Cross Section Minima in Elastic Nd Scattering: Possible Evidence for Three-Nucleon Force Effects

H. Witała,[2] W. Glöckle,[1] D. Hüber,[3] J. Golak,[2] and H. Kamada[1]
[1]*Institut für theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany*
[2]*Institute of Physics, Jagellonian University, PL-30059 Cracow, Poland*
[3]*Los Alamos National Laboratory, M.S. B283, Los Alamos, New Mexico 87545*
(Received 14 January 1998)

Neutron-deuteron elastic scattering cross sections are calculated at different energies using modern nucleon-nucleon (NN) interactions and the Tucson-Melbourne three-nucleon force adjusted to the triton binding energy. Predictions based on NN forces only underestimate nucleon-deuteron data in the minima at higher energies starting around 60 MeV. Adding the three-nucleon forces fills up those minima and reduces the discrepancies significantly. [S0031-9007(98)06806-9]

PACS numbers: 21.30.–x, 21.45.+v, 24.10.–i, 25.10.+s

Substantial progress has been made recently in the study of the three-nucleon ($3N$) system both experimentally and theoretically. The set of data is being significantly enriched for cross sections and spin observables in elastic neutron-deuteron (nd) and proton-deuteron (pd) scattering and in the $3N$ breakup process. Theoretical formulations and numerical algorithms have been significantly improved, with the result that $3N$ bound and nd scattering states can be solved exactly. Recently, in the still pending pp Coulomb force problem for the pd system, a step forward has been achieved below the deuteron breakup threshold [1]. In addition, the nucleon-nucleon (NN) system is still very intensively investigated and the increased data set provides a sound foundation for reliable modern phase-shift analysis [2]. Based on these phases, modern NN forces have been constructed by different groups [3–5]. These interactions reproduce the NN data set with unprecedented accuracy as measured by a χ^2/datum very close to 1. Although those forces are not yet linked to the underlying quantum chromodynamics (QCD) due to well-known reasons, they cover a wide spectrum of expected properties and form an interesting basis to study few-nucleon systems. Thus theoretical tools and data are available to probe the dynamics of three interacting nucleons. In the future, QCD should provide theoretically consistent NN and $3N$ forces and specifically the relative importance of the latter ones for binding energies and scattering matrices. The first steps on that ground are being done in chiral perturbation theory [6]. Despite the still restricted theoretical insight from QCD, one can go ahead and compare the theoretical predictions obtained with modern NN interactions and model $3N$ forces to experimental $3N$ data. There might be a clear-cut signal coming from certain observables which cannot be explained by $3N$ Hamiltonians based on modern NN interactions only. Such a "smoking gun" observable would then put limits on present day $3N$ force models and would also be of great importance to test the future QCD-based dynamics.

The three-nucleon binding energy by itself is a first signature. The modern NN interactions underbind ^{3}H, but to a different extent [7]. The essentially local ones lack binding energy of about 800 keV out of 8.48 MeV, whereas the nonlocal CD Bonn interaction [5] underbinds only by $\approx$500 keV. That information from ^{3}H on insufficient dynamics based on present day NN forces only should be enriched by further evidence from the $3N$ continuum.

Such a search for $3N$ continuum observables, which could serve as possible evidence for $3NF$ effects, has been pursued since $3N$ continuum calculations have become feasible [8]. With the advent of the optimally tuned NN forces and the feasibility to also include three-nucleon forces ($3NF$'s) into $3N$ continuum calculations, the conclusive power of such calculations has increased tremendously. It is the aim of this article to point to such a smoking gun in the $3N$ continuum based on modern $3N$ Faddeev calculation.

Before coming to that, let us briefly describe the situation in $3N$ continuum studies. A detailed overview has been given recently [9]. The bulk of $3N$ scattering observables below about 100 MeV nucleon lab energy can be described quite well in the NN force picture only. A beautiful example is the total nd cross section [10]. This most simple picture is also quite stable in the sense that the most modern phase-equivalent NN force models yield essentially the same predictions. But there are exceptions, "time dependent ones," which were removed by subsequent measurements [11], and more important true ones, where the data are reconfirmed by independent measurements. Such a distinguished case is the low energy vector analyzing power A_y in elastic Nd scattering [12]. A drastic discrepancy between the predictions based on NN forces only, and both nd and pd data, has been found. Present day $3NF$ models have insignificant effects and do not remove that discrepancy. It is known, that A_y depends very sensitively on the ^3P_j NN forces. Thus a trivial explanation might be that the ^3P NN phase-shift parameters from modern phase-shift

analysis have not been settled to the true ones [13]. Presently, it is an unsolved puzzle. If the reason does not lie in the *NN* forces, a 3*NF* of still unknown properties will be responsible. In Ref. [14] arguments are given for that scenario since the considered changes in the *NN* forces, excluding the well-established propriety of the one pion exchange, were not capable of solving that puzzle. The closely related deuteron vector analyzing power iT_{11} is equally not understood [13].

Another possible signature for 3*NF* effects is the space star configuration in the 3*N* breakup process at 13 MeV [11]. Two *nd* measurements agree essentially with each other but deviate from theory (in the *NN* picture only and including 3*NF* models). The situation poses even more questions since *pd* data deviate very severely from the *nd* data pointing to unexpectedly large Coulomb force effects [15].

In the present study we investigate the angular distribution in elastic *Nd* scattering. The transition amplitude for this process is composed of the nucleon exchange part (PG_0^{-1}), the direct action of a 3*NF* and a part having its origin in the multiple interactions of three nucleons through 2*N* and 3*N* forces:

$$\langle \phi' | U | \phi \rangle = \langle \phi' | PG_0^{-1} + V_4^{(1)}(1 + P) + P\tilde{T} + V_4^{(1)}(1 + P)G_0\tilde{T} | \phi \rangle . \quad (1)$$

That rescattering part is expressed in terms of a $\tilde{T}$ operator which sums up all multiple scattering contributions through the integral equation [16]

$$\tilde{T}|\phi\rangle = tP|\phi\rangle + (1 + tG_0)V_4^{(1)}(1 + P)|\phi\rangle + tPG_0\tilde{T}|\phi\rangle + (1 + tG_0)V_4^{(1)}(1 + P)G_0\tilde{T}|\phi\rangle . \quad (2)$$

Here G_0 is the free 3*N* propagator, t is the *NN* t matrix, and P is the sum of a cyclical and anticyclical permutation of three objects. The 3*NF* V_4 is split into three parts

$$V_4 = \sum_{i=1}^{3} V_4^{(i)} , \quad (3)$$

where each one is symmetrical under exchange of two particles. For the π-π exchange 3*NF*, for instance [17], this corresponds to the three possible choices of the nucleon, which undergoes the (off-shell) π-*N* scattering. The asymptotic state $|\phi\rangle$ ($|\phi'\rangle$) is a product of the deuteron wave function and the momentum eigenstate of the third particle.

The exchange part comprises two processes where the incoming nucleon ends up as a constituent of the final deuteron, and the constituents of the initial deuteron are free in the final state. Because of the nature of this term its contribution to the elastic scattering cross section is peaked at backward angles. The contribution from the driving term $tP|\phi\rangle$ and the rescattering terms in t (*NN* force contributions only) are peaked at forward angles. Therefore the elastic scattering cross section exhibits a characteristic minimum in the angular range, where the contributions of the exchange and the rescattering terms are of comparable order and both are small. This angular range around the minimum could thus be a place where the 3*NF* signal, if sufficiently strong, should appear. It would happen at those energies where the pure 3*NF* contribution to the elastic scattering amplitude in that minimum is comparable or larger than the contributions of the exchange part and the pure 2*N* rescattering terms.

The pure 3*NF* contribution to the transition operator U results from Eqs. (1) and (2) when only the 3*NF* is active:

$$U^{3NF} = P\tilde{T}^{3NF} + V_4^{(1)}(1 + P) + V_4^{(1)}(1 + P)G_0\tilde{T}^{3NF} \quad (4)$$

with

$$\tilde{T}^{3NF}|\phi\rangle = V_4^{(1)}(1 + P)|\phi\rangle + V_4^{(1)}(1 + P)G_0\tilde{T}^{3NF}|\phi\rangle . \quad (5)$$

We expect that the contribution of U^{3NF} alone is uniformly distributed over all angles.

In order to check these expectations we solved Eqs. (2) and (5) at the nucleon laboratory energies of 12, 65, 140, and 200 MeV using the modern *NN* interactions: AV18 [4], CD Bonn [5], Nijm I, and Nijm II [3]. As the 3*NF* we took the 2π-exchange Tucson-Melbourne (TM) model [17], where the strong cutoff parameter Λ has been adjusted individually together with each *NN* force to the experimental triton binding [7]. In the calculations including 3*NF*'s, all partial wave states with total angular momenta in the two-nucleon subsystem up to $j_{\max} = 3$ were taken into account. It is the most extensive calculation with 3*NF*'s in the continuum which we can presently perform. At the higher energies they are not fully converged with respect to $j_{\max}$. The importance of partial waves with higher two-nucleon angular momenta is illustrated in fully converged solutions in the case when only 2*N* forces are active. Then we included all states up to $j_{\max} = 5$. Our theoretical results are shown in Figs. 1–4 in comparison to data. Our theory does not include the *pp* Coulomb force. Therefore we should compare to *nd* data. This is only possible at rather low energies, where *nd* data exist and which agree perfectly with *NN* force predictions only [9]. The *pd* data also existing there agree with the *nd* data, except at very forward angles, where Rutherford scattering has to show up. That interference with Rutherford scattering can clearly be seen in Figs. 1 and 2 at forward angles, where the data bend towards smaller values. Aside from that, there is a very good agreement at 12 MeV with theory. This, together with the smallness of the Coulomb force effects on the elastic scattering cross section in the region of its minimum, as shown by exact calculations under the deuteron breakup threshold [1], supports the conjecture that a comparison of *nd* theory with *pd* data at even higher energies makes sense. Figures 1–4 show the expected result, that the pure 3*NF* contribution is

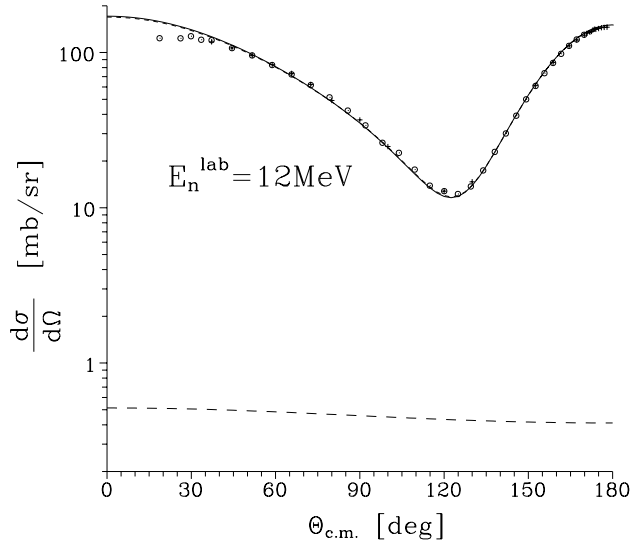

FIG. 1. The differential *Nd* cross section at $E_N^{lab} = 12$ MeV. The prediction of the CD Bonn *NN* interaction without (short-dashed curve) and with 3*NF* (solid curve) is compared to *pd* data (circles (○) from [21] and crosses (+) from [22]). The long-dashed curve is the pure 3*NF* prediction. All of the calculations are truncated at $j_{max} = 3$.

essentially uniform in its angular dependence, and we see that at 12 MeV it is totally negligible. At 65 MeV there are also a few *nd* data [18] and, as shown in Fig. 2, they come close to *NN* force predictions only, whereas the *pd* data [19] deviate strongly in the minimum. Without a rigorous calculation, including the *pp* Coulomb force, it has to remain an open question whether the deviation

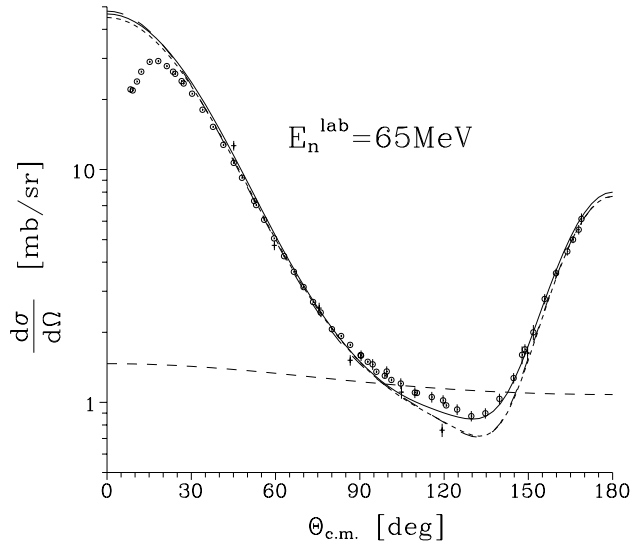

FIG. 2. The differential *Nd* cross section at $E_N^{lab} = 65$ MeV. The prediction of the CD Bonn *NN* interaction for $j_{max} = 3$ (short-dashed curve) and $j_{max} = 5$ (long-dashed curve) is compared to 64.5 MeV *pd* data [(○) from [19]] and *nd* data [(+) from [18]]. The CD Bonn calculation including the 3*NF* for $j_{max} = 3$ fills the minimum (solid curve). The pure 3*NF* prediction is shown as intermediately long-dashed curve.

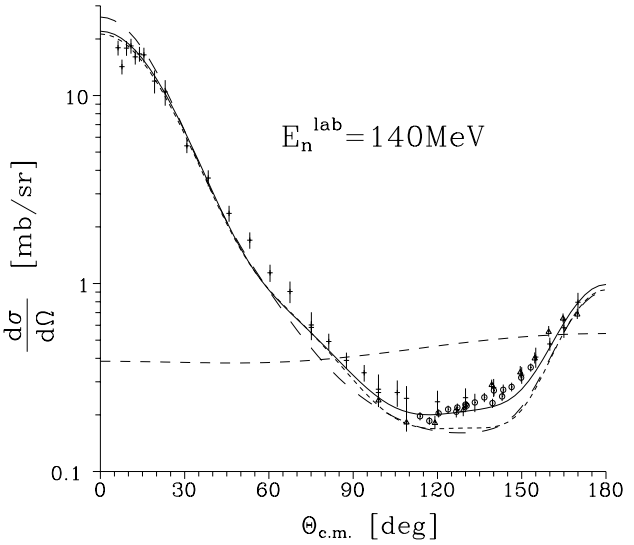

FIG. 3. The differential *Nd* cross section at $E_N^{lab} = 140$ MeV. Curve descriptions are the same as in Fig. 2. The *pd* data are 145.5 MeV (○) from [23] and 146 MeV (+) from [24]. The triangles (△) are 152 MeV *nd* data from [25].

between the *pd* data and the *NN* force predictions is due only to our neglect of Coulomb forces in the theoretical calculations. On the other hand, the *nd* data of Fig. 3 are compatible with *pd* data in this energy range and indicate only small Coulomb force effects corresponding to our conjecture. Apparently, precise *nd* data in the angular range of the minima for 65 MeV and higher would be highly desirable. Independent of that important issue, we can go ahead and display possible 3*NF* effects in these minima. The discrepancy of the theory based on *NN* forces only to the *pd* data increases with energy, as seen in

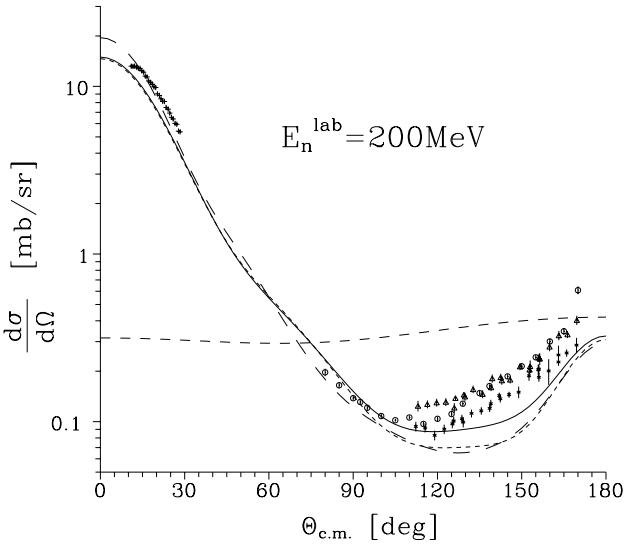

FIG. 4. The differential *Nd* cross section at $E_N^{lab} = 200$ MeV. Curve descriptions are the same as in Fig. 2. The *pd* data are 198 MeV (○) from [26], 200 MeV (+) from [20], 181 MeV (△) from [23], and 216.5 MeV (×) from [23].

Figs. 2–4. Higher angular momentum states do not cure that discrepancy. They are a significant contribution, however, to the cross section at the higher energies at forward angles [20], as seen especially in Figs. 3 and 4. As expected, the pure $3NF$ contribution remains essentially uniform also at the higher energies. With increasing energy, however, this contribution becomes significant in relation to the minimum value of the cross section. Being totally negligible at 12 MeV, it overshoots the minimum value by a factor of ≈ 6 at 200 MeV. At 65 MeV, the $3NF$ signal becomes sufficiently large to be seen in the minimum region. Indeed, as shown in Figs. 2–4, including the $3NF$ in addition to the $2N$ interactions in the $3N$ Hamiltonian removes a large part of the discrepancy in the cross section minimum at the higher energies. We consider that filling of the minima as a smoking gun for $3NF$ effects. Very precise data, in both the nd and the pd systems, would therefore be highly valuable.

We have to expect additional modifications, especially at the highest energies, due to relativistic effects, which have not been taken into account in our calculation. First estimates just based on kinematical factors indicate indeed a small shift of all angular distribution at higher energies toward higher values.

Finally, we want to emphasize that our conclusions do not depend on the particular NN interaction used. Taking different modern NN interactions and the corresponding TM $3NF$ leads to practically the same results.

In summary, we have shown that the minima of the elastic Nd scattering cross sections are probably a smoking gun for $3NF$ effects. A large part of the discrepancy between modern NN potential predictions and data in this angular range can be removed when the TM $3NF$, properly adjusted to the triton binding, is included in the $3N$ Hamiltonian. In order to check more accurately this conclusion, precise Nd elastic scattering data at different energies in the region of the cross section minima are required. The optimal data would be in the nd system to avoid the theoretical uncertainty of pp Coulomb force effects.

This work was supported by the Deutsche Forschungsgemeinschaft under Project No. Gl87/24-1. The work of D. H. was partially supported by the Deutsche Forschungsgemeinschaft under Project No. Hu 746/1-2 and partially by the U.S. Department of Energy. The numerical calculations have been performed on the CRAY T90 and the CRAY T3E of the Höchstleistungsrechenzentrum in Jülich, Germany, and on the 3840 of the ACK in Cracow, Poland (KBN/SPP/UJ/046/1996).

[1] A. Kievsky, M. Viviani, and S. Rosati, Phys. Rev. C **52**, R15 (1995).
[2] V. G. J. Stoks *et al.*, Phys. Rev. C **48**, 792 (1993); R. A. Arndt *et al.*, Phys. Rev. C **50**, 2731 (1994).
[3] V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen, and J. J. de Swart, Phys. Rev. C **49**, 2950 (1994).
[4] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).
[5] R. Machleidt, F. Sammarruca, and Y. Song, Phys. Rev. C **53**, R1483 (1996).
[6] U. van Kolck, Phys. Rev. C **49**, 49 (1994); E. Epelbaoum, W. Glöckle, and Ulf-G. Meissner, Nucl. Phys. A (to be published); nucl-th/9804nnn.
[7] A. Nogga, D. Hüber, H. Kamada, and W. Glöckle, Phys. Lett. B **409**, 19 (1997); J. L. Friar, G. L. Payne, V. G. J. Stoks, and J. J. de Swart, Phys. Lett. B **311**, 4 (1993).
[8] A. C. Phillips, Phys. Rev. **142**, 984 (1966).
[9] W. Glöckle, H. Witała, D. Hüber, H. Kamada, and J. Golak, Phys. Rep. **274**, 107 (1996).
[10] W. P. Abfalterer *et al.*, Phys. Rev. Lett. **81**, 57 (1998).
[11] H. R. Setze *et al.*, Phys. Lett. B **388**, 229 (1996).
[12] H. Witała, D. Hüber, and W. Glöckle, Phys. Rev. C **49**, R14 (1994).
[13] W. Tornow, H. Witała, and A. Kievsky, Phys. Rev. C **57**, 555 (1998).
[14] D. Hüber and J. L. Friar, Phys. Rev. C (to be published).
[15] E. O. Alt and M. Rauh, Phys. Rev. C **49**, R2285 (1994).
[16] D. Hüber, H. Kamada, H. Witała, and W. Glöckle, Acta Phys. Pol. B **28**, 1677 (1997).
[17] S. A. Coon and W. Glöckle, Phys. Rev. C **23**, 1790 (1981); S. A. Coon *et al.*, Nucl. Phys. **A317**, 242 (1979).
[18] H. Rühl *et al.*, Nucl. Phys. **A524**, 377 (1991).
[19] H. Shimizu *et al.*, Nucl. Phys. **A382**, 242 (1982).
[20] H. Rohdjess *et al.*, Phys. Rev. C **57**, 2111 (1998).
[21] W. Grübler *et al.*, Nucl. Phys. **A398**, 445 (1983).
[22] G. Rauprich *et al.*, Few-Body Syst. **5**, 67 (1988).
[23] G. Igo *et al.*, Nucl. Phys. **A195**, 33 (1972).
[24] H. Postma and R. Wilson, Phys. Rev. **121**, 1229 (1961).
[25] J. N. Palmieri, Nucl. Phys. **A188**, 72 (1972).
[26] R. E. Adelberger and C. N. Brown, Phys. Rev. D **5**, 2139 (1972).