Differences between the Pole and On-Shell Masses and Widths of the Higgs Boson

Bernd A. Kniehl* and Alberto Sirlin
Department of Physics, New York University, 4 Washington Place, New York, New York 10003
(Received 14 May 1998)

The differences between the on-shell mass and width of the Higgs boson and their pole counterparts are evaluated in leading order. For a heavy Higgs boson, they are found to be sensitive functions of the gauge parameter and become numerically large over a class of gauges that includes the unitary gauge. For a light Higgs boson, the differences remain small in all gauges. The pinch-technique mass and width are found to be close to their pole counterparts over a large range of Higgs boson masses. [S0031-9007(98)06892-6]

PACS numbers: 11.15.Bt, 12.15.Lk, 14.80.Bn

The mass and width of an unstable scalar particle are conventionally defined by the expressions [1]

$$M^2 = M_0^2 + \mathrm{Re}\,A(M^2), \qquad M\Gamma = -\frac{\mathrm{Im}\,A(M^2)}{1 - \mathrm{Re}\,A'(M^2)}, \tag{1}$$

where M_0 is the bare mass, $A(s)$ is the self-energy, and the prime indicates differentiation with respect to s. Different and, in fact, more fundamental definitions are based on the complex-valued position of the propagator's pole [2]:

$$\bar{s} = M_0^2 + A(\bar{s}). \tag{2}$$

Writing $\bar{s} = m_2^2 - im_2\Gamma_2$, in this formulation one may identify the mass and width of the unstable particle with m_2 and Γ_2, respectively, so that

$$m_2^2 = M_0^2 + \mathrm{Re}\,A(\bar{s}), \qquad m_2\Gamma_2 = -\mathrm{Im}\,A(\bar{s}). \tag{3}$$

Given m_2 and Γ_2, other definitions are possible. For instance, it has been shown that, in the Z-boson case, the alternative expressions

$$m_1 = \sqrt{m_2^2 + \Gamma_2^2}, \qquad \Gamma_1 = \frac{m_1}{m_2}\Gamma_2 \tag{4}$$

lead to a Breit-Wigner resonance with an s-dependent width and can be identified with the mass and width measured at the CERN e^+e^- collider (LEP) [3]. We will refer to Eq. (1) as the on-shell definitions of mass and width, and to Eq. (3) or Eq. (4) as their pole counterparts. Identical formulas hold for spin-1 particles if $A(s)$ is identified with their transverse self-energy, and analogous expressions can be written down for spin-$1/2$ particles. Most calculations of radiative corrections and widths in the literature employ the on-shell formulation of Eq. (1). On the other hand, in the case of gauge theories, the pole definitions have an important advantage: general arguments imply that the pole position $\bar{s}$ and, therefore, also m_2, Γ_2, m_1, and Γ_1 are gauge invariant. By contrast, it has been shown that the on-shell definitions of M_W, M_Z, and unstable-quark masses become gauge dependent in $\mathcal{O}(g^4)$ and $\mathcal{O}(\alpha_s g^2)$ [3–5]. It has also been pointed out that the on-shell definition of width is inadequate if $A(s)$ is not analytic in the neighborhood of M^2. This occurs, for example, when the mass of the decaying particle lies very close to a threshold [6] or, in the resonance region, when the unstable particle is coupled to massless quanta, such as in the case of the W boson and unstable quarks [5].

The aim of this Letter is to discuss, in leading order, the differences between the on-shell mass and width and their pole counterparts for a very important case, namely, the Higgs boson. The fact that the width difference may be numerically large for a heavy Higgs boson over a large class of gauges is strongly suggested by preliminary arguments in Ref. [7].

Expanding Eqs. (1) and (3) about $s = m_2^2$ and combining the results, one readily finds

$$\begin{aligned}\frac{M - m_2}{m_2} &= -\frac{\Gamma_2}{2m_2}\,\mathrm{Im}\,A'(m_2^2) + \mathcal{O}(g^6),\\ \frac{\Gamma - \Gamma_2}{\Gamma_2} &= \mathrm{Im}\,A'(m_2^2)\left(\frac{\Gamma_2}{2m_2} + \mathrm{Im}\,A'(m_2^2)\right)\\ &\quad - \frac{m_2\Gamma_2}{2}\,\mathrm{Im}\,A''(m_2^2) + \mathcal{O}(g^6),\end{aligned} \tag{5}$$

where g^2 is a generic coupling of $\mathcal{O}(\Gamma_2/m_2)$. As the right-hand sides of Eq. (5) are of $\mathcal{O}(g^4)$, we may evaluate them using the lowest-order expressions for Γ_2, $\mathrm{Im}\,A'(m_2^2)$, and $\mathrm{Im}\,A''(m_2^2)$.

In the Higgs-boson case, the one-loop bosonic contribution to $\mathrm{Im}\,A(s)$ in the R_ξ gauge is given by

$$\begin{aligned}\mathrm{Im}\,A_{\mathrm{bos}}(s) = \frac{G}{4}s^2\Bigg[&-\left(1 - \frac{4M_W^2}{s} + \frac{12M_W^4}{s^2}\right)\left(1 - \frac{4M_W^2}{s}\right)^{1/2}\theta(s - 4M_W^2)\\ &+ \left(1 - \frac{M_H^4}{s^2}\right)\left(1 - \frac{4\xi_W M_W^2}{s}\right)^{1/2}\theta(s - 4\xi_W M_W^2) + \frac{1}{2}(W \to Z)\Bigg],\end{aligned} \tag{6}$$

where $G = G_\mu/(2\pi\sqrt{2})$, ξ_W is a gauge parameter, $(W \to Z)$ represents the sum of the preceding terms with the substitutions $M_W \to M_Z$ and $\xi_W \to \xi_Z$, and we have omitted gauge-invariant terms proportional to $\theta(s - 4M_H^2)$. The one-loop contribution due to a fermion f is

$$\mathrm{Im}\,A_f(s) = -\frac{G}{2}\, sN_f m_f^2 \left(1 - \frac{4m_f^2}{s}\right)^{3/2} \theta(s - 4m_f^2)\,, \tag{7}$$

where $N_f = 1$ (3) for leptons (quarks). As expected, Eq. (6) is gauge invariant if $s = M_H^2$, but it depends on ξ_W and ξ_Z off shell. The ξ_W dependence in Eq. (6) is due to the fact that a Higgs boson of mass $s^{1/2} > 2\xi_W^{1/2} M_W$ has nonvanishing phase space to "decay" into a pair of "particles" of mass $\xi_W^{1/2} M_W$. The first term in Eq. (6) can be verified by a very simple argument [7]: only the unphysical longitudinal excitations have M_H-dependent couplings with the Higgs boson; therefore, if the unphysical particles decouple, which happens for $\xi_W > s/(4M_W^2)$ and similarly for the Z boson, $\mathrm{Im}\,A(s)$ can be obtained by substituting $M_H^2 \to s$ in the well-known expressions for the Higgs-boson partial widths multiplied by M_H. Using Eqs. (6) and (7), we find at the one-loop level

$$\begin{aligned}
\mathrm{Im}\,A'_{\mathrm{bos}}(M_H^2) &= \frac{G}{2}\, M_H^2 \bigg[-\left(1 - \frac{5}{4}\, x_W + \frac{x_W^2}{4} + \frac{3}{16}\, x_W^3\right)(1 - x_W)^{-1/2}\theta(1 - x_W) \\
&\qquad + (1 - \xi_W x_W)^{1/2}\theta(1 - \xi_W x_W) + \frac{1}{2}\,(W \to Z)\bigg], \\
\mathrm{Im}\,A''_{\mathrm{bos}}(M_H^2) &= \frac{G}{2} \bigg[-\left(1 - \frac{3}{2}\, x_W + \frac{3}{8}\, x_W^2 - \frac{x_W^3}{4} + \frac{9}{32} x_W^4\right)(1 - x_W)^{-3/2}\theta(1 - x_W) \\
&\qquad + (1 - \xi_W x_W)^{-1/2}\theta(1 - \xi_W x_W) + \frac{1}{2}\,(W \to Z)\bigg], \\
\mathrm{Im}\,A'_f(M_H^2) &= -\frac{G}{2}\, N_f m_f^2 \left(1 + \frac{x_f}{2}\right)(1 - x_f)^{1/2}\theta(1 - x_f)\,, \\
\mathrm{Im}\,A''_f(M_H^2) &= -\frac{3}{32}\, G N_f x_f^3 (1 - x_f)^{-1/2}\theta(1 - x_f)\,,
\end{aligned} \tag{8}$$

where $x_a = 4M_a^2/M_H^2$. Equations (6), (7), and (8) permit us to evaluate Eq. (5). We also evaluate $(M^{\mathrm{PT}} - m_2)/m_2$ and $(\Gamma^{\mathrm{PT}} - \Gamma_2)/\Gamma_2$, where M^{PT} and Γ^{PT} are the pinch-technique (PT) on-shell mass and width obtained from Eq. (1) by employing the PT self-energy $a(s)$. We recall that the PT is a prescription that combines conventional self-energies with "pinch parts" from vertex and box diagrams in such a manner that the modified self-energies are independent of ξ_i $(i = W, Z, \gamma)$ and exhibit desirable theoretical properties [8]. In the Higgs-boson case, $a(s)$ can be extracted from Ref. [9], and we find

$$\begin{aligned}
\mathrm{Im}\,a'_{\mathrm{bos}}(M_H^2) &= \frac{3}{2}\, G M_W^2 \left(1 - x_W - \frac{x_W^2}{4}\right) \\
&\quad \times (1 - x_W)^{-1/2}\theta(1 - x_W) \\
&\quad + \frac{1}{2}\,(W \to Z)\,, \\
\mathrm{Im}\,a''_{\mathrm{bos}}(M_H^2) &= \frac{G}{4}\, x_W \left(1 + \frac{x_W}{4} - \frac{x_W^2}{2} - \frac{9}{16}\, x_W^3\right) \\
&\quad \times (1 - x_W)^{-3/2}\theta(1 - x_W) \\
&\quad + \frac{1}{2}\,(W \to Z)\,.
\end{aligned} \tag{9}$$

Identifying M_H with m_2 and, for simplicity, setting $\xi = \xi_W = \xi_Z$, our results for $(M - m_2)/m_2$ and $(\Gamma - \Gamma_2)/\Gamma_2$ are illustrated in Figs. 1(a)–1(c) as functions of ξ, for three values of m_2. We have employed $M_W = 80.375$ GeV, $M_Z = 91.1867$ GeV, and $m_t = 175.6$ GeV and have neglected contributions from fermions other than the top quark. The two deep abysses in the figures are associated with the unphysical thresholds $\xi = m_2^2/(4M_Z^2), m_2^2/(4M_W^2)$, where the expansions in Eq. (5) obviously fail. For small Higgs mass ($m_2 = 200$ GeV), we see from Fig. 1(a) that, aside from the neighborhoods of the abysses, M and Γ remain numerically very close to m_2 and Γ_2. In the intermediate case ($m_2 = 400$ GeV), the relative differences reach 0.6% in the mass and 3.3% in the width. However, for a heavy Higgs boson ($m_2 = 800$ GeV), the differences become very large, reaching 11% in the mass and 44% in the width. The largest differences occur for $\xi > m_2^2/(4M_W^2)$, i.e., when the unphysical excitations decouple, a range that includes the unitary gauge. We recall that the latter retains only the physical degrees of freedom and, in this sense, it may be regarded as the most physical of all gauges. The large effects can be easily understood from Eq. (6). If $\xi > s/(4M_W^2)$, the second term in Eq. (6) does not contribute, so that $\mathrm{Im}\,A_{\mathrm{bos}}(s) \propto s^2$. For a heavy Higgs boson, this implies large values of $\mathrm{Im}\,A'(m_2^2)$ and $\mathrm{Im}\,A''(m_2^2)$. For $\xi < s/(4M_Z^2)$, the gauge-dependent terms contribute and cancel the leading s^2 dependence of $\mathrm{Im}\,A_{\mathrm{bos}}(s)$, so that the

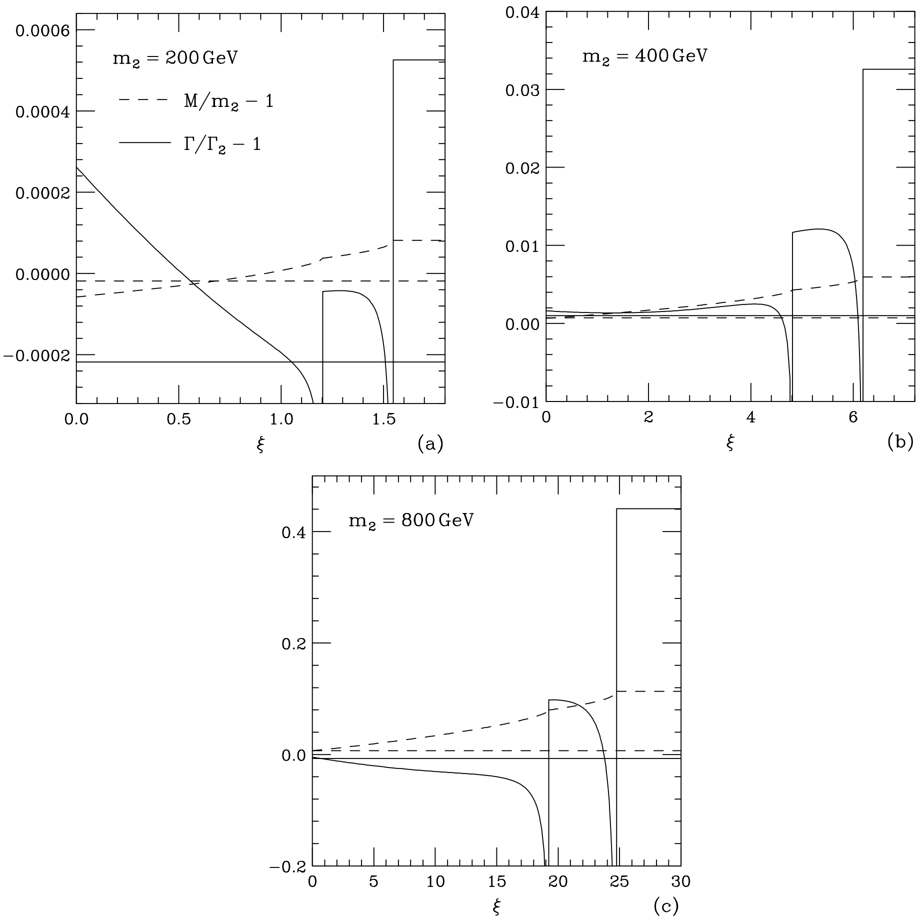

FIG. 1. Relative deviations of M and Γ from m_2 and Γ_2, respectively, as functions of $\xi = \xi_W = \xi_Z$ in the R_ξ gauge, assuming (a) $m_2 = 200$ GeV, (b) 400 GeV, and (c) 800 GeV. The horizontal lines across the figures indicate the corresponding deviations in the PT framework.

magnitudes of $\mathrm{Im}\, A'(m_2^2)$ and $\mathrm{Im}\, A''(m_2^2)$ drop sharply and the differences become much smaller. Of course, the 44% effect in the width for $\xi > m_2^2/(4M_W^2)$ may cast doubts on the convergence of the expansions in Eq. (5). As this is a leading-order calculation and higher-order terms may be not negligible, we regard this result as an indication of large corrections, rather than a precise evaluation of $(\Gamma - \Gamma_2)/\Gamma_2$.

Our results go beyond those reported in the literature [10]. The reason is easy to understand: in Ref. [10], the limits $M_W \to 0$ and $g \to 0$ are simultaneously considered keeping the Higgs self-coupling $\lambda \propto g^2 M_H^2/M_W^2$ fixed. If the gauge parameter ξ is also kept fixed, the gauge dependence of Eq. (6) is lost, and one obtains an s-independent result for $\mathrm{Im}\, A_{\mathrm{bos}}(s)$, which does not contribute to the right-hand sides of Eq. (5). Thus, the above approximation, although interesting and useful, does not exhibit the gauge dependence and the large effects discussed here.

From the horizontal lines across Figs. 1(a)–1(c), we see that the PT mass and width remain very close to m_2 and Γ_2 for all values of m_2, the maximum departures being 0.7% for M^{PT} and -0.7% for Γ^{PT} at $m_2 = 800$ GeV. The differences vary somewhat if M and Γ are compared with m_1 and Γ_1. Through $\mathcal{O}(g^6)$, $(M - m_1)/m_1$ and $(\Gamma - \Gamma_1)/\Gamma_1$ are obtained from $(M - m_2)/m_2$ and $(\Gamma - \Gamma_2)/\Gamma_2$ by subtracting the gauge-invariant term $\Gamma_2^2/(2m_2^2)$. For $m_2 = 800$ GeV, $(M - m_1)/m_1$ and $(\Gamma - \Gamma_1)/\Gamma_1$ amount to 5.6% and 38% in the unitary gauge (rather than 11% and 44%) and to -4.8% and -6.6% in the 't Hooft-Feynman gauge (rather than 0.9% and -0.8%). For the same value of m_2, the differences $(M^{\mathrm{PT}} - m_1)/m_1$ and $(\Gamma^{\mathrm{PT}} - \Gamma_1)/\Gamma_1$ are -5.1% and -6.5% (rather than 0.7% and -0.7%).

In summary, we have shown that, in leading order, the differences between the on-shell mass and width of a heavy Higgs boson and their pole counterparts are sensitive functions of the gauge parameter, and reach large numerical values in a class of gauges that includes the unitary gauge. For other frequently employed gauges, such as $\xi = 1$ ('t Hooft-Feynman gauge) and $\xi = 0$ (Landau gauge), the differences are very small with respect to m_2 and Γ_2, but are not negligible relative to m_1 and Γ_1. For intermediate (light) Higgs bosons, the differences are reasonably (very) small for all values of

ξ, except in the abysses described above. The PT on-shell mass and width remain close to m_2 and Γ_2 in the range $200 \leq m_2 \leq 800$ GeV. These results give further support to the proposition that a consistent definition of two of the most important concepts in particle physics, namely, those of mass and width of an unstable particle, must ultimately be based on the pole position rather than the on-shell approach [3–7,11]. For many purposes, the well-known and convenient machinery of the latter can be employed, but physicists should become aware of its limitations and potential pitfalls.

B. A. K. thanks the NYU Physics Department for the hospitality extended to him during a visit when this manuscript was prepared. This research was supported in part by NSF Grant No. PHY-9722083.

*Permanent address: Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, 80805 Munich, Germany

[1] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, Massachusetts, 1995), p. 236.

[2] R. E. Peierls, in *The Proceedings of the 1954 Glasgow Conference on Nuclear and Meson Physics*, edited by E. H. Bellamy and R. G. Moorhouse (Pergamon Press, London and New York, 1955), p. 296; M. Lévy, Nuovo Cimento **XIII**, 115 (1959); R. J. Eden, P. V. Landshoff, D. I. Olive, and J. C. Polkinghorne, *The Analytic S-Matrix* (Cambridge University Press, Cambridge, England, 1966), p. 247.

[3] A. Sirlin, Phys. Rev. Lett. **67**, 2127 (1991).

[4] M. Passera and A. Sirlin, Phys. Rev. Lett. **77**, 4146 (1996).

[5] M. Passera and A. Sirlin, Report No. NYU-TH/98-04-01; hep-ph/9804309.

[6] T. Bhattacharya and S. Willenbrock, Phys. Rev. D **47**, 4022 (1993).

[7] A. Sirlin, in *Proceedings of the Ringberg Workshop: The Higgs Puzzle—What Can We Learn from LEP2, LHC, NLC and FMC?, Ringberg Castle, Germany, 1996*, edited by B. A. Kniehl (World Scientific, Singapore, 1997), p. 39.

[8] J. M. Cornwall, Phys. Rev. D **26**, 1453 (1982); J. M. Cornwall and J. Papavassiliou, Phys. Rev. D **40**, 3474 (1989); J. Papavassiliou, Phys. Rev. D **41**, 3179 (1990); G. Degrassi and A. Sirlin, Phys. Rev. D **46**, 3104 (1992).

[9] A. Pilaftsis, Nucl. Phys. **B504**, 61 (1997).

[10] S. Willenbrock and G. Valencia, Phys. Lett. B **247**, 341 (1990); G. Valencia and S. Willenbrock, Phys. Rev. D **46**, 2247 (1992); A. Ghinculov and T. Binoth, Phys. Lett. B **394**, 139 (1997); K. Riesselmann and S. Willenbrock, Phys. Rev. D **55**, 311 (1997); T. Binoth and A. Ghinculov, Phys. Rev. D **56**, 3147 (1997).

[11] M. Consoli and A. Sirlin, in *Physics at LEP*, CERN Yellow Report No. 86-02, 1986, Vol. 1, p. 63; S. Willenbrock and G. Valencia, Phys. Lett. B **259**, 373 (1991); R. G. Stuart, Phys. Lett. B **262**, 113 (1991); **272**, 353 (1991); Phys. Rev. Lett. **70**, 3193 (1993); H. Veltman, Z. Phys. C **62**, 35 (1994).