Quantum Decoherence in Disordered Mesoscopic Systems

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We point out that the low temperature saturation of the electron phase decoherence time in a disordered conductor can be explained within the existing theory of weak localization provided the effect of quantum (high frequency) fluctuations is taken into account. Making use of the fluctuation-dissipation theorem we evaluate the quantum decoherence time, the crossover temperature below which thermal effects become unimportant, and the weak localization correction $\delta\sigma$ at $T = 0$. For 1D systems the latter is found to be $\delta\sigma/\sigma \propto 1/\sqrt{N}$, where N is the number of conducting channels. [S0031-9007(98)06762-3]

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Quantum interference between electrons has a strong impact on electron transport in a disordered metal, leading to the so-called weak localization correction to the system conductance [1]. This correction is large provided the electrons moving in the metal remain coherent. On the other hand, this phase coherence can persist only for a finite time and is eventually destroyed due to various processes, such as electron-electron and electron-phonon interactions, spin-flip scattering, etc. This characteristic decoherence time τ_φ plays a prominent role in the theory of weak localization [1,2].

In the absence of magnetic impurities and if the temperature of the system is sufficiently low, the decoherence time τ_φ is determined by electron-electron interactions. It was demonstrated in Ref. [3] (see also [2,4,5]) that for this dephasing mechanism the decoherence time increases with temperature as $\tau_\varphi \propto T^{2/(d-4)}$, d is the system dimension. This theoretical prediction was verified in the experiments [6,7] over a certain temperature interval.

Does the divergence of τ_φ in the zero temperature limit imply that coherence is not destroyed at $T = 0$? Recent experiments [8] clearly suggest a negative answer, indicating that at very low temperatures the time τ_φ saturates at a finite level showing no tendency for further increase with decreasing T. The authors [8,9] argued that this saturation is not caused by heating or magnetic impurities, but rather is a fundamental consequence of zero-point fluctuations of electrons. A saturation of τ_φ at low T was also observed in earlier works (see, e.g., [6,7]).

The aim of this paper is to demonstrate that the observed saturation of τ_φ at lowest temperatures [8] can be explained within the existing theory of weak localization [2] if one takes into account quantum fluctuations of the electric field in a disordered conductor.

We essentially follow the analysis elaborated by Chakravarty and Schmid [2] and consider the propagation of an electron with the kinetic energy $m\dot{\boldsymbol{r}}^2/2$ in a potential of randomly distributed impurities $U_{\rm imp}(\boldsymbol{r})$. In addition to that the electron interacts with other electrons and experiences two additional forces: (i) the damping force $\boldsymbol{F}$ due to electron-electron collisions and (ii) the stochastic force due to the fluctuating electric field $\boldsymbol{E}(\boldsymbol{r}, t) = -\nabla V(\boldsymbol{r}, t)$ produced by other electrons. These two forces are related to each other by the fluctuation-dissipation theorem (FDT) [10]. The force $\boldsymbol{F}[\boldsymbol{r}(t)]$ acting on the electron with the coordinate $\boldsymbol{r}(t)$ is given by the equation

$$\boldsymbol{F}[\boldsymbol{r}(t)] = \tanh\left(\frac{\xi}{2T}\right) e\nabla V_0(\boldsymbol{r}, t)|_{\boldsymbol{r}=\boldsymbol{r}(t)}\,, \tag{1}$$

where $\xi = m\dot{\boldsymbol{r}}^2/2 - \mu$, μ is the chemical potential, and the self-induced potential $V_0(\boldsymbol{r}, t)$ obeys the equation

$$\nabla(\hat{\boldsymbol{\epsilon}}\nabla V_0(t', \boldsymbol{r}')) = 4\pi e\delta(\boldsymbol{r} - \boldsymbol{r}(t))\,. \tag{2}$$

Here $\hat{\boldsymbol{\epsilon}}$ is the dielectric susceptibility operator. One can show [11] that the factor $\tanh(\xi/2T)$ in Eq. (1) appears due to the Pauli principle.

Let us express the propagating electron amplitude in terms of the Feynman path integral. Within the quasiclassical approximation (sufficient as long as the elastic mean free path l exceeds the Fermi wavelength $p_F l \gg 1$) the path integral can be replaced by the sum over the classical trajectories obeying the equation of motion

$$m\ddot{\boldsymbol{r}} = -\nabla U_{\rm imp}(\boldsymbol{r}) + \boldsymbol{F}(\boldsymbol{r}(t)) - e\nabla V(\boldsymbol{r}, t) \tag{3}$$

for each realization of random potentials $U_{\rm imp}(\boldsymbol{r})$ and $V(\boldsymbol{r}, t)$. Averaging over disordered configurations of impurities [2] yields the effective picture of electron diffusion at the scales bigger than l.

Let us estimate the phase difference between a classical electron path $\boldsymbol{r}(t')$ and a time reversed path $\boldsymbol{r}(t - t')$ induced by the two last terms in Eq. (3). Considering the effect of the force $\boldsymbol{F}$ we find

$$\delta\varphi_{\boldsymbol{F}} = -e\tanh\frac{\xi}{2T}\int_0^t dt'[V_0(\boldsymbol{r}(t')) - V_0(\boldsymbol{r}(t - t'))]\,. \tag{4}$$

Since the kernel of the operator $\hat{\boldsymbol{\epsilon}}$ is symmetric with respect to its spacial arguments, one can easily observe that $\delta\varphi_F$ is identically zero provided V_0 obeys Eq. (2).

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The phase difference due to the stochastic potential $V(\boldsymbol{r},t)$ is, on the contrary, nonzero. It is defined as

$$\delta\varphi(\boldsymbol{r},t) = -e\int_0^t dt'[V(\boldsymbol{r}(t'),t') - V(\boldsymbol{r}(t-t'),t')]. \tag{5}$$

Averaging with respect to fluctuations of V, for not very small t one gets [2]

$$\langle[\delta\varphi(\boldsymbol{r},t)]^2\rangle/2 = t/\tau_\varphi(T), \tag{6}$$

where

$$\frac{1}{\tau_\varphi(T)} = \frac{e^2}{a^{3-d}}\int dt\int\frac{d\omega\, d^dq}{(2\pi)^{d+1}}\langle|V_{q,\omega}|^2\rangle e^{-Dq^2|t|-i\omega t}, \tag{7}$$

a is the film thickness for $d=2$, and $a^2 = s$ is the wire cross section for $d=1$.

The correlation function for voltages in (7) can be determined with the aid of FDT [10]. Let us first consider a quasi-1D conductor. In this case one finds

$$\frac{\langle|V_{q,\omega}|^2\rangle}{a^2} = \frac{\omega\coth(\frac{\omega}{2T})}{\frac{\omega^2C^2}{\sigma q^2} + \sigma q^2(1+\frac{CD}{\sigma})^2}. \tag{8}$$

Here $\sigma = 2e^2N_0Ds$ is the classical Drude conductance, D is the diffusion coefficient, and C is the capacitance of a linear conductor per unit length. In (8) we neglected retardation and skin effects which may become important only at very high frequencies. Substituting (8) into (7) and integrating over t and q we find

$$\frac{1}{\tau_\varphi(T)} = \frac{e^2\sqrt{2D}}{\sigma}\int_{1/\tau_\varphi}^{1/\tau_e}\frac{d\omega}{2\pi}\frac{\coth(\omega/2T)}{\sqrt{\omega}}. \tag{9}$$

The low frequency cutoff in the integral (9) is chosen in a standard manner [1–3], at high frequencies the integral is cut at $1/\tau_e = v_F/l$ because at even higher ω a diffusion approximation is inapplicable and Eq. (8) becomes incorrect. In Eq. (9) we made use of the condition $C \ll \sigma/D$ which is usually well satisfied (perhaps except for extremely thin wires) indicating the smallness of capacitive effects in our system. Equation (9) yields

$$\frac{1}{\tau_\varphi} = \frac{e^2}{\pi\sigma}\sqrt{\frac{2D}{\tau_e}}[2T\sqrt{\tau_e\tau_\varphi} + 1]. \tag{10}$$

The first term in the square brackets comes from the low frequency modes $\omega < T$, whereas the second term is due to high frequency ($\omega > T$) fluctuations of the electric field in a disordered conductor. At sufficiently high temperature the first term dominates and the usual expression [3] $\tau_\varphi \sim (\sigma/e^2D^{1/2}T)^{2/3}$ is recovered. As T is lowered the number of the low frequency modes decreases and eventually vanishes in the limit $T \to 0$. At $T \lesssim T_q \sim 1/\sqrt{\tau_\varphi\tau_e}$ the expression (10) is dominated by the second term and τ_φ saturates at the value

$$\tau_\varphi \approx \pi\sigma/e^2v_F. \tag{11}$$

The estimate for the crossover temperature T_q reads

$$T_q \sim ev_F/\sqrt{\sigma l}. \tag{12}$$

Making use of Eq. (11) it is also easy to find the weak localization correction $\delta\sigma$ to the Drude conductance in the limit $T=0$. For $T \lesssim T_q$ we obtain

$$\frac{\delta\sigma}{\sigma} = -\frac{e^2}{\pi\sigma}\sqrt{D\tau_\varphi} \approx -\frac{1}{p_Fs^{1/2}}, \tag{13}$$

i.e., $\delta\sigma \approx -\sigma/\sqrt{N}$, where $N \sim p_F^2s$ is the effective number of conducting channels in a 1D mesoscopic system.

For 2D and 3D systems the same analysis yields

$$\frac{1}{\tau_\varphi} = \frac{e^2}{4\pi\sigma\tau_e}[1 + 2T\tau_e\ln(T\tau_\varphi)], \quad \text{2D},$$
$$\frac{1}{\tau_\varphi} = \frac{e^2}{3\pi^2\sigma\sqrt{2D}\,\tau_e^{3/2}}[1 + 6(T\tau_e)^{3/2}], \quad \text{3D}, \tag{14}$$

where $\sigma = 2e^2N_0Da^{3-d}$ is the conductance of a d-dimensional system. The result (14) demonstrates that for 2D and 3D systems saturation of τ_φ is expected already at relatively high temperatures: the crossover temperature T_q is of the order of the inverse elastic time in the 3D case and $T_q \sim v_F/l\ln(p_F^2al)^2$ for a 2D system. The latter value agrees well with the experimental results [6].

Our analysis clearly demonstrates that—in accordance with previous considerations [3,12]—the decoherence time τ_φ is *entirely different* from the inelastic mean free time τ_i, which is known to become infinite at zero temperature for almost all processes, including electron-electron interaction. In order to find τ_i it is sufficient to proceed within the standard quasiclassical approach and to solve the kinetic equation for the electron distribution function. The collision integral in this equation contains the product of the occupation numbers for different energy levels $n_k(1-n_q)$, which vanishes at $T \to 0$ due to the Pauli principle. Hence, τ_i becomes infinite at $T=0$.

In terms of the path integral analysis this procedure amounts to expanding the electron effective action on the Keldysh contour in the parameter $\boldsymbol{r}_-(t') = \boldsymbol{r}_1(t') - \boldsymbol{r}_2(t')$ assuming this parameter to be small [$\boldsymbol{r}_{1(2)}(t')$ is the electron coordinate on the forward (backward) part of the Keldysh contour]. The quasiclassical equation (3) for the "center of mass" $\boldsymbol{r}_+(t') = [\boldsymbol{r}_1(t') + \boldsymbol{r}_2(t')]/2$ follows from this procedure. Both forces $\boldsymbol{F}$ and $-e\nabla V$ are important for such classical paths, and the combination $\coth(\omega/2T) + \tanh[(\xi-\omega)/2T]$ [from Eqs. (1) and (8)] enters the expression for the inelastic time τ_i (see, e.g., [5]). For a detailed calculation of the inelastic scattering time in various limits we refer the reader to Ref. [12].

The above quasiclassical procedure is formally very different from one used to calculate the weak localization correction to conductivity. In the latter case the time reversed paths $\boldsymbol{r}_1(t')$ and $\boldsymbol{r}_2(t - t')$ are assumed to be close to each other [2], whereas $\boldsymbol{r}_-(t')$ can be arbitrarily large. This formal difference is just an illustration of the well know fact, that weak localization is an essentially quantum phenomenon. In this case the contribution of the force $\boldsymbol{F}$, containing $\tanh(\xi/2T)$, is zero as it was discussed above. Therefore, the quasiclassical kinetic analysis of τ_i in terms of the collision integral is principally insufficient for calculation of the decoherence time τ_φ.

We would like to emphasize that our results are obtained within the standard theoretical treatment of weak localization effects [2] combined with FDT. One can elaborate a more general real time analysis based on the Keldysh technique [11]. Starting from the microscopic Hamiltonian for electrons in a disordered metal with Coulomb interaction, one can introduce two quantum fields V_1, and V_2, respectively, on the forward and backward parts of the Keldysh contour by means of a standard Hubbard-Stratonovich transformation. The initial problem is then mapped onto that of a single electron propagating in a random potential of impurities in a metal and interacting with an effective fluctuating matrix external field $V_{ij}(\boldsymbol{r}, t) = V_i(\boldsymbol{r}, t)\delta_{ij}$ $(i, j = 1, 2)$ which is in turn produced by fluctuating electrons. One can show [11] that the effective density matrix of this electron $\rho_V(t, \boldsymbol{r}_1, r_2)$ obeys the *exact* equation of motion

$$i\,\frac{\partial \rho_V}{\partial t} = [H_0 - eV^+, \rho_V] - (1 - \rho_V)\,\frac{eV^-}{2}\,\rho_V - \rho_V\,\frac{eV^-}{2}\,(1 - \rho_V), \tag{15}$$

where H_0 is the exact Hamiltonian for an electron in a metal in the presence of impurities but in the absence of the electron-electron interaction, $V^+ = (V_1 + V_2)/2$ and $V^- = V_1 - V_2$. The single electron density matrix ρ in the presence of interactions is derived by averaging over the above stochastic fields $\rho = \langle \rho_V \rangle_{V^+, V^-}$, and the correlators for these fields are determined by the influence functional obtained by integrating out all electron degrees of freedom [11]. Within this approach one arrives at the same results [11] as those obtained here.

Equation (15) allows for a simple understanding of the role of the Pauli principle in our problem. One can show [11] that fluctuations of the field V^- are responsible for collision-induced damping described by the force $\boldsymbol{F}$ in Eq. (3). The field V^- is obviously sensitive to the Pauli principle, and the factor $\tanh(\xi/2T)$ appears in Eq. (1).

In contrast to V^-, fluctuations of the field V^+, which just coincides with the quasiclassical potential V in Eq. (3), are not sensitive to the Pauli principle. It is obvious from Eq. (15) that $V^+(t, \boldsymbol{r})$ is equivalent to an external potential. All electrons "feel" the same fluctuating field V^+, and the Pauli principle plays no role in this process. This effect causes quantum decoherence which persists down to $T = 0$.

The existence of a nonzero electron dephasing rate at $T = 0$ has a transparent physical interpretation. Let us represent the fluctuating field V as a collection of oscillators with different frequencies, all being in the ground state at $T = 0$. The interference contribution to the return probability for an electron interacting with *one* oscillator with a frequency ω oscillates in time and is smaller than one for all time moments except $t = 2\pi n/\omega$ when the system returns to its initial state. In the case of *infinitely many* oscillators with a continuous distribution of frequencies the electron will *never* return exactly to its initial state. At $T = 0$ the interference contribution will be always suppressed by a factor $\exp(-\eta r^2)$, where η depends on the interaction strength and the spectrum of oscillators and $r(t)$ is the size of the return path. For an electron in a diffusive metal $r(t)$ grows with time as $r \sim \sqrt{Dt}$, and the interference contribution to the return probability will decay as $\propto \exp(-\eta Dt)$. This is the effect of quantum dephasing at $T = 0$.

Note that this effect is qualitatively different from that discussed in Ref. [13] where it was argued that zero-point motion of impurities may cause dephasing at $T = 0$. Later it was pointed out in Ref. [14]—and we fully agree with this statement [14]—that *purely elastic* scattering considered in Ref. [13] cannot cause quantum decoherence. In contrast, in our case the energy exchange between the electron and the field oscillators is possible even at $T = 0$: in the presence of interaction none of them is in its "noninteracting" eigenstate, the ground state levels get broadened, and the energy can be exchanged *without excitation* of the field oscillators.

The saturation of the dephasing rate at low T was recently discussed by the authors [9] who started from the framework similar to that of Refs. [2,4] and the present paper. However, in contrast to our analysis, the calculation [9] involves integration over 2D (not 1D) wave vector and a phenomenologically chosen high frequency cutoff. As a result the authors [9] arrived at the estimate for the decoherence length $L_\varphi \sim N_{\rm ch}/p_F$ which does not contain the elastic mean free path l. This result is not correct. Also the role of the Pauli principle in the effect of quantum dephasing was not clarified in Ref. [9].

Let us emphasize that the saturation of τ_φ at low T might not necessarily indicate the failure of the Fermi liquid hypotheses for disordered metals at $T \lesssim 1/\tau_\varphi$ (although it does not support this hypothesis either). It is because τ_φ is the dephasing time for *real electrons* and not for Landau quasiparticles. In a disordered metal interacting electrons are "bad" particles: their wave functions dephase even at $T = 0$. The possibility to construct "better" quasiparticles is questionable in this case, but anyway they are not needed within our analysis which allows one to directly calculate the

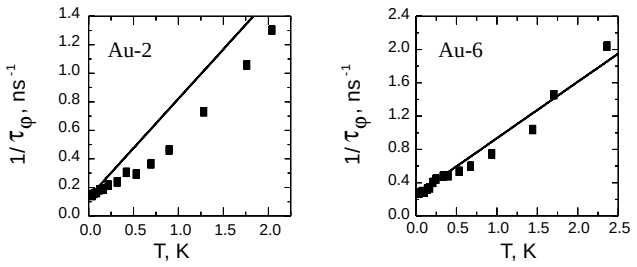

FIG. 1. The temperature dependent decoherence rate (10) (solid curves) plotted together with the experimental data for two 1D gold wires (Au-2 and Au-6) [8] (squares).

physically measurable quantities in terms of interacting electrons.

The results of our analysis agree sufficiently well with the experimental findings [8]. The corresponding comparison of our Eq. (10) with the experimental data for two gold wires (Au-2 and Au-6) [8] at low T is presented in Fig. 1. This agreement was achieved with *no fitting parameters* and is even better than one could expect within the accuracy of the cutoff procedure used in Eq. (9). The value of the decoherence length $L_\varphi = \sqrt{D\tau_\varphi}$ measured for the samples Au-3 and Au-4 (Au-1 and Au-5) [8] is 1.5–2 times (respectively, $\sim$3 times) bigger than one obtained from (10), i.e., the agreement is reasonable also for the remaining four samples [8]. Our results both for the magnitude and the temperature dependence of τ_φ also agree well with earlier experimental data [6,7]. Furthermore, in 1D wires the scaling $L_\varphi = \sqrt{D\tau_\varphi} \propto \sqrt{\sigma}$ was observed in [7]. Similarly, in [6] the linear dependence of $1/\tau_\varphi$ on the sheet resistance of 2D films was found. These observations are also consistent with our Eqs. (11) and (14).

Finally, we would like to point out that the low temperature saturation of τ_φ should cause *dramatic* consequences for the existing picture of strong localization in low dimensional metals [15,16]. Our results demonstrate that for typical metals the effective decoherence length L_φ is *always* smaller than the localization length $L_{\rm loc}$; e.g., in 1D we have $L_\varphi \sim l\sqrt{N_{\rm ch}} \ll L_{\rm loc} \sim lN_{\rm ch}$ for $N_{\rm ch} \gg 1$. This implies that localization should remain "weak" at all T, and the 1D and 2D metals (at least for $p_F l \gg 1$) *do not become insulators* even at $T = 0$ because of the effect of electron-electron interaction.

In conclusion, we point out that the low temperature saturation of the electron decoherence time found in recent experiments with mesoscopic conductors can be explained within the existing theory of weak localization provided the effect of intrinsic quantum fluctuations of the electric field is properly accounted for. Our results agree well with the experimental data.

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