

Pathways to Naturally Small Neutrino Masses

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(Received 11 May 1998)

In the minimal standard electroweak gauge model, there is an effective dimension-five operator which generates neutrino masses, and it has only three tree-level realizations. One is the canonical seesaw mechanism with a right-handed neutrino. Another is having a heavy Higgs triplet as recently proposed. The third is to have a heavy Majorana fermion triplet, an example of which is presented here in the context of supersymmetric SU(5) grand unification. The three generic one-loop realizations of this operator are also discussed. [S0031-9007(98)06809-4]

PACS numbers: 14.60.Pq, 12.10.Kt

In the minimal standard gauge model of quarks and leptons, each of the three known neutrinos (ν_e, ν_μ, ν_τ) appears only as a member of a left-handed SU(2) lepton doublet

$$\psi_i = (\nu_i, l_i)_L, \quad (1)$$

and the Higgs sector contains only one scalar doublet

$$\Phi = (\phi^+, \phi^0). \quad (2)$$

As a result, neutrinos are massless in this model. Experimentally there is now a host of evidence for neutrino oscillations, and that is most naturally explained if neutrinos are massive and mix with one another. Theoretically there is no compelling reason for massless neutrinos, and any extension beyond the minimal standard model often allows them to be massive. There exists already a vast literature on specific models of neutrino mass and mixing.

In this paper I make the following simple observation. In the minimal standard electroweak gauge model, there is a well-known effective dimension-five operator [1] which generates Majorana neutrino masses, to wit,

$$\Lambda^{-1} \phi^0 \phi^0 \nu_i \nu_j, \quad (3)$$

where Λ is a large effective mass. All models of neutrino mass and mixing (which have the same light particle content as the minimal standard model) can be summarized by this operator. Different models (among them the well-known seesaw model [2]) are merely different realizations of this operator. In the following I will show that it has only three tree-level realizations, all of which are conceptually just as simple. I will also discuss its many possible one-loop realizations, encompassing thus most previous work on radiative neutrino masses.

To obtain the effective operator (3) at tree level, using only renormalizable interactions, it is clear that there are only three ways: (I) ψ_i and Φ form a fermion singlet, (II) ψ_i and ψ_j form a scalar triplet, and (III) ψ_i and Φ form a fermion triplet. Note that the singlet combination of ψ_i and ψ_j is $\nu_i l_j - l_i \nu_j$ which does not generate (3). In each case, the complete gauge-invariant effective operator is actually the same, but how it is written reveals its possible origin:

$$(I) \Lambda^{-1} (\phi^0 \nu_i - \phi^+ l_i) (\phi^0 \nu_j - \phi^+ l_j), \quad (4)$$

$$(II) \Lambda^{-1} [\phi^0 \phi^0 \nu_i \nu_j - \phi^+ \phi^0 (\nu_i l_j + l_i \nu_j) + \phi^+ \phi^+ l_i l_j], \quad (5)$$

$$(III) \Lambda^{-1} [(\phi^0 \nu_i + \phi^+ l_i) (\phi^0 \nu_j + \phi^+ l_j) - 2\phi^+ \nu_i \phi^0 l_j - 2\phi^0 l_i \phi^+ \nu_j]. \quad (6)$$

(I) The intermediate heavy particle in this case is clearly a fermion singlet as shown in Fig. 1. Call it N and let its mass be M and its coupling to ν_i be f_i , then $\Lambda^{-1} = f_i f_j / 2M$. As ϕ^0 acquires a nonzero VEV (vacuum expectation value $\langle \phi^0 \rangle = v$), we can identify $f_i v$ as a Dirac mass m_i linking ν_i to N and the neutrino mass matrix is simply $-m_i m_j / M$. This is, of course, just the well-known seesaw mechanism, with N identified as the right-handed neutrino with a large Majorana mass. Note that with one such singlet, only one linear combination of ν_i gets a tree-level mass. Hence the usual scenario requires three N 's. This mechanism of generating naturally small neutrino masses dominates the literature, but, as I show

below, the two other alternatives are conceptually just as simple.

(II) What is needed here is a heavy scalar triplet $\xi = (\xi^{++}, \xi^+, \xi^0)$ as shown in Fig. 2. If its mass is M and its coupling to $\nu_i \nu_j$ and $\phi^0 \phi^0$ are f_{ij} and μ , respectively, then $\Lambda^{-1} = f_{ij} \mu / M^2$ and the neutrino mass matrix is given by

$$(\mathcal{M}_\nu)_{ij} = \frac{-2f_{ij} \mu v^2}{M^2}. \quad (7)$$

Note that only one ξ is required for all neutrinos to become massive. This is a simple mechanism which does not require right-handed neutrinos, and is indistinguishable

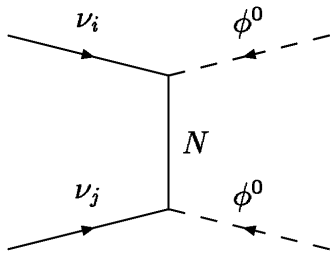


FIG. 1. Tree-level realization of the effective operator (3) with heavy fermion singlet.

from (I) as far as the low-energy limit of the theory is concerned. As already discussed recently [3], another way of understanding the above is to consider the VEV of ξ . Although ξ is very heavy, it acquires a tiny VEV given by $u = -\mu v^2/M^2$; hence the neutrino mass matrix is equal to $2f_{ij}u$ as expected from the direct coupling of ξ to $\nu_i\nu_j$. The idea that a heavy Higgs scalar could have a naturally small VEV was known but not widely appreciated, and this mechanism has largely been neglected.

(III) We replace N of (I) here with a heavy Majorana fermion triplet ($\Sigma^+, \Sigma^0, \Sigma^-$). Again a seesaw mass is obtained [4], and there is no low-energy distinction between this and the other two mechanisms. However, each has its own unique implications about physics beyond the standard model. In (I), the addition of three N 's argues favorably for the efficacy of $SO(10)$ instead of $SU(5)$ as a suitable unifying symmetry, whereas in (II) and (III), $SU(5)$ by itself is sufficient. The **15** representation of $SU(5)$ would contain ξ , whereas the **24** representation would contain both N and Σ .

As an example, consider a supersymmetric $SU(5)$ model of grand unification [5]. The breaking of $SU(5)$ to the standard $SU(3) \times SU(2) \times U(1)$ gauge group is accomplished using the **24** supermultiplet,

$$\mathbf{24} = (1, 1, 0) + (8, 1, 0) + (1, 3, 0) + (3, 2, -5/6) + (3^*, 2, 5/6), \quad (8)$$

where the scalar component of $(1, 1, 0)$ acquires a large VEV. The fermionic components of $(1, 1, 0)$ and $(1, 3, 0)$ are exactly N and Σ of (I) and (III). However, a $\nu_i\phi^0N$ coupling is not desirable because the scalar partner of N has a large VEV and ν_i must then combine with the

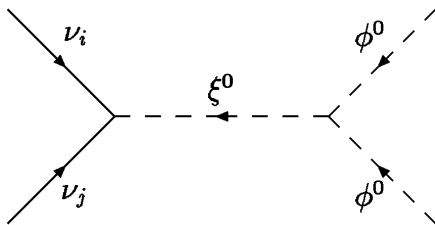


FIG. 2. Tree-level realization of the effective operator (3) with heavy scalar triplet.

fermion partner of ϕ^0 to form a superheavy Dirac particle. On the other hand, since the scalar partner of Σ^0 has no VEV, a $\nu_i\phi^0\Sigma^0$ coupling is permissible in principle.

Now ψ_i belongs to the $\mathbf{5}^*$ representation, and there are two scalar doublets $\Phi_1 = (\phi_1^0, \phi_1^-)$ and $\Phi_2 = (\phi_2^+, \phi_2^0)$ belonging to the $\mathbf{5}^*$ and $\mathbf{5}$ representations, respectively. With only one $\mathbf{5}$ and one $\mathbf{24}$, there can be only one $\mathbf{5}^*$ which appears in the singlet decomposition of $\mathbf{5}^* \times \mathbf{5} \times \mathbf{24}$. This $\mathbf{5}^*$ is defined to be the one containing Φ_1 . Hence ψ_i does not couple to N or Σ of Eq. (8). To obtain a neutrino mass, we need another $\mathbf{24}$ which has no VEV. Consider then a discrete Z_2 symmetry, under which all the quark and lepton supermultiplets are odd, and all others are even except for this additional $\mathbf{24}$ which is odd as well. In that case, one linear combination of ν_i gets a seesaw mass from the N and Σ of this odd $\mathbf{24}$. For $M_N \sim M_\Sigma \sim 10^{16}$ GeV and $\langle \phi_2^0 \rangle \sim 10^2$ GeV, a neutrino mass of order 10^{-3} eV is then very natural and suitable for solar neutrino oscillations [6].

Consider now the MSSM (minimal supersymmetric standard model [7]) which pervades the present literature on particle physics. The neutrinos of this model are massless. However, if the MSSM is the low-energy remnant of supersymmetric $SU(5)$, then an additional superheavy $\mathbf{24}$ naturally yields one massive neutrino which could explain the solar data. However, to accommodate either the atmospheric data [8] or the LSND (Liquid Scintillator Neutrino Detector) data [9] as well, we need another massive neutrino. We now have the option of using any one of the above three mechanisms. For example, if we would add another odd $\mathbf{24}$ with a mass of order 10^{14} GeV, we could get a neutrino mass of about 0.1 eV, which would be suitable for atmospheric neutrino oscillations.

The effective operator (3) may be realized also radiatively in one loop. There is, in fact, one well-known generic mechanism [10] as shown in Fig. 3. The fermions ω and ω^c must couple to ϕ^0 ; hence one of them has to belong to a doublet. Without loss of generality, we choose

$$\omega \sim (q_3, 2, q_1) \quad (9)$$

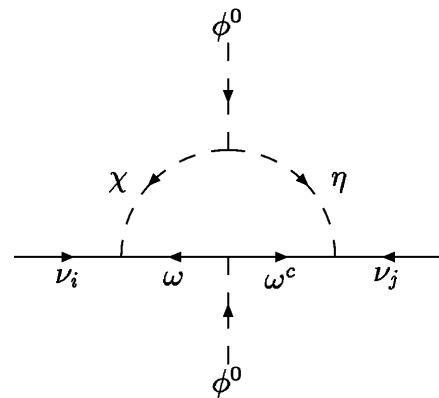


FIG. 3. First one-loop realization of the effective operator (3).

under $SU(3) \times SU(2) \times U(1)$. We then must have

$$\omega^c \sim (q_3^*, q_2, -q_1 + \frac{1}{2}), \quad (10)$$

where $q_2 = 1$ or 3 . As we go around the loop, we see that

$$\eta \sim (q_3, 2, q_1) \quad (11)$$

as well, and

$$\chi \sim (q_3^*, q_2', -q_1 + \frac{1}{2}), \quad (12)$$

where $q_2' = 1$ or 3 also. For a given choice of q_3 and q_1 , there are then four variations, corresponding to the choice of q_2 and q_2' . Most specific proposals for the one-loop radiative generation of Majorana neutrino masses are contained in the above.

The fermions ω and ω^c may, in fact, be the usual quark or lepton doublet and singlet. For example, if we choose $q_3 = 1$, $q_1 = 1$, and $q_2 = q_2' = 1$, then $\omega \sim (1, 2, -1/2) \sim l_L$ and $\omega^c \sim (1, 1, 1) \sim \tilde{l}_L^c$. Now $\eta \sim (1, 2, -1/2)$ and $\chi \sim (1, 1, 1)$ may be arbitrary new scalar particles, in which case we have the Zee model [11], or supersymmetric scalar leptons \tilde{l} and \tilde{l}^c , in which case we have the R -parity violating model [12]. In the latter case, we may also use the quarks and their supersymmetric scalar partners, i.e., $q_3 = 3$, $q_1 = 1/6$, and $q_2 = q_2' = 1$.

For simplicity, both q_2 and q_2' are usually chosen to be one, but $q_2' = 3$ has also been considered [13]. The observation that the effective operator (3) comes from a specific model has also been made [14]. Here I start with (3) and show how all specific models are extracted from it. This approach leads one naturally to another one-loop diagram which generates (3) as shown in Fig. 4. This mechanism has rarely been used, and only in scenarios [15] where one neutrino already has a tree-level mass.

The fermions ω and ω^c of Fig. 4 must combine to form an invariant mass, hence

$$\omega \sim (q_3, q_2, q_1), \quad \omega^c \sim (q_3^*, q_2, -q_1). \quad (13)$$

As we go around the loop, we see that

$$\eta \sim (q_3, q_2', q_1 + \frac{1}{2}), \quad \chi \sim (q_3^*, q_2'', -q_1 + \frac{1}{2}). \quad (14)$$

If $q_2 = 1$, then $q_2' = q_2'' = 2$. Otherwise, q_2' and q_2'' may be either $q_2 - 1$ or $q_2 + 1$ independently, except that

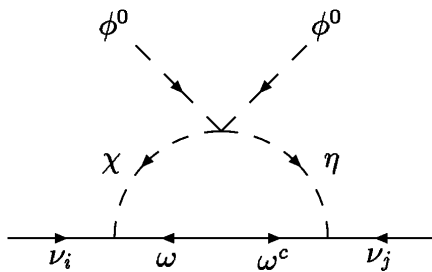


FIG. 4. Second one-loop realization of the effective operator (3).

$q_2' \times q_2''$ must contain the triplet representation; hence $q_2' = q_2'' = 1$ is not allowed. Consider the following specific example. Add to the standard model just one right-handed neutrino singlet N with a Majorana mass, then we can set $\omega = \omega^c = N$, i.e., $q_3 = q_2 = 1$ and $q_1 = 0$. In that case, both η and χ are $(1, 2, 1/2)$ doublets, so they can be the same extra scalar doublet we add to the standard model. If we did not have the second doublet, then only one linear combination of ν_i 's would get a tree-level seesaw mass from N , and the rest would become massive [16] only at the two-loop level through the exchange of 2 W bosons.

In the above example with one N , one neutrino mass is obtained at tree level and the others are radiative. If there is no N , the mechanism of Fig. 4 can still be used to find radiative masses for all three neutrinos. As an illustration, let ω and ω^c be charged fermion singlets: $\omega \sim (1, 1, -1)$, $\omega^c \sim (1, 1, 1)$. We must now have $\eta \sim (1, 2, -1/2) = (\eta^0, \eta^-)$ and $\chi \sim (1, 2, 3/2) = (\chi^{++}, \chi^+)$. There are then the following invariant interaction terms:

$$\begin{aligned} &(\nu_i \chi^+ - l_i \chi^{++})\omega, \quad (\nu_j \eta^- - l_j \eta^0)\omega^c, \\ &\chi^+ \eta^- \bar{\phi}^0 \bar{\phi}^0 + (\chi^{++} \eta^- + \chi^+ \eta^0) \bar{\phi}^0 \phi^- \\ &+ \chi^{++} \eta^0 \phi^- \phi^-, \end{aligned} \quad (15)$$

which allow Fig. 4 to generate radiative neutrino masses as shown. A trivial variation of Fig. 4 is to replace the quartic $\chi \eta \bar{\phi}^0 \bar{\phi}^0$ coupling with two cubic couplings $\chi \bar{\phi}^0 \zeta$ and $\eta \bar{\phi}^0 \bar{\zeta}$, where ζ is an extra complex scalar multiplet.

Finally, consider Fig. 5 which requires only one complex scalar multiplet ζ but four fermion multiplets ω , ω^c , σ , and σ^c . This mechanism is largely known [10] only for generating masses for quarks and charged leptons. A variation of it was applied [12] to Majorana neutrinos in the supersymmetric R -parity violating model, but there the scalar neutrinos have VEV's, whereas the assumption here is that only ϕ^0 has a VEV.

The fermions σ and σ^c of Fig. 5 must combine to form an invariant mass. The simplest case is to let them be singlets: $\sigma \sim (q_3, 1, q_1)$ and $\sigma^c \sim (q_3^*, 1, -q_1)$. Then $\omega \sim (q_3, 2, q_1 + 1/2)$, $\omega^c \sim (q_3^*, 2, -q_1 + 1/2)$, and

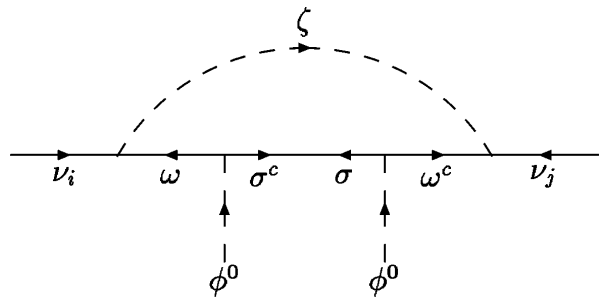


FIG. 5. Third one-loop realization of the effective operator (3).

$\zeta \sim (q_3, 1, q_1)$ or $(q_3, 3, q_1)$. If we choose $q_3 = 1$ and $q_1 = -1$, then ω is a doublet with charges 0 and -1 , whereas ω^c is a doublet with charges 2 and 1. We see that, as in the case of Fig. 4, exotic representations are needed for the implementation of this mechanism. We can generate all other solutions systematically by starting with a given SU(2) representation for σ and σ^c . Two other variations of Fig. 5 are also possible. We simply place the invariant mass to the other side of one or the other of the two $\phi^{0\prime}$ s.

In the three tree-level realizations of the effective operator (3) for naturally small Majorana neutrino masses, the mass scale of the heavy particles involved should be very large: 10^{13} to 10^{16} GeV. Looking at the three identical effective interactions of Eqs. (4)–(6), we see that there would be no other observable effect except for nonzero neutrino masses and mixing. In the many one-loop realizations, the mass scale of the new particles involved depends on the model, but there is a general rule. If there is a fermion doublet which does not have an invariant mass, then the masses of its components must come from the VEV of ϕ^0 ; hence they should be of order 100 GeV and be accessible experimentally in the near future. If a new particle is found, then we can use Figs. 3–5 to check if the other new particles are there or not.

In conclusion, the important issue of naturally small Majorana neutrino masses in any extension of the standard model, which has the same light particle content, can be synthesized in terms of a single effective operator: $\Lambda^{-1} \phi^0 \phi^0 \nu_i \nu_j$. There are three tree-level realizations of this operator: one is the well-known seesaw mechanism with a heavy singlet fermion, another is having a heavy Higgs triplet which naturally acquires a tiny vacuum expectation value, a third is to replace the singlet in the seesaw with a triplet. The literature on neutrino masses is dominated by the first mechanism, but the other two are just as conceptually simple and would open up the options available for physics beyond the standard model. There are also three one-loop realizations (plus variations) of this operator as shown in Figs. 3–5. Almost all previous specific models of radiative Majorana neutrino masses are embodied in Fig. 3. The detailed structures of this and the other two diagrams are systematically described here for the first time. The new particles involved are possibly

of order 100 GeV, in which case experimental verification is within reasonable reach in the near future.

This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG03-94ER40837. I thank K. S. Babu, R. Foot, A. S. Joshipura, H. Murayama, and J. Pantaleone for communications.

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