Minimality Condition and Atmospheric Neutrino Oscillations

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A structure is proposed for the mass matrices of the quarks and leptons that arises in a natural way from the assumption that the breaking of SO(10) gauge symmetry is achieved by the smallest possible set of vacuum expectation values. This structure explains well many features of the observed spectrum of quarks and leptons. It reproduces the Georgi-Jarlskog mass relations and leads to a charm quark mass in reasonable agreement with data. It also predicts a large mixing angle between ν_μ and ν_τ, as suggested by atmospheric neutrino data. The mixing angles of the electron neutrino are predicted to be small. [S0031-9007(98)06830-6]

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In this Letter we propose a structure for the quark and lepton mass matrices that arises naturally in supersymmetric SO(10) from the simple assumption that SO(10) *is broken to the standard model by the smallest possible set of vacuum expectation values* (VEVs). This structure reproduces many of the features of the known fermion mass spectrum. It also predicts a large value for the $\nu_\mu - \nu_\tau$ mixing angle, as is suggested by the atmospheric neutrino data [1]. Usually this angle is small (or not predicted) in grand unified models, but in the present model its large value has a simple group-theoretical explanation.

The smallest set of vacuum expectation values that can break SO(10) to the standard model consists of one adjoint (**45**) and one pair of spinors ($\mathbf{16} + \overline{\mathbf{16}}$) [2]. The spinor pair breaks the rank of the group from 5 to 4 and provides superlarge masses for the right-handed neutrinos. The adjoint completes the breaking of SO(10) to the standard model (SM) group $SU(3) \times SU(2) \times U(1)$ and produces the "doublet-triplet splitting"—that is, gives superlarge mass to the color-triplet partners of the SM Higgs doublets, while leaving those doublets light.

Our assumption of minimality requires that there is only **one** adjoint Higgs. It has recently been shown that this is enough to break SO(10) with no fine-tuning, while preserving gauge-coupling unification [3]. Besides its economy, having only one adjoint seems to be desirable in the context of perturbative heterotic string theory where there are limitations on multiple adjoints [4]. If there is only one adjoint, its VEV is fixed to be in the *B-L* direction, as required by the Dimopoulos-Wilczek mechanism for doublet-triplet splitting [3,5]. This severely constrains the possibilities for constructing realistic quark and lepton masses. (For other approaches that generate mass matrix textures in SO(10) utilizing an extended Higgs sector see [6].)

In "minimal SO(10)" the quark and lepton masses come from the operators $\mathbf{16}_i \mathbf{16}_j \mathbf{10}_H$, where i and j are family indices and subscript H denotes a Higgs field. This leads to the "naive SO(10) relations": $N = U \propto D = L$. Here U, D, L, and N denote, respectively, the Dirac mass matrices for the up quarks, down quarks, charged leptons, and neutrinos. $U \propto D$ would imply vanishing Cabibbo-Kobayashi-Maskawa angles and $m_c^0/m_t^0 = m_s^0/m_b^0$, which is off by 1 order of magnitude. (Superscript zero refers to parameters evaluated at the unification scale.) The weaker "naive SU(5) relation" $D = L^T$ would imply $m_s^0 = m_\mu^0$, and $m_d^0 = m_e^0$. All these bad predictions can be avoided if the quark and lepton mass matrices depend on both superlarge SO(10)-breaking VEVs, $\langle \mathbf{16}_H \rangle$ and $\langle \mathbf{45}_H \rangle$. [The latter breaks SU(5) as well.] Empirically, one finds the so-called Georgi-Jarlskog relations [7], $m_s^0 \cong m_\mu^0/3$ and $m_d^0 \cong 3m_e^0$. Since $\langle \mathbf{45}_H \rangle \propto B - L$, a natural explanation of the Georgi-Jarlskog factors of 3 and $1/3$ will emerge.

The assumption of minimal VEVs for SO(10) breaking leads naturally, as will be seen, to the following forms for the quark and lepton mass matrices at the unification scale (with the convention that the left-handed fermions multiply them from the right, and the left-handed antifermions from the left):

$$U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} m, \qquad N = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} m,$$
$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \rho + \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} \tilde{m}, \qquad (1)$$
$$L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \rho + \epsilon & 1 \end{pmatrix} \tilde{m},$$

These matrices leave u, d, and e massless and are obviously not the whole story. At the end of this Letter, we will discuss extending the model to include the first generation. However, since $m_e \ll m_\mu$, $m_d \ll m_s$, and $m_u \ll m_c$, the effects of such first-generation physics should be quite small on the second and third generation

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parameters that we wish to fit. It turns out that with only two parameters, ϵ and ρ, one can get a good fit for five quantities that involve the second and third generations: m_c/m_t, m_s/m_b, m_μ/m_τ, m_b/m_τ, and V_{cb}. (There is also a phase parameter between ϵ and ρ, but it enters weakly in these quantities.) The other mass ratio, m_b/m_t, depends on an unknown ratio of VEVs.

The forms of the above Dirac matrices can be understood to arise naturally from the assumption of minimal VEVs. A heavy third generation with $m_b^0 \cong m_\tau^0$ naturally suggests the term $\mathbf{16}_3\mathbf{16}_3\mathbf{10}_H$, giving the "1" entries in Eq. (1). However, the second-generation masses must depend on $\langle \mathbf{45}_H \rangle$ because of the Georgi-Jarlskog factors. The simplest such operators are of the form $\mathbf{16}_2\mathbf{16}_3\mathbf{10}_H\mathbf{45}_H$. This yields the "$\epsilon$" entries in (1). Both the antisymmetry of these entries and the factors of $1/3$ are consequences of $\langle \mathbf{45}_H \rangle \propto B - L$.

With only these "1" and "ϵ" entries, U would be proportional to D, implying $V_{cb} = 0$ and $m_c^0/m_t^0 = m_s^0/m_b^0$ (which is off by an order of magnitude), and m_s^0/m_b^0 would be $\cong 1/9$ instead of $\cong 1/3$ of m_μ^0/m_τ^0.

All three of these unrealistic features are cured in a single stroke by introducing the group-theoretically simplest dimension-4 operator involving the $\langle \mathbf{16}_H \rangle$, i.e., $\mathbf{16}_2\mathbf{16}_3\mathbf{16}_H\mathbf{16}'_H$. The $\mathbf{16}'_H$ is some spinor Higgs, distinct from $\mathbf{16}_H$, which breaks the electroweak symmetry but does *not* participate in the breaking of SO(10) down to the standard model group [8]. That is, the components that get VEVs are $\mathbf{1}(\mathbf{16}_H)$ and $\overline{\mathbf{5}}(\mathbf{16}'_H)$, where $\mathbf{p}(\mathbf{q})$ denotes a $\mathbf{p}$ of SU(5) contained in a $\mathbf{q}$ of SO(10).

This term arises most naturally from "integrating out" $\mathbf{10}$'s of SO(10), as shown in Fig. 1. The resulting operator is $\overline{\mathbf{5}}(\mathbf{16}_2)\mathbf{10}(\mathbf{16}_3)\langle\overline{\mathbf{5}}(\mathbf{16}'_H)\rangle\langle\mathbf{1}(\mathbf{16}_H)\rangle$. Note that this contributes to L and D, but not to U and N, and that it lopsidedly contributes to D_{23} and L_{32} but not to D_{32} and L_{23}. This lopsidedness, which is the group-theoretical origin of the "ρ" entries in Eq. (1), will explain why the 2–3 mixing is small for the quarks ($V_{cb} \ll 1$) but large for the leptons ($\sin^2 2\theta_{\mu\tau} \sim 1$), provided $\rho \gg \epsilon$.

There can be a relative phase, α, between the parameters ϵ and ρ. As is apparent from Eq. (1), this phase only enters at order ϵ/ρ, which will be seen to be a small parameter. (Henceforth the symbols ρ and ϵ will denote $|\rho|$ and $|\epsilon|$, and the phase will appear explicitly as α.)

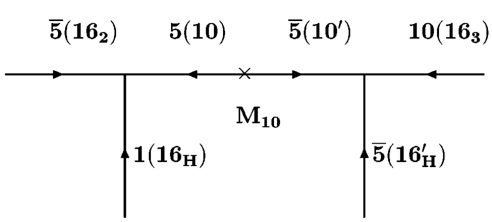

FIG. 1. A diagram that shows how vectors of fermions may be integrated out to produce the "ρ" terms in the mass matrices in (1). For group-theoretical reasons these produce lopsided contributions to the charged-lepton and down-quark mass matrices that explain why V_{cb} is small while $\sin^2 2\theta_{\mu\tau}$ is large.

Diagonalizing the matrices in (1), one finds

$$\begin{aligned}
m_b^0/m_\tau^0 &\cong 1 - \tfrac{2}{3}\,\tfrac{\rho}{\rho^2+1}\,(\epsilon\cos\alpha)\,,\\
m_\mu^0/m_\tau^0 &\cong \epsilon\,\tfrac{\rho}{\rho^2+1}\Big[1 - \tfrac{\rho^2-1}{\rho(\rho^2+1)}\,(\epsilon\cos\alpha)\Big],\\
m_s^0/m_b^0 &\cong \tfrac{1}{3}\,\epsilon\,\tfrac{\rho}{\rho^2+1}\Big[1 - \tfrac{1}{3}\,\tfrac{\rho^2-1}{\rho(\rho^2+1)}\,(\epsilon\cos\alpha)\Big], \qquad (2)\\
m_c^0/m_t^0 &\cong \epsilon^2/9\,,\\
V_{cb}^0 &\cong \tfrac{1}{3}\,\epsilon\,\tfrac{\rho^2}{\rho^2+1}\Big[1 + \tfrac{2}{3}\,\tfrac{1}{\rho(\rho^2+1)}\,(\epsilon\cos\alpha)\Big],
\end{aligned}$$

where $O(\epsilon^2)$ terms have been dropped, since they affect the results at the fraction of a percent level. The following features of the observed masses and mixings have been reproduced by the model: that $m_b^0 \cong m_\tau^0$; that V_{cb}^0, m_μ^0/m_τ^0, and m_s^0/m_b^0 are all of the same order $[O(\epsilon)]$, whereas m_c^0/m_t^0 is much smaller $[O(\epsilon^2)]$; and that $m_s^0/m_b^0 \cong \frac{1}{3}\,m_\mu^0/m_\tau^0$. Also explained is the hierarchy among generations, which arises from the smallness of ϵ and from the rank-2 nature of the matrices.

Since there are five observables in terms of the two parameters ϵ and ρ in Eq. (2) (noting that the dependence on α is rather weak), the model predicts three relations among the charged fermions. To study them we use the following input parameters: $m_\mu = 105.66$ MeV, $m_\tau = 1.777$ GeV, $m_s(1\text{ GeV}) = (180 \pm 50)$ MeV, $m_b(m_b) = (4.26 \pm 0.11)$ GeV, $m_c(m_c) = (1.27 \pm 0.1)$ GeV [9], $V_{cb} = 0.0395 \pm 0.0017$, and $M_t = 174.1 \pm 5.4$ GeV [10], which corresponds to $m_t(m_t) = 165 \pm 5$ GeV.

Various renormalization factors are needed. Those that run the masses from the low scales up to the supersymmetry scale, M_{SUSY} (taken to be at m_t), are denoted η_i and computed using 3-loop QCD and 1-loop QED or electroweak renormalization group equations (RGE), with inputs $\alpha_s(M_Z) = 0.118$, $\alpha(M_Z) = 1/127.9$, and $\sin^2\theta_W(M_Z) = 0.2315$. The relevant RGE can be found, for instance, in [11]. The results are $(\eta_\mu, \eta_\tau, \eta_s, \eta_b, \eta_c, \eta_t) = (0.982, 0.984, 0.426, 0.654, 0.473, 1.0)$.

The renormalization factors from M_{SUSY} up to the unification scale M_G are calculated using the 2-loop MSSM beta functions for all parameters [11], with $M_G = 2 \times 10^{16}$ GeV, and all SUSY thresholds taken to be at M_{SUSY}. These factors also depend on the value of $\tan\beta$, allowed *a priori* (by the perturbativity of the Yukawa couplings up to M_G) to be in the range $1.5 \le \tan\beta \le 65$, but favored by the fit in this model to be between about 10 and 40. We use $\tan\beta = 30$. [In this model, since the light doublet, H' is a linear combination of $\overline{\mathbf{5}}(\mathbf{10})$ and $\overline{\mathbf{5}}(\mathbf{16}')$, $\tan\beta \ne m_t/m_b$. It is also not expected to be very small, since the same Yukawa coupling contributes to both the top and the bottom quark masses.] The running factors for $\tan\beta = 30$ are $(\eta_{\mu/\tau}, \eta_{s/b}, \eta_{c/t}, \eta_{b/\tau}, \eta_{cb}) = (0.956, 0.840, 0.691, 0.514, 0.873)$, where $\eta_{i/j} \equiv (m_i^0/m_j^0)/(m_i/m_j)_{M_{\text{SUSY}}}$, and $\eta_{cb} \equiv V_{cb}^0/(V_{cb})_{M_{\text{SUSY}}}$.

Aside from the running of the couplings described by the η's, there are finite corrections [12] to m_s, m_b, and V_{cb} from gluino and chargino loops, which are proportional to $\tan\beta$ and thus sizable for moderate to large $\tan\beta$. These will be denoted by the factors $(1 + \Delta_s)$, $(1 + \Delta_b)$, and $(1 + \Delta_{cb})$, which depend on the supersymmetric spectrum: $\Delta_b \simeq \tan\beta\{\frac{2\alpha_3}{3\pi}\frac{\mu M_{\tilde g}}{m^2_{\tilde b_L} - m^2_{\tilde b_R}}[f(m^2_{\tilde b_L}/M^2_{\tilde g}) - f(m^2_{\tilde b_R}/M^2_{\tilde g})] + \frac{\lambda_t^2}{16\pi^2}\frac{\mu A_t}{m^2_{\tilde t_L} - m^2_{\tilde t_R}}[f(m^2_{\tilde t_L}/\mu^2) - f(m^2_{\tilde t_R}/\mu^2)]\}$, where $f(x) \equiv x\log(x)/(1-x)$. Δ_s is given by the same expression but without the chargino contribution (the second term) and with $\tilde b \rightarrow \tilde s$. $\Delta_{cb} = -\Delta_b^{\text{chargino}}$. One sees that even for $\tan\beta \approx 10$ these corrections are of order 10%. The analogous corrections to m_μ and m_τ arise only from Bino loops, while those to m_c and m_t lack the $\tan\beta$ enhancement, and so these are all negligible.

To fit for ρ and ϵ it is convenient to use the second and fifth relations of Eq. (2), since there is very little experimental uncertainty in m_μ, m_τ, and V_{cb}. This gives $\rho = [3V_{cb}/(m_\mu/m_\tau)](\frac{\eta_\tau\eta_{cb}}{\eta_\mu\eta_{\mu/\tau}})(1 - \frac{\epsilon\cos\alpha}{3}\frac{3\rho^2-1}{\rho(\rho^2+1)})(1 - \Delta_{cb})$, and $\epsilon = [\frac{\rho^2+1}{\rho}(m_\mu/m_\tau)](\frac{\eta_\mu\eta_{\mu/\tau}}{\eta_\tau})(1 + \epsilon\cos\alpha\frac{\rho^2-1}{\rho(\rho^2+1)})$. One finds, for $\cos\alpha = 1$, that

$$\rho = 1.73(1 - \Delta_{cb}), \qquad \epsilon = 0.136(1 - 0.5\Delta_{cb}). \quad (3)$$

The dependence on $\cos\alpha$, arising only at order ϵ/ρ, is rather weak: for $\cos\alpha = -1$, $\rho = 1.92\,(1 - \Delta_{cb})$, and $\epsilon = 0.134\,(1 - 0.5\Delta_{cb})$. The dependence on $\tan\beta$, because it is only through the renormalization factors, is also fairly weak for $10 \leq \tan\beta \leq 40$. Increasing $\tan\beta$ to 40 changes ρ by +0.7% and ϵ by −3%. Changing M_{SUSY} from m_t to 500 GeV changes ρ by +3% and ϵ by +2%.

Having determined ρ and ϵ from V_{cb} and m_μ/m_τ, one can predict m_b, m_s, m_c, and $\sin^2 2\theta_{\mu\tau}$.

(i) m_b prediction—The first relation of Eq. (2) implies $m_b(m_b) = m_\tau(m_\tau)(\frac{\eta_\tau}{\eta_b\eta_{b/\tau}})(1 - \frac{2}{3}\frac{\rho}{\rho^2+1}\epsilon\cos\alpha)(1 + \Delta_b)$. For $\cos\alpha = 1$, this gives $m_b(m_b) = 5.0\,(1 + \Delta_b)$ GeV. Comparing this with the experimental value 4.26 ± 0.11 GeV, one sees that $\Delta_b \cong -0.15$. This is quite a reasonable value if $\tan\beta \approx 30$. (With supergravity boundary conditions and a generic sparticle spectrum, the gluino loops contribute $\sim\pm0.2$ to Δ_b, while the charginos contribute roughly a quarter as much and with the opposite sign [13]. We shall keep these numbers as a rough guide to estimate the corrections.) It should be noted that if $\tan\beta$ is close to 1.6 or near 60, $m_b(m_b)$ will be in the acceptable range even with $\Delta_b = 0$. However, these extreme values of $\tan\beta$ lead to wrong predictions of the charm mass [$m_c(m_c) \simeq 1.57$ GeV when $\tan\beta \simeq 1.6$] and are thus disfavored within the model. An interesting consequence is that the model predicts the sign of μ (and A_t) to be such that it decreases the b quark mass through the gluino and chargino graphs.

(ii) m_s prediction—The first and third relations of Eq. (2) yield $m_s(1\text{ GeV}) = m_\tau(m_\tau)\frac{1}{3}\epsilon\frac{\rho}{\rho^2+1}(\frac{\eta_\tau}{\eta_s\eta_{s/b}\eta_{b/\tau}}) \times (1 - \frac{1}{3}\epsilon\cos\alpha\frac{3\rho^2-1}{\rho(\rho^2+1)})(1 + \Delta_s)$. For $\cos\alpha = 1$ this gives $m_s(1\text{ GeV}) = 176\,(1 + \Delta_s)$ MeV. Taking $\Delta_s \simeq \Delta_b \cong -0.15$ (justified if the gluino loop dominates and $m_{\tilde s} \cong m_{\tilde b}$), one finds $m_s(1\text{ GeV}) = 150$ MeV, in excellent agreement with the experimental value of 180 ± 50 MeV.

(iii) m_c prediction—The fourth relation of Eq. (2) implies $m_c(m_c) = m_t(m_t)\frac{1}{9}\epsilon^2(\frac{\eta_t}{\eta_c\eta_{c/t}})$. For $\cos\alpha = 1$, this gives $m_c = (1.05 \pm 0.11)(1 - \Delta_{cb})$ GeV. The error reflects the 1σ uncertainties in the experimental values of m_t, α_s (= 0.118 ± 0.004), and V_{cb}. [These lead, respectively, to 6.5%, 7%, and 4% uncertainties for $m_c(m_c)$. It should also be noted that changing $\tan\beta$ by ±10 changes the m_c prediction by ∓4%, changing M_{SUSY} to 500 GeV has less than a 2% effect, and changing $\cos\alpha$ to 0 reduces m_c by 3%.] Since $\Delta_{cb} \simeq -\Delta_b|^{\text{chargino}}$, it is reasonable to take $\Delta_{cb} \simeq -0.05$, using the supergravitylike spectrum as a guideline. This gives $m_c = 1.10 \pm 0.11$ GeV, which is in quite reasonable agreement with the experimental value $m_c(m_c) = (1.27 \pm 0.1)$ GeV. It is interesting that the sign of the correction term Δ_{cb} suggested by the supergravity spectrum is such that it improves the agreement of $m_c(m_c)$ with the experimental value.

(iv) $\sin^2 2\theta_{\mu\tau}$ prediction.—The neutrino-mixing matrix U_ν is defined by $\nu_f = \sum_m (U_\nu)_{fm}\nu_m$, where ν_f and ν_m are the flavor and mass eigenstates, respectively. $f = e, \mu, \tau$, and $m = 1, 2, 3$. $U_\nu = U^{(L)\dagger}U^{(N)}$, where $U^{(L)}$ and $U^{(N)}$ are the unitary transformations of the left-handed fermions required to diagonalize, respectively, L and $M_\nu = -N^T M_R^{-1} N$. (M_R is the superheavy Majorana mass matrix of the right-handed neutrinos.)

The crucial point, easily seen from Eq. (1), is that a large rotation in the 2–3 plane will be required to diagonalize L. Calling this angle $\theta_{23}^{(L)}$, $\tan\theta_{23}^{(L)} \cong |L_{32}/L_{33}| \cong \rho + \epsilon\cos\alpha$. The actual $\nu_\mu - \nu_\tau$ mixing angle is the difference between $\theta_{23}^{(L)}$ and the corresponding rotation angle, $\theta_{23}^{(N)}$, required to diagonalize M_ν.

It might appear that one can know nothing about M_ν, and therefore about $\theta_{23}^{(N)}$, without knowing the precise form of M_R. This is not so. From Eq. (1) one sees that in the limit $\epsilon \rightarrow 0$ both N and $M_\nu = -N^T M_R^{-1} N$ are proportional to diag(0, 0, 1), so that $\theta_{23}^{(N)} \rightarrow 0$. Thus, formally, $\theta_{23}^{(N)} = O(\epsilon)$. If M_R^{-1} is parametrized by $(M_R^{-1})_{22} = (M_R^{-1})_{33}Y/\epsilon^2$, and $(M_R^{-1})_{23} = (M_R^{-1})_{32} = (M_R^{-1})_{33}X/\epsilon$, one finds (ignoring the first generation) that $\tan 2\theta_{23}^{(N)} \cong 2\epsilon|(1 - X)/(1 - 2X + Y)|$. Unless X and Y are fine-tuned, this is indeed of order ϵ. Define κ by $\text{Re}(U_{23}^{(N)}) = \kappa\epsilon\cos\alpha$, in a phase convention where $U_{23}^{(L)}$ is real. If it is required that $m_{\nu_\mu}/m_{\nu_\tau} \approx 0.05$, as suggested by the atmospheric and solar neutrino data, then $|\kappa| \lesssim 2$. The $\mu - \tau$ mixing angle at the unification scale is then given by

$$\tan\theta_{\mu\tau} = \frac{\rho + (1 - \kappa)\epsilon\cos\alpha}{1 + \kappa\rho\epsilon\cos\alpha}. \quad (4)$$

The one-loop renormalization group equation for this quantity [14] has the simple form $d(\ln\tan\theta_{\mu\tau})/d(\ln\mu) = -h_\tau^2/16\pi^2$. For $\tan\beta = 30$, $\tan\theta_{\mu\tau} = 1.03\ \tan\theta^0_{\mu\tau}$.

Unlike the quark masses, the $\nu_\mu - \nu_\tau$ mixing angle is very sensitive to $\cos\alpha$, and therefore $\sin^2 2\theta_{\mu\tau}$ can be in a large range, from 1 down to about 1/4. Values >0.7 obtain for most of the parameter range. For example, if $\cos\alpha = 0$, Eq. (4) simplifies to $\tan\theta_{\mu\tau} = \rho$, giving $\sin^2 2\theta_{\mu\tau} = 0.78$, independent of κ. If $\kappa = 0$, then $\sin^2 2\theta_{\mu\tau} > 0.7$ for all $\cos\alpha$. $\sin^2 2\theta_{\mu\tau}$ reaches 1 for

$\cos\alpha = 1$ and $\kappa = 2$, and reaches $\approx 1/4$ for $\cos\alpha = 1$ and $\kappa = -2$. (For other ways to generate large $\nu_\mu - \nu_\tau$ mixing see Ref. [15].)

There is not a unique way to extend this model to include the first generation. A simple possibility that gives a reasonable fit to the first-generation masses and mixings is to add (12) and (21) entries symmetrically to all the mass matrices. This would give several new predictions: (i) $\frac{m_d^0}{m_e^0} = 3(1 + \frac{2}{3\rho}\epsilon\cos\alpha)$ (a Georgi-Jarlskog relation); (ii) $|V_{us}^0| = |\sqrt{m_d^0/m_s^0}\,\frac{1}{(\rho^2+1)^{1/4}} - \sqrt{m_u^0/m_c^0}\,e^{i\phi}|$; (iii) $|V_{ub}^0| \simeq |\sqrt{m_d^0/m_s^0}\,\frac{m_s^0}{m_b^0}\,\frac{\rho}{(\rho^2+1)^{1/4}} - \sqrt{m_u^0/m_c^0}\,e^{i\phi} \times (\sqrt{m_c^0/m_t^0} - \frac{m_s^0}{m_b^0}\frac{1}{\rho})|$. If ϕ, which is a phase parameter, is near π, acceptable $|V_{us}|$ and $|V_{ub}|$ result. The leptonic mixing angles involving the electron are given by $|(U_\nu)_{e\nu_2}^0| \simeq |\sqrt{m_e^0/m_\mu^0}\,(\rho^2 + 1)^{1/4} + O(\epsilon)|$, and $|(U_\nu)_{e\nu_3}^0| \simeq |\sqrt{m_e^0/m_\mu^0}\,\frac{m_\mu^0}{m_\tau^0}\,\frac{(\rho^2+1)^{3/4}}{\rho} + O(\epsilon^2)|$ where the $O(\epsilon)$ and $O(\epsilon^2)$ terms represent corrections from the neutrino sector. Since these mixing angles are both small, their precise values are sensitive to the structure of M_R. These values are consistent with the small angle matter oscillations for the solar neutrinos [10].

The model presented here can be tested in future experiments in several ways. (i) The prediction of $\tan\beta = 10-40$ can be tested once supersymmetric particles are discovered. (ii) The spectrum of the sparticles is predicted to be such that the gluino and the chargino corrections to m_b decrease its value by about 15%. (iii) More precise determinations of m_t, $\alpha_3(M_Z)$, and V_{cb} and information about the sparticle spectrum will sharpen the model's prediction of $m_c(m_c)$. (iv) Large angle $\nu_\mu - \nu_\tau$ oscillations should be seen in long baseline experiments, but not in the ongoing accelerator experiments. The interpretation of the atmospheric neutrino anomaly in terms of $\nu_\mu - \nu_\tau$ oscillations should be confirmed. (v) There are also predictions in the model for proton decay branching ratios [16] and rare decays such as $\mu \to e\gamma$ [17].

In this Letter we have studied a simple form for the mass matrices that is motivated by general group-theoretical considerations, without examining a particular underlying unified model in great detail. That has been done in [18], where it is found that mass matrices of the type discussed here can arise in realistic models.

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Note added.—After submission of this paper, Super-Kamiokande has reported unambiguous evidence for neutrino oscillations in their atmospheric neutrino data [19].

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[2] In order to break the gauge symmetry at the unification scale while preserving supersymmetry, it is generally necessary in SO(10) that for every **16** that has a superlarge VEV, there be a corresponding $\overline{\mathbf{16}}$ with an equal VEV.

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