Channel Drop Tunneling through Localized States

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We present a general analysis of the tunneling process through localized resonant states between one-dimensional continuums. We show that complete transfer can occur between the continuums by creating resonant states of different symmetry, and by forcing an *accidental* degeneracy between them. The degeneracy must exist in both the real and imaginary parts of the frequency. We illustrate the results of the analysis by performing computational simulations on the transport properties of electromagnetic waves in a two-dimensional photonic crystal. [S0031-9007(97)05091-6]

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Resonant tunneling processes can occur between states when they interact through a coupling element which supports localized resonances. Of particular interest is the complete channel drop tunneling between one-dimensional continuums, i.e., the selective transfer of a single propagating state (i.e., monoenergy electron, or single-frequency photon) from one continuum to the other, leaving all other states unaffected. Examples include the transfer of states between electron waveguides [1,2] through a quantum dot device, and the transfer of photonic states between dielectric waveguides through an optical resonator system. Such transfer processes are important for single-energy electron spectroscopy or wavelength demultiplexing in optical communication systems [3,4]. However, to our knowledge, the general conditions needed to realize optimal transfer until now have not been recognized.

In this Letter, we determine the general characteristics of the coupling element required to achieve complete channel drop tunneling. We begin by presenting a qualitative analysis using symmetry and energy conservation arguments which identifies the important ingredients needed in constructing an analytical theory. Using a rigorous mathematical formalism, we then demonstrate that complete transfer can occur by creating resonant states of different symmetry, and by forcing an *accidental* degeneracy of both the real and imaginary parts of the frequency between the resonant states. We illustrate the results of the analysis by simulating the transport properties of electromagnetic waves in a two-dimensional photonic crystal.

The schematic diagram of a generic coupled system is shown in Fig. 1. The two continuums are labeled C and $\overline{C}$. We consider the gedanken experiment where we excite a propagating state in C and study how it is affected by the coupling element. At resonance, the propagating state excites the resonant modes, which in turn decay into both continuums. The transmitted signal in C is made up of the input signal and the signal which originates from the decay of the localized states. In order to achieve complete transfer, these two components must be made to interfere destructively. The reflected amplitude in C, on the other hand, originates entirely from the decay of the localized states. Hence, at least two states are needed for the decaying amplitudes to cancel in the backward direction.

To ensure the cancellation of the reflected signal, we consider a structure with a mirror plane symmetry perpendicular to both C and $\overline{C}$, and assume that there exist two localized states with different symmetry with respect to the mirror plane, one even labeled $|e\rangle$, and one odd labeled $|o\rangle$. The incoming wave $\exp(ikx)$ can then be decomposed into the form $\cos(kx) + i\ \sin(kx)$, where x corresponds to the direction along both C and $\overline{C}$. The $\cos(kx)$ part, which is even with respect to the mirror plane, couples only to the even resonant state, and the $\sin(kx)$ part, which is odd, couples only to the odd state. In the specific case where the coupling constants and the frequencies are equal for both modes, a resonant state of the form $|e\rangle + i|o\rangle$ is excited, which in turn decays in C only along the forward direction. As a result, reflection is completely absent.

From conservation of energy, the state is completely transferred to $\overline{C}$ by eliminating both the reflection and the transmission in C. The amplitude of the transferred wave and that of the input wave therefore must be equal, which implies that the resonances must decay equally into

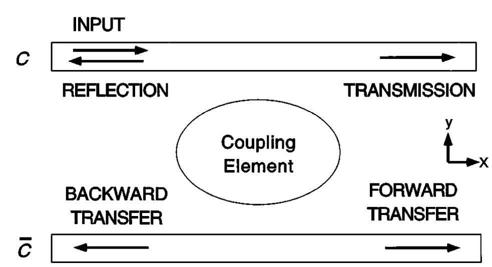

FIG. 1. Schematic diagram of two continuums coupled through an element which supports localized resonant states.

C and $\overline{C}$. This requirement can be satisfied by imposing an additional mirror-plane symmetry parallel to both C and $\overline{C}$ halfway between the two continuums.

Based on the qualitative arguments presented above we construct an analytical theory by considering a structure that possesses two mirror planes, one parallel to the continuums and one perpendicular. The structure also supports two resonant states of opposite symmetry with respect to the mirror plane perpendicular to C and $\overline{C}$. This structure can be characterized by the propagating modes in C and $\overline{C}$, which are labeled by their wave vectors k and $\overline{k}$, respectively, and by the even and odd resonant states which are labeled $|e\rangle$ and $|o\rangle$. The interaction between these states determines the transport properties of the structure. We can describe the interactions by a Hamiltonian H which can be written as the sum of four parts, namely $H = H_{\text{continuum}} + H_{\text{coupling-element}} + V_e + V_o$, where

$$H_{\text{continuum}} = \sum_k \omega(k)|k\rangle\langle k| + \sum_{\overline{k}} \omega(\overline{k})|\overline{k}\rangle\langle\overline{k}|, \qquad (1)$$

$$H_{\text{coupling-element}} = \omega_e|e\rangle\langle e| + \omega_o|o\rangle\langle o|, \qquad (2)$$

$$V_e = \sqrt{1/L}\sum_k[E(k)|e\rangle\langle k| + E^*(k)|k\rangle\langle e|] + \sum_{\overline{k}}[\overline{E}(\overline{k})|e\rangle\langle\overline{k}| + \overline{E}^*(\overline{k})|\overline{k}\rangle\langle e|], \qquad (3)$$

$$V_o = \sqrt{1/L}\sum_k[O(k)|o\rangle\langle k| + O^*(k)|k\rangle\langle o|] + \sum_{\overline{k}}[\overline{O}(\overline{k})|o\rangle\langle\overline{k}| + \overline{O}^*(\overline{k})|\overline{k}\rangle\langle o|], \qquad (4)$$

and $\omega(k)$ and $\omega(\overline{k})$ are the dispersion relations in C and $\overline{C}$, respectively. The coefficients $E(k)$, $O(k)$, $\overline{E}(\overline{k})$, $\overline{O}(\overline{k})$ are the coupling constants between the resonances and the propagating states. The $\sqrt{1/L}$ factor in Eqs. (3) and (4) arises from a box normalization of length L. Similar Hamiltonians have been used by Fano [5] and by Anderson [6] to describe the interaction between localized resonances and continuums in different contexts.

The coupling constants are not independent variables, but rather they are related to each other through the symmetry operations. We note that the Hamiltonian H is invariant with respect to the two mirror operators Π_x and Π_y, i.e.,

$$[\Pi_x, H] = 0, \qquad [\Pi_y, H] = 0, \qquad (5)$$

where Π_x is perpendicular to the continuums while Π_y is parallel. In addition, the states transform under the mirror operators according to

$$\Pi_x|k\rangle = |-k\rangle, \qquad \Pi_x|e\rangle = |e\rangle,$$
$$\Pi_x|o\rangle = -|o\rangle, \qquad \Pi_y|k\rangle = |\overline{k}\rangle. \qquad (6)$$

Here, the two states $|e\rangle$ and $|o\rangle$ are chosen to be even with respect to the mirror operator Π_y, i.e.,

$$\Pi_y|e\rangle = |e\rangle, \qquad \Pi_y|o\rangle = |o\rangle. \qquad (7)$$

With these conditions, it can easily be shown that the coupling constants satisfy the following constraints:

$$E(k) = E(-k), \quad \overline{E}(\overline{k}) = \overline{E}(-\overline{k}), \quad E(k) = \overline{E}(\overline{k}), \qquad (8)$$

$$O(k) = O(-k), \quad \overline{O}(\overline{k}) = \overline{O}(-\overline{k}), \quad O(k) = \overline{O}(\overline{k}). \qquad (9)$$

The symmetries of the structure also allow us to block diagonalize the Hamiltonian H using the following linear transformations:

$$|k_e\rangle = \frac{1}{\sqrt{2}}(|k\rangle + |-k\rangle), \quad |k_o\rangle = \frac{1}{\sqrt{2}i}(|k\rangle + |-k\rangle), \qquad (10)$$

$$|\overline{k}_e\rangle = \frac{1}{\sqrt{2}}(|\overline{k}\rangle + |-\overline{k}\rangle), \quad |\overline{k}_o\rangle = \frac{1}{\sqrt{2}i}(|\overline{k}\rangle + |-\overline{k}\rangle), \qquad (11)$$

The Hamiltonian H can then be written as the sum of two independent parts $H = H^e + H^o$, where H^e and H^o are defined as

$$\begin{aligned} H^e = &\sum_{\overline{k}_e>0} \omega(\overline{k}_e)|\overline{k}_e\rangle\langle\overline{k}_e| \\ &+ \sum_{k_e>0} \omega(k_e)|k_e\rangle\langle k_e| + \omega_e|e\rangle\langle e| \\ &+ \sum_{k_e>0}\sqrt{\frac{2}{L}}(E(k)|k_e\rangle\langle e| + E^*(k)|e\rangle\langle k_e|) \\ &+ \sum_{\overline{k}_e>0}\sqrt{\frac{2}{L}}(E(k)|\overline{k}_e\rangle\langle e| + E^*(k)|e\rangle\langle\overline{k}_e|) \end{aligned} \qquad (12)$$

$$\begin{aligned} H^o = &\sum_{\overline{k}_o>0} \omega(\overline{k}_o)|\overline{k}_o\rangle\langle\overline{k}_o| \\ &+ \sum_{k_o>0} \omega(k_o)|k_o\rangle\langle k_o| + \omega_o|o\rangle\langle o| \\ &+ \sum_{k_o>0}\sqrt{\frac{2}{L}}(iO(k)|k_o\rangle\langle o| - iO^*(k)|o\rangle\langle k_o|) \\ &+ \sum_{\overline{k}_o>0}\sqrt{\frac{2}{L}}(iO(k)|\overline{k}_o\rangle\langle o| - iO^*(k)|o\rangle\langle\overline{k}_o|). \end{aligned} \qquad (13)$$

Equations (12) and (13) describe two independent scattering processes. The T matrix of each process can be related to the single particle Green's function of the resonances using the standard techniques involving the Lippman-Schwinger formalism [7]. By summing the T matrices of the two processes, the scattering wave function can be obtained, which has the following asymptotic behavior: Transmitted amplitude ($x \gg 0$),

$$\psi(x) = \frac{1}{\sqrt{L}}\left[1 - \frac{1}{2}\left(\frac{i\nu_e}{\omega - \tilde{\omega}_e + i\nu_e}\right) - \frac{1}{2}\left(\frac{i\nu_o}{\omega - \tilde{\omega}_o + i\nu_o}\right)\right]e^{ikx}; \qquad (14)$$

reflected amplitude ($x \ll 0$),

$$\psi(x) = \frac{1}{\sqrt{L}}\left[-\frac{1}{2}\left(\frac{iv_e}{\omega - \tilde{\omega}_e + iv_e}\right) + \frac{1}{2}\left(\frac{iv_o}{\omega - \tilde{\omega}_o + iv_o}\right)\right]e^{ikx}; \quad (15)$$

transferred amplitude in the forward direction ($\overline{x} \gg 0$),

$$\psi(\overline{x}) = \frac{1}{\sqrt{L}}\left[-\frac{1}{2}\left(\frac{iv_e}{\omega - \tilde{\omega}_e + iv_e}\right) - \frac{1}{2}\left(\frac{iv_o}{\omega - \tilde{\omega}_o + iv_o}\right)\right]e^{ik\overline{x}}; \quad (16)$$

transferred amplitude in the backward direction ($\overline{x} \ll 0$),

$$\psi(\overline{x}) = \frac{1}{\sqrt{L}}\left[-\frac{1}{2}\left(\frac{iv_e}{\omega - \tilde{\omega}_e + iv_e}\right) + \frac{1}{2}\left(\frac{iv_o}{\omega - \tilde{\omega}_o + iv_o}\right)\right]e^{ik\overline{x}}; \quad (17)$$

where $\tilde{\omega}_e$ and $\tilde{\omega}_o$ are the "renormalized" frequencies of the resonator in the presence of the continuums, and v_e and v_o are the linewidths of the even and odd resonances, respectively. These four parameters are related to the dispersion relation of the guided modes and the coupling constants. Detailed calculations are presented elsewhere [8]. For the purpose of this discussion, it is sufficient to note that the transport properties of the structures depend only on the frequencies and the linewidths of the even and the odd resonances.

Of particular interest is the case where $\tilde{\omega}_e = \tilde{\omega}_o$ and $v_e = v_o$, which corresponds to the even and odd resonances having the same frequency and the same linewidth. Under these conditions it follows from Eqs. (15) and (17) that the reflected wave in C and the transferred wave to $\overline{C}$ along the backward direction vanish over the entire frequency range, while Eq. (16) reveals that the wave transferred to $\overline{C}$ along the forward direction has a Lorentzian line shape with a 100% transfer efficiency at resonance. This is consistent with the qualitative discussion presented above.

In general, the symmetry of the channel drop systems is low such that only one-dimensional irreducible representations are allowed. Hence, the even and odd resonances belong to different irreducible representations and an accidental degeneracy between the resonances must be forced.

To realize the results of the analytic theory, we consider the case of two photonic crystal waveguides and two coupled single-mode high-Q microcavities, as shown in Fig. 2. The photonic crystal is made of a square lattice of high-index dielectric rods with radius $0.20a$ and dielectric constant 11.56, where a is the lattice constant. The waveguides are formed by removing two rows of dielectric rods, and the cavities are introduced between the waveguides by reducing the radius of two rods. Each cavity supports a localized monopole state which is singly degenerate [9].

The even and odd states are made up of linear combinations of the two monopoles which are coupled indirectly through the waveguide and directly through the crystal.

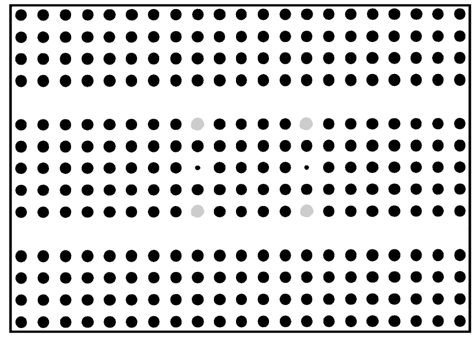

FIG. 2. Photonic crystal structure with two waveguides and two cavities. The black circles correspond to rods with a dielectric constant of 11.56, while the gray circles correspond to rods with a dielectric constant of 9.5. The two smaller rods have a dielectric constant of 6.6, and a radius of $0.05a$, where a is the lattice constant.

Each coupling mechanism splits the frequency of the even and odd states, but with opposite sign. An accidental degeneracy, caused by an exact cancellation between the two coupling mechanisms, is enforced by reducing the dielectric constant of four specific rods in the photonic crystal to 9.5, as shown in Fig. 2. The cancellation could equally have been accomplished by reducing the size of the rods instead of their dielectric constant.

Analytically, we can show that the quality factor of the two states can be made equal provided that the wave vector k of the guided mode satisfies the relation $kd = n\pi + \pi/2$, where d is the distance between the two defects, and n is an integer [8]. This condition can be reached by separating the two defects by five lattice constants, and by choosing the size and dielectric constant of the defect posts in such a way that the guided mode at the resonant frequency has a wave vector of $0.25(2\pi a^{-1})$.

We simulate the filter response of the structure shown in Fig. 2 using a finite-difference time-domain scheme [10] with perfectly matched layer absorbing boundary condition [11]. A pulse is sent down one of the waveguides and excites both the even and odd states. These two states then decay exponentially into the waveguides. By Fourier transforming the decaying amplitudes, we obtain the frequency spectrum of the even and odd states, each with a Lorentzian line shape, as shown in Fig. 3. The two line shapes overlap almost perfectly, as desired.

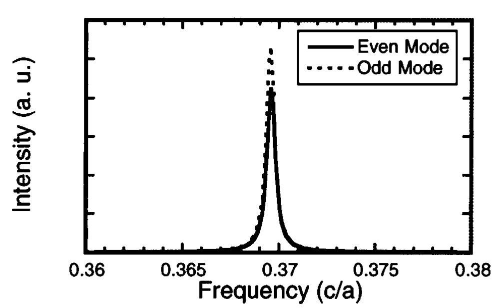

FIG. 3. Spectrum of the even and odd modes for the structure shown in Fig. 2.

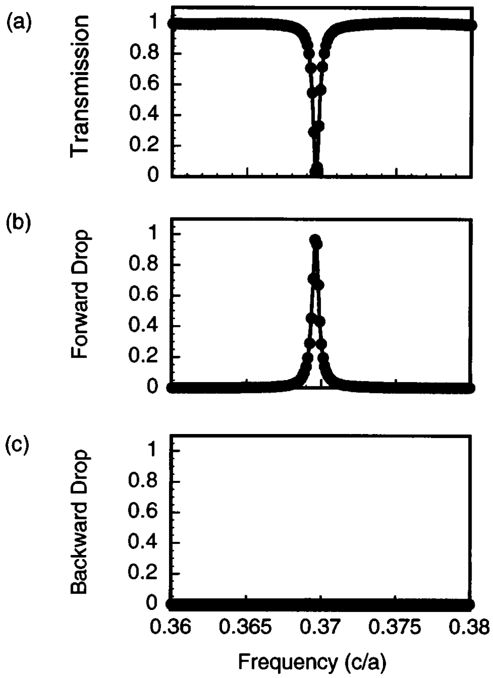

FIG. 4. (a) Intensity spectrum of the transmitted signal in the structure shown in Fig. 2. (b) Intensity spectrum of the transferred signal in the forward direction. (c) Intensity spectrum of the transferred signal in the backward direction. The solid dots are obtained from computer simulations. The lines result from analytical theory.

From the line shape, we can determine the widths and the frequencies of both resonances, and calculate analytically the spectrum of the transmitted signal and that of the transferred signals using Eqs. (14)–(17). These spectra are shown as solid lines in Fig. 4 and are compared to those obtained by Fourier transforming the computational data (solid circles). Excellent agreement is obtained between theory and simulation. Figure 4 shows that the transmission is close to 100% over the entire spectrum, except at the resonant frequency, where it drops to 0%. The forward transferred signal shows a Lorentzian line shape with a maximum close to 99% at resonance. The quality factor is larger than 1000, as seen in Fig. 4(b). The backward transferred signal is almost completely absent over the entire frequency range [Fig. 4(c)]. The steady-state field pattern at maximum transfer efficiency is shown in Fig. 5. The simulation does indeed demonstrate complete channel drop tunneling via localized states.

Instead of using two defects as the coupling element, each supporting a singly degenerate monopole state, we could also use a single defect that supports two modes with opposite symmetries. The defect could be introduced, for example, by increasing the radius of a single rod in the crystal [9]. The accidental degeneracy is then forced by changing the dielectric constant of the neighboring rods. Equal decay of the even and odd modes into the waveguides is best achieved with a cavity that supports photonic states with large orbital angular momentum. In a structure that supports defect modes with hexapole char-

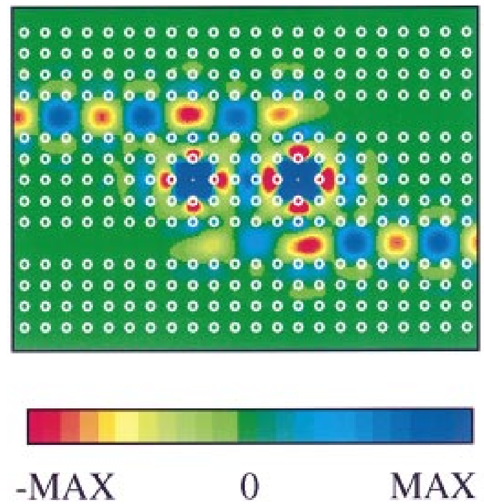

FIG. 5(color). Electric field pattern of the structure shown in Fig. 2 at the resonant frequency. The white circles indicate the position of the rods.

acteristics, for example, complete transfer with the quality factor exceeding 6000 can be achieved when the defects are separated from the center of the waveguides by three lattice constants [8]. We also note that, in this case, the two resonances have *opposite* symmetry properties with respect to the mirror plane *parallel* to the waveguides. Consequently, when the two resonant peaks coincide, the transferred signal propagates along the backward direction instead of the forward direction.

In summary, we have presented the criteria for complete transfer between continuums through localized states. We have demonstrated these criteria by simulating the propagation of electromagnetic waves in a two-dimensional photonic crystal.

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