Mass Inflation in Dynamical Gravitational Collapse of a Charged Scalar Field

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During the last ten years evidence has been mounting that generically the Cauchy horizon inside a charged or a spinning black hole becomes a null, weak singularity which is a precursor of a strong, spacelike singularity along the $r = 0$ hypersurface. We present here the missing link in this picture: A complete calculation from a *regular* initial data to the formation of a black hole and its inner singularities. We follow the gravitational collapse of a self-gravitating charged massless scalar field and observe the formation of an apparent horizon, a null, weak, mass-inflation singularity along the Cauchy horizon, and a final, spacelike, central singularity.   [S0031-9007(98)06951-8]

The no-hair conjecture, introduced by Wheeler [1] in the early 1970s, states that the external field of a black-hole relaxes to a Kerr-Newman field characterized solely by the black-hole's mass, charge and angular-momentum. This simple picture describing the exterior of a black hole is in dramatic contrast with its interior. The singularity theorems of Penrose and Hawking [2] predict the occurrence of inevitable spacetime singularities as a result of a gravitational collapse in which a black hole forms. According to the weak cosmic censorship conjecture [3] these spacetime singularities are hidden beneath the black hole's event horizon. However, these theorems tell us nothing about the nature of these spacetime singularities. In particular, the final outcome of a generic gravitational collapse is still an open question in general relativity.

Until recently, our physical intuition regarding the nature of these inner singularities was largely based on the known static or stationary black-hole solutions: Schwarzschild (spacelike, strong, and unavoidable central singularity), Reissner-Nordström and Kerr (timelike, strong singularity). Further insight was gained from the work of Belinsky, Khalatnikov, and Lifshitz [4] who found a strong oscillatory spacelike singularity. However, a new and drastically different picture of these inner black-hole singularities has emerged in the last few years, according to which the Cauchy horizon (CH) inside charged or spinning black holes is transformed into a null, weak singularity [5–9]. The CH singularity is weak in the sense that an infalling observer which hits this null singularity experiences only a finite tidal deformation [7,8]. Nevertheless, curvature scalars (namely, the Newman-Penrose Weyl scalar Ψ_2) diverge at the CH, a phenomena known as *mass inflation* [6]. The physical mechanism which underlies this CH singularity is actually quite simple: small perturbations, which are remnants of the gravitational collapse outside the collapsing object are gravitationally blueshifted as they propagate in the black hole's interior parallel to the CH [10] (the mass-inflation scenario itself includes in addition an outgoing radiation flux which irradiates the CH. This outgoing flux represents a portion of the ingoing radiation which is scattered inside the black hole).

Yet, it should be emphasized that despite the remarkable progress in our physical understanding of the inner structure of black holes, the evidence supporting the existence of a null, weak CH singularity is largely based on perturbative analysis. Thus, it is of interest to perform a *full* nonlinear investigation of the inner structure of black holes. The pioneering work of Gnedin and Gnedin [11] was a first step in this direction. They have demonstrated the existence of a central spacelike singularity deep inside a charged black hole coupled to a (neutral) scalar field. Much insight was gained from the numerical work of Brady and Smith [9] who studied the nonlinear evolution of a (neutral) scalar field on a spherical charged black hole. These authors established the existence of a null mass inflation singularity along the CH. Furthermore, they have shown that the singular CH contracts to meet the central $r = 0$ spacelike singularity. More recently, Burko [12] studied the same model problem. His work improves the numerical results given in [9]; namely, he found a good agreement between the nonlinear numerical results and the predictions of the perturbative approach.

Despite the important results achieved in these numerical investigations the mass-inflation scenario has never been demonstrated explicitly in a *collapsing* situation. It should be emphasized that these numerical works begin on a singular Reissner-Nordström spacetime and the black hole formation was *not* calculated there. The main goal of this paper is to demonstrate *explicitly* that mass inflation takes place during a dynamical charged gravitational collapse.

We consider the gravitational collapse of a self-gravitating charged massless scalar field ϕ. Massless charged fields are not expected in nature and charged collapse in general is not expected in a realistic gravitational collapse. Still we expect that this is a good toy model. It is generally accepted that the similarity between the inner structure of a Reissner-Nordström black hole and a Kerr black hole indicates that a charged collapse might be a

simple (spherical) toy model for the more realistic generic rotating collapse. Furthermore, is it expected that the inner structure of the black hole will not depend on the details of the collapsing matter field. In previous papers we have investigated the linear [13,14] and nonlinear [15] evolution of a charged scalar field outside a charged black hole. The results given in these papers, and in particular the existence of oscillatory inverse power-law charged tails along the black hole outer horizon suggest the occurrence of mass inflation along the CH of a *dynamically* formed charged black hole. Thus, this model is suitable to establish our main goal.

Our scheme is based on double null coordinates. This allows us to begin with *regular* initial spacetime (at approximately past null infinity), calculate the *formation* of the black hole's event horizon, and follow the evolution *inside* the black hole all the way to the central singularity, which is formed *during* the collapse. Thus, this numerical scheme makes it possible to test *directly* (and for the first time) the conjecture that the mass-inflation scenario is an inevitable feature of a generic gravitational collapse.

The physical model is described by the coupled Einstein-Maxwell-charged scalar equations. We express the metric of a spherically symmetric spacetime in the form [16]

$$ds^2 = -\alpha(u,v)^2\, du\, dv + r(u,v)^2\, d\Omega^2, \qquad (1)$$

where u is a retarded time null coordinate and v is an advanced time null coordinate. We fix the axis $r = 0$ to be along $u = v$. The remaining coordinate freedom is the freedom of the choice of v along some future null cone, i.e., along some fixed $u = $ const outgoing null ray. It should be noted that for $v \gg M$ our null ingoing coordinate v is proportional to the Eddington-Finkelstein null ingoing coordinate v_e.

Following Hamade and Stewart [17] we formulate the problem as a system of first-order coupled partial differential equations. To do so we define auxiliary variables d, f, g, s, x, and y:

$$d = \frac{\alpha_v}{\alpha}, \qquad f = r_u, \qquad g = r_v,$$
$$ \qquad (2)$$
$$s = \sqrt{4\pi}\, \phi, \qquad x = s_u, \qquad y = s_v.$$

Using these variables one can generalize the neutral Hamade and Stewart scheme [17]. In terms of these new variables the Einstein equations expand to

$$d_u - \frac{fg}{r^2} - \frac{\alpha^2}{4r^2} + \frac{\alpha^2 q^2}{2r^4} +$$

$$\frac{1}{2}(xy^* + x^*y) + \frac{1}{2}iea(sy^* - s^*y) = 0, \qquad (3)$$

$$rf_v + fg + \frac{1}{4}\alpha^2 - \frac{\alpha^2 q^2}{4r^2} = 0, \qquad (4)$$

and

$$g_v - 2dg + ry^*y = 0, \qquad (5)$$

where $a(u,v) \equiv A_u$ is the u component of the vector potential A, and otherwise the indices u and v denote

partial derivatives. The electromagnetic potential is given by the Maxwell equations

$$a_v - \frac{\alpha^2 q}{2r^2} = 0, \qquad (6)$$

where the charge $q(u,v)$ is given by

$$q_v - ier^2(s^*y - sy^*) = 0. \qquad (7)$$

The Hawking mass function $m(u,v)$ is given by

$$m = \frac{r}{2}\left(1 + \frac{q^2}{r^2} + \frac{4}{\alpha^2} r_u r_v\right). \qquad (8)$$

Finally, the wave equation for the charged scalar field becomes

$$ry_u + fy + gx + ieary + ieags + \frac{ie}{4r}\alpha^2 qs = 0. \qquad (9)$$

The initial conditions include the specification of $y(0,v)$ and $d(0,v)$ along an outgoing $u = 0$ null ray. We assume $d(0,v) = 0$, which fixes the remaining freedom in the coordinates. The boundary conditions on the axis $r = 0$ ($u = v$) are $g = -f = \frac{1}{2}\alpha$, $x = y$, $a = q = 0$, and $\alpha_r = s_r = 0$ (on axis).

The evolution of the quantities d and y are determined by Eqs. (3) and (9), respectively. We then integrate Eq. (2) outwards from the axis along an outgoing ($u = $ const) null ray to find α. Equations (5) and (2) are used to obtain g and r. Finally, we evaluate the quantities s, q, a, f, and x by integrating Eqs. (2), (7), (6), (4), and (9), respectively [x is evaluated from (9) using the relation $x_v = y_u$]. The integration in the u direction is carried out using a fifth-order Runge-Kutta method, while the integrals in the v direction are discretized using the three-point Simpson method [18].

The qualitative nature of the results was found to be the same for all numerical models computed. Figure 1 displays the radius $r(u,v)$ as a function of the ingoing null coordinate v along a sequence of outgoing ($u = $ const) null rays. All the outgoing null rays originate from the *non*singular axis $r = 0$, i.e., we start with a regular spacetime (this situation is in contrast with previous numerical works, where the evolution begins on a Reissner-Nordström spacetime). One can distinguish between three types of outgoing null rays in the rv plane: (i) The outermost (small-u) rays, which escape to infinity. (ii) The intermediate outgoing null rays approach a fixed radius $r_{CH}(u)$ at late times $v \rightarrow \infty$. This indicates the existence of a CH in these spacetimes. (iii) The innermost (large-u) rays, which originate on the nonsingular axis $r = 0$ and terminate at the singular section of the $r = 0$ hypersurface. These outgoing rays reach the $r = 0$ singularity in a finite v, without intersecting the CH. This situation is in contrast with the Reissner-Nordström spacetime, in which it is well known that all the outgoing null rays which originate inside the black hole intersect the CH. One should also note
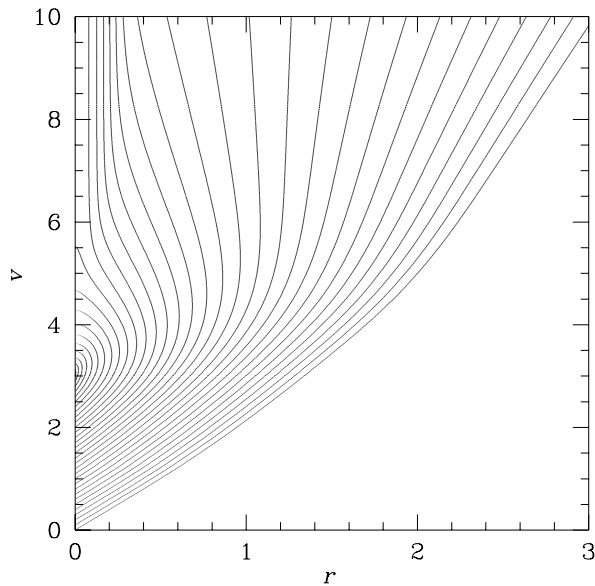
FIG. 1. Null rays in the rv plane. One can distinguish between three types of outgoing null rays: The outermost, which escape to infinity, the innermost, which terminate at the singular section of the $r = 0$ hypersurface and the intermediate outgoing null rays which approach a (u-dependent) finite radius, indicating the existence of a CH. All the null rays originate from the nonsingular axis $r = 0$.

FIG. 2. Contour lines of the coordinate r in the vu plane. The $r = 0$ contour line is indicated by a thicker curve. Its left section ($u = v$) represents the nonsingular axis, while its right section corresponds the the central spacelike singularity. The apparent horizon is indicated by the vanishing of r_v (a thicker curve). The (singular) CH (a null hypersurface, located at $v \rightarrow \infty$, and indicated by the approach of outgoing null rays to finite values of r) contracts to meet the central spacelike singularity (in a finite proper time).

that in contrast with the Reissner-Nordström spacetime, where the CH is a stationary null hypersurface, here $r_{CH}(u)$ depends on the outgoing null coordinate u, i.e., the CH contracts [9]. This dramatic difference in the causal structure of the present *collapsing* spacetime compared with the Reissner-Nordström spacetime is attributed to the outgoing flux of energy momentum carried by the charged scalar field which crosses the CH and forces it to contract.

To understand better the causal structure of our dynamical spacetime we display in Fig. 2 the contour lines of $r(u, v)$ in the vu plane. The outermost contour line corresponds to $r = 0$, where its left section (a straight line $u = v$) is the nonsingular axis, and its right section corresponds to the central singularity at $r = 0$. It should be emphasized that this central singularity forms during the gravitational collapse. The singularity at the $r = 0$ hypersurface is clearly a spacelike one, for r_v is negative along this section. The vanishing of r_v indicates the existence of an apparent horizon (which is first formed at $u \approx 1.05$ for this specific solution). The CH itself is a null hypersurface which is located at $v \rightarrow \infty$. Its existence is indicated by the fact that the intermediate outgoing null rays (in the range $1.05 \lesssim u \lesssim 2.1$ for this specific solution) terminate at a finite (u-dependent) radius $r_{CH}(u)$. The singular CH contracts to meet the central ($r = 0$) spacelike singularity (along the $u \simeq 2.1$ outgoing null ray). Thus, the null CH singularity is a precursor of the final spacelike singularity along the $r = 0$ hypersurface [9].

The behavior of the mass function $m(u, v)$ along the outgoing null rays is displayed in the top panel of Fig. 3. This figure establishes *explicitly* the exponential diver-
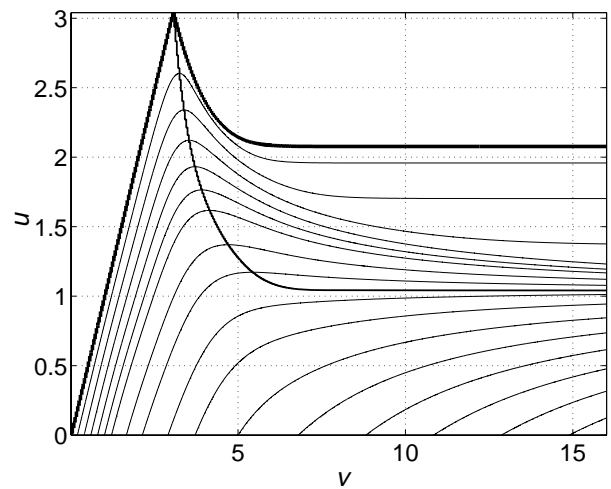
gence of the mass function (and the curvature scalars, namely the Newman-Penrose-Weyl scalar Ψ_2 which diverges like m/r^3 along the CH in a dynamically collapsing spacetime. These results agree with the mass-inflation
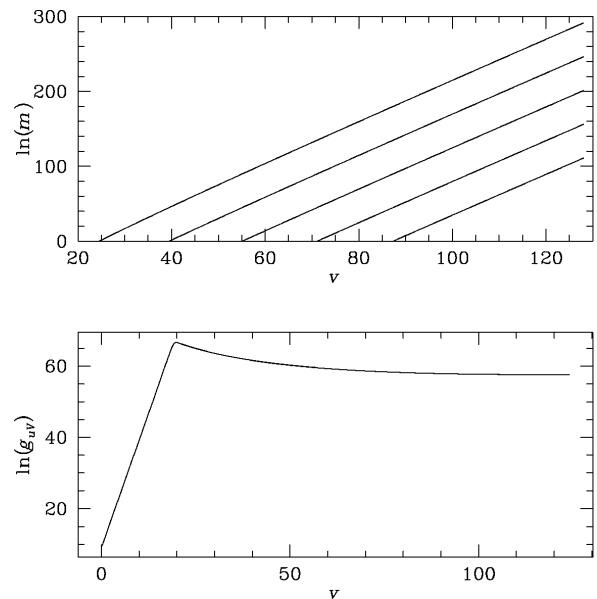

FIG. 3. The CH singularity. The top panel displays $\ln(m)$ vs advanced time v, along a sequence of outgoing null rays. The exponential growth of the mass function demonstrates the appearance of the mass-inflation scenario [6]. The bottom panel displays the metric function g_{uv} along an outgoing null ray. The finite value approached by the metric functions is in agreement with the simplified model of Ori [7,8], and demonstrates the *weakness* of the null mass-inflation singularity.
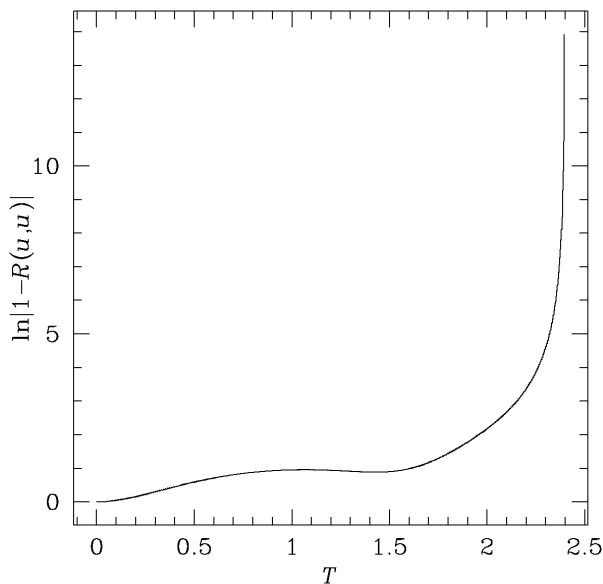
FIG. 4. The Ricci curvature scalar $R(u, u)$ as a function of the proper time on axis. The divergence of the curvature on the axis indicates that the *initially regular* axis is replaced in a finite proper time by a central (spacelike) *singularity*.

scenario to within 4%. The mass function increases not only along the outgoing ($u = $ const) null rays (as v increases) but also along ingoing ($v = $ const) null rays (as u increases).

The *weakness* of the null mass-inflation singularity was first discussed by Ori [7,8]. It is demonstrated in the bottom panel of Fig. 3. This figure displays the metric function g_{uV} along an outgoing null ray, where V is a Kruskal-like ingoing null coordinate. Clearly, g_{uV} approaches a finite value as the CH is approached ($V \to 0$). This confirms the analytical analysis of Ori, according to which a suitable coordinate transformation can produce a nonsingular metric.

Figure 4 displays the Ricci curvature scalar (on the axis):

$$R(u, v) = -\frac{8}{\alpha^2} \left[\text{Re}(y^* x) + ea \, \text{Im}(s^* y) \right], \qquad (10)$$

as a function of T, the proper time on axis [17]:

$$T(u) = \int_0^u \alpha(w, w) \, dw. \qquad (11)$$

The curvature on the axis diverges in a finite proper time, where R as a function of T grows faster than exponential.

Thus, the initially regular axis is replaced by a strong (spacelike) singularity along the $r = 0$ hypersurface.

In summary, we have studied the gravitational collapse of a self-gravitating charged scalar field. We calculate the formation of an apparent horizon, followed by a *weak,* null, *mass-inflation* singularity along the contracting CH, which precedes a *strong, spacelike* singularity along the $r = 0$ hypersurface. Our results give a first *explicit* confirmation of the mass-inflation scenario in a dynamical collapse that begins with *regular* initial conditions.

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