three-body forces are needed in nuclear matter [14] was therefore also confirmed.

However, the sensitivity of the results to the choice of the auxiliary single particle potential has never been tested. This point must be checked and clarified before any firm conclusion is drawn, and only a substantial independence of the results from the auxiliary potential U can provide a strong indication of the convergence of the BBG expansion at this level of approximation. Since the microscopic determination of nuclear matter EOS is a fundamental problem of nuclear physics, it appears desirable to study the EOS in the BBG expansion at the three-hole level of approximation within the continuous choice, to be compared both with the BHF (two-hole) approximation and with the previous results within the standard choice.

In this Letter we present the nuclear matter EOS calculated up to the three-hole lines within the continuous choice and make a systematic comparison with the results of Refs. [12] and [14] and with previous BHF calculations. This will allow us to establish the convergence of the BBG expansion up to about 3 times the saturation density.

Two hole-line contribution.—In a preceding paper [8] we have developed a method for solving the BHF equations both in the continuous and in the standard choices. More recently [12] the method was applied to symmetric nuclear matter in the case of the v_{14} Argonne potential [15], with particular attention to the convergence of the results with respect to the number of two-body channels and to the momentum cutoff k_c in the single particle spectrum. All the channels up to total angular momentum $j = 4$ have been included, and a value of $k_c = 6\ \text{fm}^{-1}$ was found to be appropriate. For later comparison, the nuclear matter EOS is reported in Fig. 1 (BHF-C), together with the corresponding EOS calculated within the standard choice (BHF-G). The discrepancy between the two saturation curves indicates to what extent the EOS still depends on the choice of the auxiliary potential at BHF level. Full convergence is, therefore, not yet reached. It has to be stressed that the discrepancy (5–6 MeV from Fig. 1) has to be compared with the value of the potential energy per particle, as calculated by the BHF procedure, which turns out to be about -40 MeV around saturation. The discrepancy, and the approximate degree of convergence, is therefore of the order of 10%–15%.

We do not report here other results of the calculations, like the single particle potential as a function of density or other quantities, since they are not pertinent to the main points of the paper. They will be presented elsewhere.

Three hole-line contribution.—Let us now turn to the three hole-line contributions. They can be obtained by solving the Bethe-Fadeev equations. We followed closely the method described in detail by Day in Ref. [13],

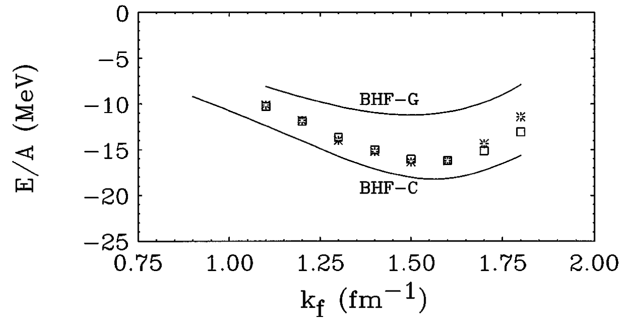

FIG. 1. Equation of state (EOS) of symmetric nuclear matter in different approximations. Full lines indicate the Brueckner-Hartree-Fock approximation within the gap (BHF-G) and the continuous choice (BHF-C). The squares and the stars correspond to the EOS which include up to the three-hole line contribution within the standard and the continuous choices, respectively.

where the equations were solved within the standard choice for the single particle auxiliary potential. The whole set of three hole-line diagrams can be divided into three main contributions. The “bubble” diagram and the “ring” diagram are the diagrams with three particle lines only. The remaining set of diagrams, the so-called “higher order” contributions, with an arbitrary larger number of particle lines, can indeed be summed up by the Bethe-Fadeev equations for the three-body scattering matrix $T^{(3)}$ inside nuclear matter [13]. These diagrams are depicted in Fig. 2, where, for completeness, the two hole-line diagrams (BHF) are also reported. In all the diagrams a wavy line represents a Brueckner G-matrix. Figure 2(f) represents schematically the higher order diagrams, where, however, $T^{(3)}$ does not include the first order (one G-matrix) contributions, since they give the just discussed bubble and ring diagrams. Each one of these three hole-line contributions is quite large, but strong cancellation occurs since the three terms turn out to be of different signs. Therefore the overall contribution of the three hole-line diagrams to the nuclear matter potential energy turns out to be substantially smaller than the two hole-line contribution. This point is illustrated in Fig. 3(a), where the values of the three individual contributions as a function of k_F are reported, together with their total sum. One can notice the above-mentioned strong compensation between the three terms, in particular, between the bubble and the ring diagrams. This is hardly a surprise, since the ring diagram is the exchange of the bubble diagram. These results, as discussed in Ref. [12], are in close agreement with the ones of Ref. [13], and are reported here for comparison. The corresponding EOS up to this order of approximation is also reported in Fig. 1 (squares). Notice that in Ref. [12] an estimate of the four hole-line contribution was also included in the reported EOS, which here we neglect. This contribution is expected to be small [2,13].

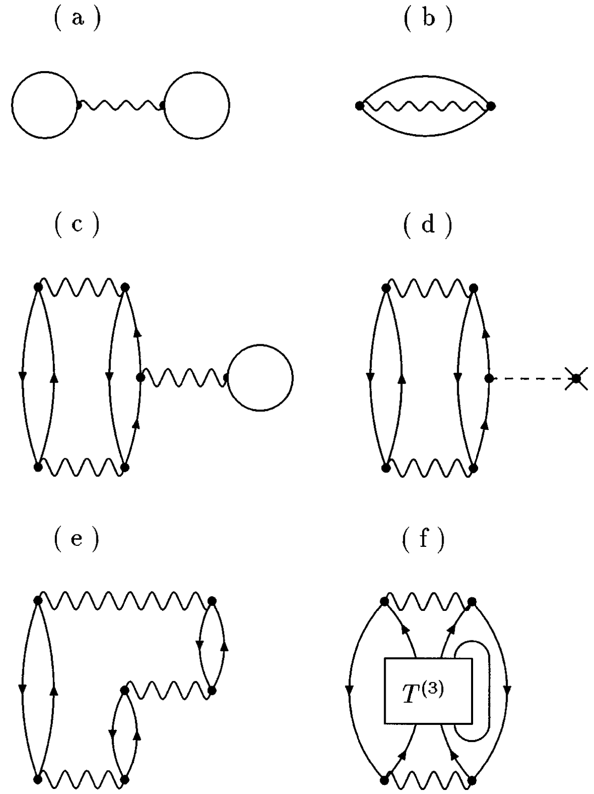

FIG. 2. Different diagrams which contribute to the equation of state as discussed in the text. Diagrams (a) and (b) give the Brueckner-Hartree-Fock (two hole-line) approximation. Diagrams (c) and (e) are the bubble and the ring diagrams, respectively, while diagram (f) represents schematically the contribution of the higher order diagrams, obtained by solving the Bethe-Fadeev equation for the scattering three-body matrix $T^{(3)}$ (exchange terms not indicated for simplicity). Finally diagram (d) is the potential insertion diagram to be included in the case of the continuous choice. The sum of the diagrams from (c) to (f) gives the three hole-line contribution discussed in the text.

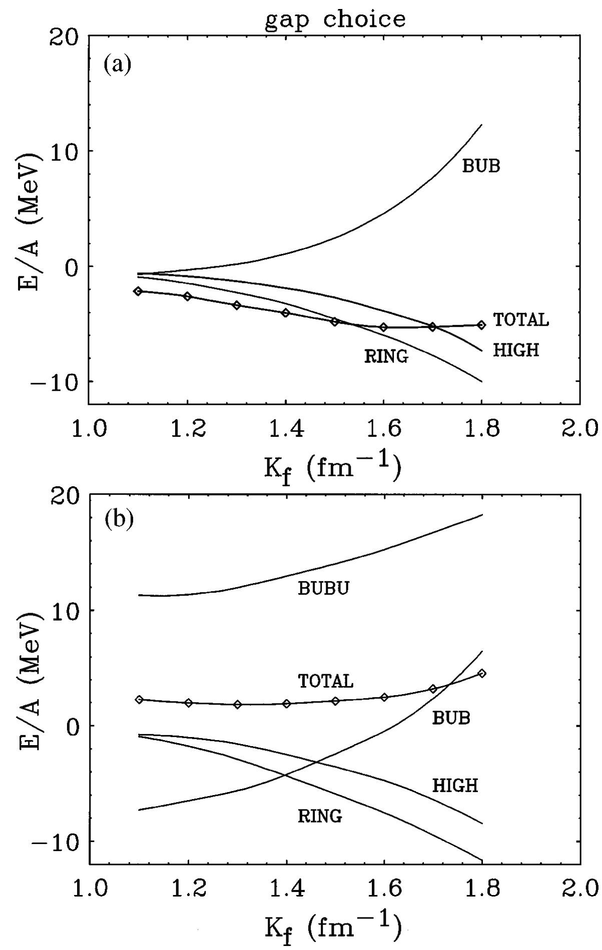

FIG. 3. (a) The contributions of the bubble (BUB), ring (RING), and higher order (HIGH) diagrams to the binding energy of symmetric nuclear matter as a function of Fermi momentum, calculated within the gap choice. The line denoted by TOTAL is the sum of all these contributions and gives the overall three hole-line contribution to the EOS. (b) The same as in (a), but within the continuous choice. Here the line denoted by BUBU is the contribution of the potential insertion diagram of Fig. 2(d).

In the case of the continuous choice, the additional "potential insertion" diagram of Fig. 2(d) has to be included, since in this case $U(k)$ is different from zero also for single particle momenta $k > k_F$. The two diagrams, corresponding to Figs. 2(c) and 2(d), in which the bubble and the potential insertions are attached to a hole line, exactly cancel out each other, in virtue of the Bethe-Brandow-Petschek theorem [16], for both choices of the auxiliary potential $U(k)$. Of course, the change of single particle spectrum introduced by the continuous choice alters both the two-body Brueckner G-matrix and the energy denominators appearing in the diagrams of Fig. 2. Correspondingly, the bubble, ring, and higher order diagrams change their values accordingly, as reported in Fig. 3(b), where also the contribution of the potential insertion diagram is shown. The latter is quite large and essential in making the overall three hole-line contribution still relatively small. This result was already anticipated in Ref. [17], where a semiquantitative estimate of such a diagram was given.

In all the calculations within the continuous choice of the three hole-line diagrams the same procedure of angle and total momentum averaging in the energy denominators of Ref. [12,13] was used. More details will be given elsewhere.

Finally, the EOS up to the three hole-line order of approximation within the continuous choice is reported also in Fig. 1 (stars).

Within the BBG expansion of the nuclear matter binding energy, we have reported the EOS at two and three hole-line levels of approximation. The calculations have been done within the standard as well as the continuous choice for the single particle auxiliary potential. At the two hole-line level, one can see from Fig. 1 that the standard and continuous choices produce nuclear matter binding energies per particle which differ by 5–6 MeV in the

TABLE I. Energy per particle E/A in the standard (s) and in the continuous (c) choices as a function of the Fermi momentum k_F of symmetric nuclear matter.

k_F (fm^{-1})	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
E/A (MeV)s	−10.28	−11.88	−13.63	−15.00	−16.03	−16.19	−15.12	−13.04
E/A (MeV)c	−10.14	−11.97	−13.96	−15.30	−16.34	−15.96	−14.34	−11.40

Fermi momentum range $1.2 < k_F < 1.8$ fm^{-1}. This corresponds to a discrepancy of about 10% in the potential energy per particle. This result, consistent with the previous calculations of Ref. [8], shows that the BBG expansion has not yet reached convergence at this level of approximation, at least not with an accuracy better than the above-mentioned discrepancy.

A decisive improvement seems to occur when the three hole-line contribution is included in the EOS. The two saturation curves, the one calculated within the standard choice and the one calculated within the continuous choice, appear now to be very close. The discrepancy does not exceed 1 MeV, except for $k_F = 1.8$ fm^{-1}, and it is actually substantially smaller than 1 MeV at lower densities, as can be seen from Table I, where the standard and continuous choices are compared. The agreement can be considered within the numerical accuracy of the calculations. The conclusion one can draw from these results is that there is a strong evidence of convergence in the BBG expansion for the considered density range and therefore Fig. 1 establishes the nuclear matter EOS, for the Argonne v_{14} potential, with a high degree of accuracy. Furthermore, the three hole-line contribution appears substantially smaller in the continuous choice than in the standard choice for the single particle auxiliary potential. A cubic spline fit to the EOS in the standard choice (squares in Fig. 1) gives the saturation point at $k_F = 1.565$ fm^{-1} and $E/A = -16.18$ MeV, with the corresponding compressibility $K = k_F^2 d^2(E/A)/dk_F^2 \approx 234$ MeV. As already mentioned, this implies the necessity of introducing three-body forces. An estimate of the strength of the needed three-body forces can be found in Ref. [18], where a phenomenological three-body force [19] has been included in the calculation of the EOS within the continuous BHF approximation. The reported calculations do not include the so-called hole-hole diagram [1,2], which, however, has been estimated [2] to be very small and of the same sign in both the standard and the continuous choices. Its inclusion can hardly alter the graphs of Fig. 1 in a sizable way. Only rough estimates of the four hole-line contribution has been reported in the literature [13,14,20]. It appears that they cannot change the main conclusions of this work. A systematic extension of the analysis to other realistic nucleon-nucleon interactions is in progress.

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