Phase Coexistence in Transitional Nuclei and the Interacting-Boson Model

F. Iachello
Center for Theoretical Physics, Sloane Physics Laboratory, Yale University, New Haven, Connecticut 06520-8120

N. V. Zamfir
WNSL, Yale University, New Haven, Connecticut 06520-8124
Clark University, Worcester, Massachusetts 01610
Institute of Atomic Physics, Bucharest-Magurele, Romania

R. F. Casten
WNSL, Yale University, New Haven, Connecticut 06520-8124
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Recent data provide evidence for coexisting phases at low energy in the spherical-deformed transitional nucleus ^{152}Sm. The nature of the wave functions at the spherical-deformed phase transition in nuclei is analyzed within the framework of the interacting-boson model. It is shown that in the U(5)-SU(3) transition, two phases coexist in a very small region of parameter space around the critical value of the control parameter ξ. The coexistence region with two minima in the potential shrinks to zero as one moves to the U(5)-O(6) transition. Implications for other systems are briefly mentioned. [S0031-9007(98)06839-2]

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The nature of "shape" phase transitions in finite many-body systems is a fundamental issue and has been the subject of many investigations. Recently, new data on transition rates in one of the best-known regions of rapid structural change in nuclei—the Sm isotopes—has shed new light on phase transitional behavior. In view of its interest in many areas of physics and chemistry, it is important to understand the precise mechanisms by which structural transitions occur.

The study of phase-shape transitions in nuclei can be best done in the interacting-boson model [1] (IBM) which reproduces well the data in the transitional Nd-Sm-Gd region [2]. It is the purpose of this Letter to (i) show that phase transitions in the IBM display the phenomenon of phase coexistence, (ii) determine the region of phase coexistence, and (iii) show that the newly obtained data provide evidence for phase coexistence. We will extend the discussion to comment more generally on phase transitional behavior in algebraic Hamiltonians that describe other physical systems.

The possible phases that can occur in the IBM have been classified previously. They can be depicted in a triangular diagram, shown in Fig. 1. The three phases correspond to the breaking of U(6) into its three subalgebras (I) U(5) (Ref. [3], (II) SU(3) (Ref. [4]), and (III) O(6) (Ref. [5]).

We begin by considering the U(5)-SU(3) transition. A quantum calculation of this transition can be done by using the Hamiltonian

$$H = \epsilon \hat{n}_d - \kappa \hat{Q}^\chi \cdot \hat{Q}^\chi \tag{1}$$

with $\chi = -\sqrt{7}/2$. The meaning and definitions of the various terms are given in Ref. [1]. Here it suffices to say that the Hamiltonian (1) is the combination of two invariant operators, C_1(U5) and C_2(SU3), having U(5) and SU(3) symmetry, respectively. The U(5) to SU(3) leg of the symmetry triangle is traversed by varying the ratio ϵ/κ. In discussing phase transitions, it is convenient to reparametrize this ratio as

$$\epsilon/\kappa = (1 - \xi)/\xi\,. \tag{2}$$

The leg of the symmetry triangle from U(5) to SU(3) is then labeled by the control parameter $0 \leq \xi \leq 1$ and the wave functions, transition rates, and energy eigenvalues (apart from a scale factor) depend only on the values of ξ. A phase transition occurs in the ground state energy at a critical value of the parameter, $\xi = \xi_c$. As discussed previously [6] this phase transition is 1st order. Here we concentrate on the nature of the wave functions at (or around) the phase transition. To this end we show in

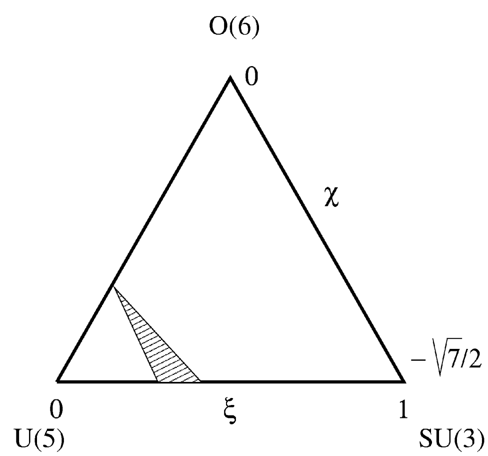

FIG. 1. The symmetry triangle for the IBM showing schematically the region of parameter space corresponding to the phase coexistence discussed in the text.

Fig. 2 both the relevant experimental low lying levels of ^{152}Sm and calculations with the IBM, using the parameters of Ref. [2]. As will be apparent later, the data for ^{152}Sm, and, in particular, the very small strength of the $2_3{}^+ \rightarrow 0_2{}^+$ transition, give evidence for phase coexistence.

The IBM calculation reproduces these data very well. The parameters for ^{152}Sm ($\epsilon/\kappa \sim 30$ or $\xi = 0.032$), $\chi = -\sqrt{7}/2$ belong to a particular small region in the overall (ξ, χ) parameter space, very close to the critical value ($\xi_c = 0.025$ for $N = 10$, $\chi = -\sqrt{7}/2$—see below).

For this unique region, the wave functions have a special character, as seen in Fig. 3, where the arresting feature is that the single IBM Hamiltonian of Eq. (1) is able to generate a coexistence of two phases with some states having (approximately since this is a finite system) wave functions appropriate to phase I [U(5)] and others having wave functions appropriate to phase II [SU(3)]. Note in particular the wide distribution in the number of d bosons, n_d (the order parameter for this transition), of the ground state of ^{152}Sm, typical of a deformed [SU(3)-like] wave function distribution, contrasted with the large amplitude for $n_d = 0$ in the $0_2{}^+$ state, typical of the ground state of a spherical [U(5)-like] wave function distribution. (The n_d distributions for the levels with higher angular momentum reflect the same separation into two classes of states.) In terms of the level scheme in Fig. 2, the states built on the $0_1{}^+$ state form a rotational-like sequence while those levels built on the $0_2{}^+$ state comprise a vibrational or phonon sequence. Also in Fig. 3 one can see how this coexistence evolves, from ^{150}Sm (spherical in its ground state) to ^{154}Sm (deformed in its ground state).

This phenomenon of coexistence (similar to liquid-gas phase transitions) appears also in a classical calculation. This calculation can be done by making use of the coherent state formalism for the IBM [6].

The scaled potential energy surface

$$E(\beta,\gamma) = \frac{N\beta^2}{1+\beta^2}[1-\xi(\chi^2-3)] - \frac{N(N-1)}{(1+\beta^2)^2}\,\xi\left[4\beta^2 - 4\sqrt{\frac{2}{7}}\,\chi\beta^3\cos 3\gamma + \frac{2}{7}\chi^2\beta^4\right], \tag{3}$$

(with $\chi = -\sqrt{7}/2$) has two shallow minima for ξ values in the narrow range 0.025–0.029, as shown in the inset in Fig. 4 (left). Here β, γ are the intrinsic shape variables and N is the total boson number. $N = 10$ is used in Fig. 4. The minima become deeper for larger N. The presence of these two minima in the energy surface occurs only for a very small region of ξ values as seen in Fig. 4. It is just these ξ values that are applicable to ^{152}Sm. The classical expression (3) allows one also to study the nature of the phase transition as a function of χ. Changing χ from $-\sqrt{7}/2$ to 0, one moves along the side of the triangle from SU(3) to O(6). In the phase transition region, one moves along the path shown in Fig. 1. The value of χ for a point inside the triangle is given by the intersection of a line, originating from the U(5) vertex and passing through the given point, with the side of the triangle extending from SU(3) to O(6). It is very interesting to note that the coexistence region shrinks with χ [as shown in the inset in Fig. 4 (right)], in accordance with the fact that the

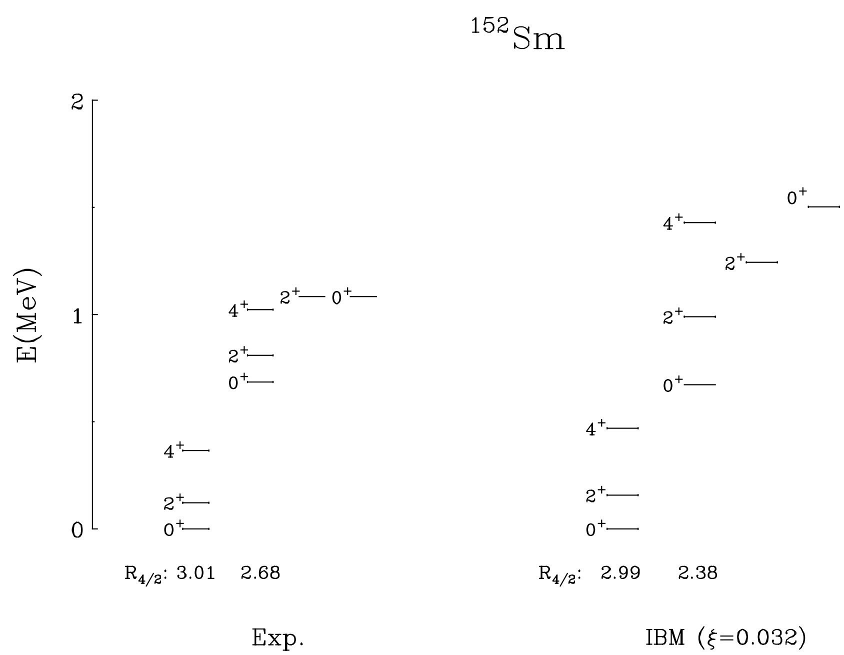

FIG. 2. Experimental and theoretical level schemes for ^{152}Sm. Note the two families of levels: a rotational band built on the 0^+ ground state and a vibrational set of levels built on the $0_2{}^+$ level. For the latter we show the levels up to the two-phonon states. The deformed and vibrational phases are highlighted by quite different $R_{4/2} \equiv E(4^+)/E(2^+)$ ratios, as indicated.

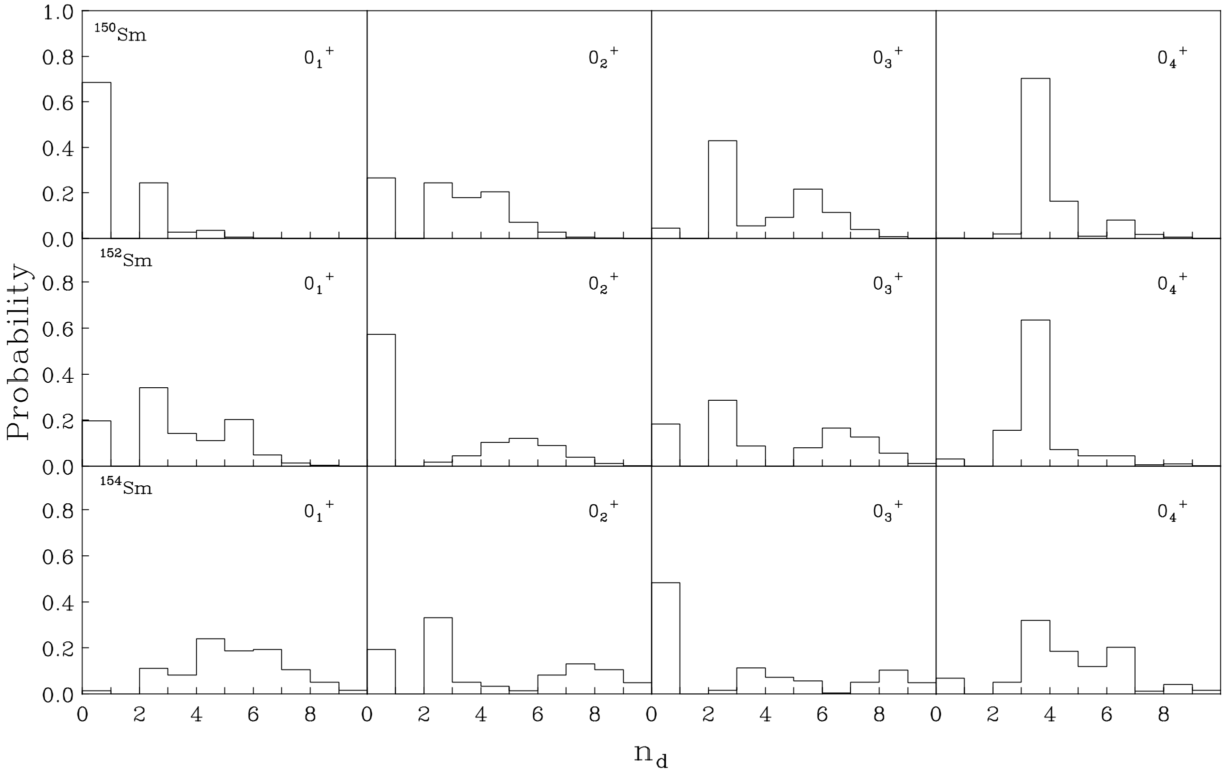

FIG. 3. Distribution of squared wave function amplitudes for 0^+ states as a function of n_d for ^{150,152,154}Sm.

phase transition is 1st order for U(5) → SU(3) but 2nd order for U(5) → O(6).

Phase transitions in nuclei can be tested experimentally by measuring observables that are particularly sensitive to them. Two observables have been used previously: (i) separation energies, S_{2n}, and (ii) isomer shift, $\delta\langle r^2\rangle$. The latter is directly related to the order parameter $\langle n_d\rangle$, but it is difficult to access experimentally. In this Letter we point out that there is another observable, namely, the electromagnetic transition rate $2_3{}^+ \rightarrow 0_2{}^+$, that is sensitive not only to the phase transition but also to coexistence. The behavior of this transition rate as a function of ξ is shown in Fig. 5. It varies extremely rapidly near the phase transitional point and has a zero at the phase transition.

The origin of this zero has an interesting physical interpretation. In a deformed nucleus [near-SU(3) in the IBM] the lowest excited bands (β and γ vibrations) belong

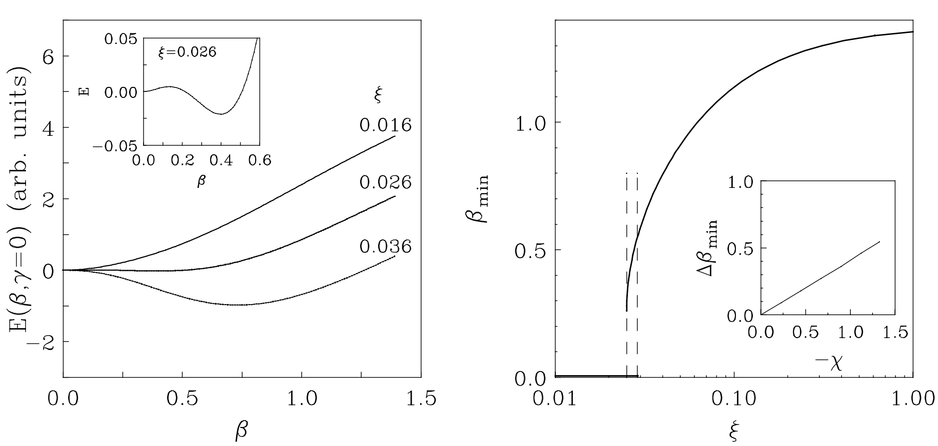

FIG. 4. Left: Energy surfaces as a function of the deformation parameter β, for three ξ values. The inset shows, on an expanded scale, the two shallow coexisting minima that arise for a narrow range of values near 0.026. The upper and lower curves have one minima only, either for a spherical or a deformed shape. Right: Value(s) of the location of the minima of the energy surfaces. Note the two minima for a small range of ξ values. The calculations are for $N = 10$ and $\chi = -\sqrt{7}/2$. The inset shows the difference in the $\beta_{\rm min}$ values, $\Delta\beta_{\rm min}$, as a function of the parameter χ.

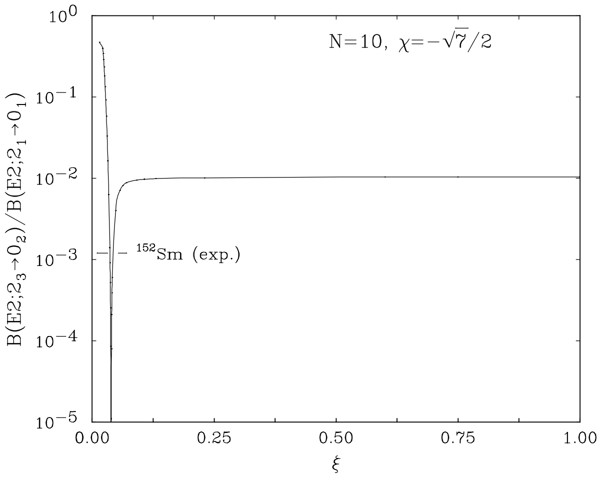

FIG. 5. Calculated values of the ratio $B(E2; 2_3{}^+ \rightarrow 0_2{}^+)/B(E2; 2_1{}^+ \rightarrow 0_1{}^+)$ showing the extremely rapid change near the critical value of ξ. The experimental value [7] for ^{152}Sm is 0.0012.

to the same SU(3) representation and hence $E2$ transitions between $2_3{}^+$ and $0_2{}^+$ are allowed $[B(E2; 2_3{}^+ \rightarrow 0_2{}^+)/B(E2; 2_1{}^+ \rightarrow 0_1{}^+) \approx 3/2N^2]$. In a spherical nucleus [near-U(5) in the IBM], the $2_3{}^+ \rightarrow 0_2{}^+$ transition is also allowed since it corresponds to a transition from a three-phonon state ($n_d = 3$) to a two-phonon state ($n_d = 2$) $[B(E2; 2_3{}^+ \rightarrow 0_2{}^+)/B(E2; 2_1{}^+ \rightarrow 0_1{}^+) = \frac{7}{5}\frac{(N-2)}{N}]$. In between, the $B(E2; 2_3{}^+ \rightarrow 0_2{}^+)$ value goes to zero due to the fact that the transition $2_3{}^+ \rightarrow 0_2{}^+$ becomes a transition in a spherical structure (the coexisting phase commencing with the $0_2{}^+$ state) involving a change in phonon number $\Delta n_d = 2$. These coexisting deformed and spherical structures are seen in both the experimental and calculated level schemes in Fig. 2, as well as in two nucleon transfer reactions [8,9]. Since the minima in Fig. 4 are shallow one expects mixing of the phases. This mixing is automatically included in the quantum IBM calculation. The $n_d = 0$ probability for the $0_2{}^+$ state in the ^{152}Sm is complemented by a smaller probability distribution (higher n_d values) typical of a deformed wave function.

The transition rates $2_3{}^+ \rightarrow 0_2{}^+$ have been very difficult to measure. However, with the development of very sensitive detectors and detector arrays [7], such measurements are now feasible. The experimental value for ^{152}Sm is shown in Fig. 5. It gives strong evidence for the occurrence of phase coexistence in nuclei.

In conclusion, we have discussed shape phase transitions [U(5) → SU(3) and U(5) → O(6)] in nuclei and shown experimental evidence for the phenomenon of coexistence. We have discussed how phase coexistence can occur for a single Hamiltonian and basis space in the IBM both at the quantum and classical levels. At the quantum level, the arguments stand as given. At the classical level, they should be complemented by the inclusion of quantum fluctuations (important in the transition region) and of mixing of the two phases. A study in the classical limit of the interacting-boson model [10] shows that the coexistence persists even in the presence of these fluctuations. The mixing of the phases has been discussed above.

The analysis of this Letter can be extended to the class of models described by algebraic Hamiltonians. A generic statement for models based on U(n) is that phase transitions between the two phases U($n-1$) and O(n) are always 2nd order and have no phase coexistence. This statement implies, among other things, that the phase transition between rigid molecules, described by the subalgebra O(4) of U(4) (Ref. [11]), and van der Waals molecules, described by the subalgebra U(3) of U(4), does not have phase coexistence. In contrast, when one of the phases corresponds to a subalgebra other than U($n-1$) or O(n), involving a change in n by more than one unit, such as the case of U(6) ⊃ SU(3) discussed here for nuclei, phase coexistence can occur.

Our analysis is also related to that of Ref. [12], where the coexistence of superconductivity and charge-density waves was described in terms of the algebra of O(6) ≈ SU(4) and its breaking into O(5) and SU(2) × SU(2).

It also has implications to phase transitions in atomic clusters (another finite quantal system) where phenomena similar to those reported here are expected to occur [13].

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