Anomalous Behavior above the Melting Point of Two-Dimensional Electrons on Liquid Helium

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(Received 2 June 1997)

Anomalous fluctuations in magnetoconductance are observed above the melting temperature T_m of two-dimensional electrons on liquid ^{4}He. In the 2D solid the low frequency Corbino magnetoconductivity $\sigma(B)$ is increased by coherent Bragg-Cerenkov coupling to ripplons on the helium surface. Above T_m (plasma parameter $\Gamma = 127$), fluctuations in $\sigma(B)$ suggest a microstructure with significant ordered and coupled regions. The fluctuations decrease continuously with temperature up to $T_f = (2.8 \pm 0.3)T_m$ at $\Gamma = 46 \pm 5$. Fluctuations also occur in the 2D electron solid at high drive amplitudes. [S0031-9007(97)05054-0]

PACS numbers: 73.20.Dx, 67.90.+z, 73.50.Jt

There is still no consensus on the mechanism for the melting of two-dimensional (2D) crystals, or even on the order of the transition. The ground state of the classical crystal is a triangular lattice with true long-range bond orientational order but only quasi-long-range translational order [1]. The most interesting theory of 2D melting (by Kosterlitz and Thouless, developed by Halperin and Nelson, and Young, the KTHNY theory [2]) predicts two second-order transitions, first to the hexatic phase by the unbinding of dislocation pairs (crystal melting at a temperature T_m) and then to an isotropic liquid by the unbinding of disclination pairs (a free dislocation is a bound disclination pair) at the hexatic phase instability at T_i. The hexatic phase has quasi-long-range orientational order but only short-range translational order and zero shear modulus. Other mechanisms for melting, such as grain-boundary melting [3] and rotational stiffness [4], would lead to a first-order transition from a solid to an isotropic liquid. The range of the 2D interaction potential is important [5,6]. For a hard-core potential, melting occurs via a weakly first-order transition, as seen in experiments and computer simulations on short-range potentials [3,7]. Longer range potentials, such as colloidal systems [8] with screened Coulomb or dipole interactions, melt into an intermediate, probably hexatic, phase. Monte Carlo calculations on Lennard-Jones and $1/r^6$ potentials [9] show a region above T_m with the algebraic decay of orientational order expected for the hexatic phase. So the hexatic phase should exist for the softer unscreened $1/r$ Coulomb potential. A classical 2D electron crystal melts at $T_m = 0.227 \times 10^{-6} n^{1/2}$ at $\Gamma = \Gamma_m = 127 \pm 3$ [the plasma parameter $\Gamma = e^2(\pi n)^{1/2}/4\pi\varepsilon_0 kT$ is the ratio of the potential and kinetic energies, n m^{-2} is the electron density]. Density functional theory [6] suggests that T_i lies in the range $104.85 > \Gamma > 24.5$, depending on the (unknown) core energy of the disclinations. Computer simulations are contradictory, showing a hexatic phase [10] or a weak first-order transition [11]. Two-dimensional electrons above liquid helium [12] are an ideal experimental model system. Experiments [13,14] are consistent with the KTHNY theory, which gives the shear modulus at T_m. But there have been no direct observations of the hexatic phase in this system. We present new experiments on 2D electrons on liquid helium which show that some aspects of the 2D crystal persist well above T_m to a temperature $T_f = (2.8 \pm 0.3)T_m$ at $\Gamma_f = 46 \pm 5$. Between T_m and T_f we observe fluctuations and nonlinearities in the magnetoconductivity which suggest significant ordered regions.

The magnetoconductivity $\sigma(B)$ in a perpendicular field B was measured using a 4 mm diameter Corbino disk [15] (Fig. 1) a distance d (typically 100 μm) beneath the helium surface. A central electrode A is surrounded by a ring E which separates the receiving electrodes $B1$, $B2$, and $B3$. Round these is a planar guard G. An ac voltage

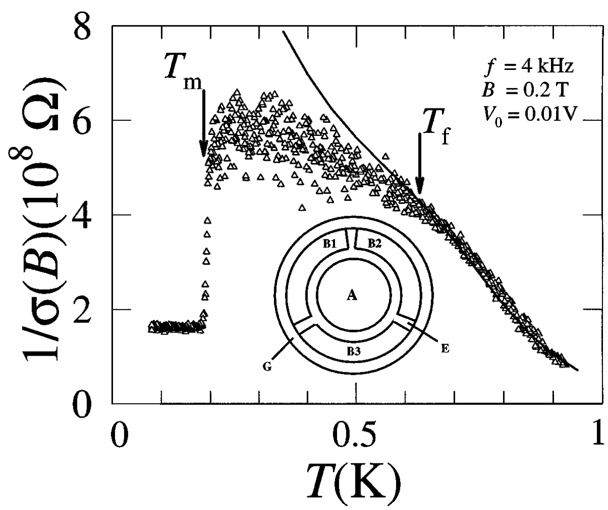

FIG. 1. The temperature dependence of $1/\sigma(B)$ for $n = 0.74 \times 10^{12}$ m^{-2} with the fluid theory (line). Fluctuations are observed between T_m and T_f. (Inset: Corbino electrodes.)

$1 < V_0 < 1000$ mV rms was applied to electrode A at a frequency $f = \omega/2\pi$ between 1 and 100 kHz. The ac current I to the electrodes B was measured. The phase shift $\phi(B)$ away from a purely capacitative current is proportional to $1/\sigma(B)$ for small angles [16].

The temperature dependence of $1/\sigma(B)$ for $n = 0.74 \times 10^{12}\ \mathrm{m}^{-2}$ at $B = 0.2$ T for $V_0 = 10$ mV at 4 kHz is shown in Fig. 1. The transition to the 2D solid is clearly seen at $T_m = 0.193$ K. In the 2D fluid, the magnetoconductivity [15,17] follows the Drude model, $\sigma(B) = \sigma_0/(1 + \mu^2 B^2)$, where $\sigma_0 = ne\mu$ is the zero field conductivity and μ is the mobility, for fields less than an onset field $B_0 \approx 0.5$ T. Hence $1/\sigma(B) \approx \mu B^2/ne$ in this region. The mobility is high, $\mu > 100\ \mathrm{m}^2/\mathrm{V\,s}$, with scattering from the ^{4}He vapor atoms and the ripplons, or surface vibrations. The line shows the theoretical $1/\sigma(B)$ from the zero field mobility [15], in good agreement with the data above $T_f = 0.63$ K.

In the electron solid, the magnetoconductivity is highly nonlinear [18,19] in low fields and is closely given by the semiempirical expression $\sigma(B) = KfV_0/v_1 Bd$, where K depends on the electrode geometry and $v_1 = (\alpha G_1/\rho)^{1/2}$ is the ripplon phase velocity at the first reciprocal lattice vector $G_1 = 2\pi(2/\sqrt{3})^{1/2} n^{1/2}$ of the crystal (α is the surface tension and ρ the helium density). This arises from Bragg-Cerenkov radiation [20] of ripplons with wave vector $\mathbf{G}_1$. Bragg reflections coherently enhance the electron-ripplon coupling and give a resonant drag force which limits the azimuthal Hall velocity of the electrons in the crossed radial electric field E and B, $v_\mathrm{H} = E/B$, to the ripplon phase velocity v_1. The magnetoconductivity at low ac voltages probes Bragg order in the 2D electrons. Above the threshold voltage (30 mV for the data in Fig. 1), $\sigma(B)$ decreases rapidly [18,19] as the electrons decouple from the surface.

The experimental transition at T_m is typically 20 mK wide. Between T_m and a higher temperature T_f new effects are observed. First, $1/\sigma(B)$ lies below the fluid theory [15] and the phase angle ϕ fluctuates by up to 30%, as shown in Fig. 1 by the scatter of data points between T_m and T_f. This is not noise. The fluctuations are in ϕ only, not the current amplitude. For Corbino electrodes, the complex ac current follows a well defined locus on an Argand diagram, as σ changes with temperature or field, along the theoretical line in Fig. 2. This locus is followed in both the high-temperature fluid and the solid at low drives. Figure 2 shows a temperature sweep of I^* (normalized to the magnitude of the zero field current) from 0.95 to 0.10 K in a fixed field of 0.4 T. From 0.95 K to $T_f = 0.57$ K, ϕ increases with the mobility as the temperature falls. Between T_f and 0.2 K (where the data becomes almost temperature independent), the fluctuations increase. At a given temperature, the complex fluctuating currents lie on a straight line on the Argand diagram, connecting two points on the normal locus. A simple interpretation

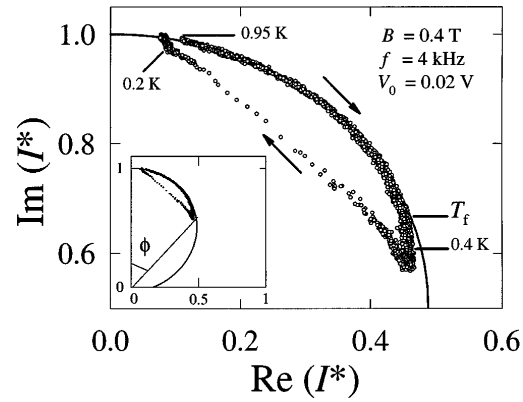

FIG. 2. Argand diagram plot of the ac current for $n = 0.85 \times 10^{12}\ \mathrm{m}^{-2}$ and $0.95 > T > 0.1$ K. The solid line is the theory for a homogeneous electron sheet. (Inset: The phase angle ϕ).

is that the measured current is a mixture of a solidlike and a fluidlike signal, $I = \beta I_s + (1 - \beta)I_f$, where the fraction β fluctuates and the 2D electron sheet behaves as a mixture of relatively ordered, coupled, regions within a relatively disordered, uncoupled matrix. The amplitude of the fluctuations will depend on the size and number of the coherent ordered regions. The Corbino disk should be sensitive to length scales less than the 200 μm width of the annular electrode E.

The fluctuations increase as the temperature falls. Figure 3 shows the temperature dependence of the standard deviation s^* of the phase angle ϕ, normalized to the value at 0.1 K. The deviation is constant (though ϕ increases) down to the onset temperature T_f below which s^* increases. A sharp drop occurs at the melting temperature T_m. These features can be observed qualitatively on Fig. 1 and have been measured in detail for electron densities from 0.3 to $1.5 \times 10^{12}\ \mathrm{m}^{-2}$. Each point in Fig. 3 comes from over 400 data points giving a statistical error in s^* of $\pm 4\%$. The temperature T_f is density

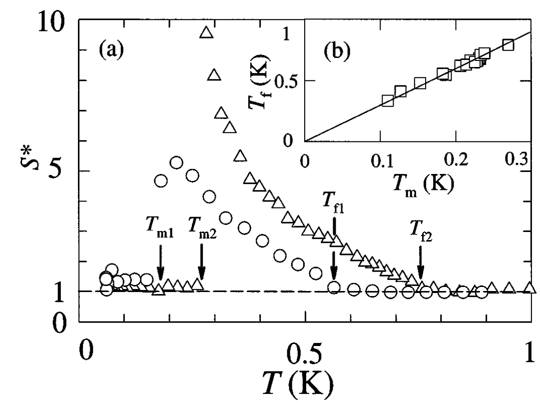

FIG. 3. (a) The standard deviation s^*, of the fluctuations in phase angle ϕ, normalized to 1.0 K, versus T for $n = 0.67 \times 10^{12}\ \mathrm{m}^{-2}$ (○) with $T_{m1} = 0.18$ K, $T_{f1} = 0.56$ K, $B = 0.2$ T, and $n = 1.44 \times 10^{12}\ \mathrm{m}^{-2}$ (△) with $T_{m2} = 0.27$ K, $T_{f2} = 0.76$ K, and $B = 0.5$ T. (b) The fluctuation onset temperature T_f versus T_m.

dependent and proportional to T_m as shown in Fig. 3, with $T_f/T_m = 2.8 \pm 0.3$ at $\Gamma = \Gamma_f = 46 \pm 5$. The onset is independent of B, though fluctuations decrease at higher fields along with nonlinearities in $\sigma(B)$ in the 2D solid.

These nonlinearities are an important indicator of electron decoupling. In the solid, ϕ decreases with the drive voltage V_0 (as the drag force increases) up to the threshold voltage, above which ϕ increases dramatically [18,19], as in Fig. 4(a). The corresponding ratio of the third and first harmonic currents $r_3 = |I_3|/|I|$ is also shown. Below the threshold, r_3 is small, due to the relatively small value of the equivalent nonlinear resistance. A sharp increase in r_3 occurs at the threshold, and the peak in r_3 suggests that the crystal cannot recover during each cycle at higher drives. In the fluctuation region, Fig. 4(b), the mean ϕ increases slowly with V_0 and the peak in r_3 is replaced by a smaller broad maximum at lower drives, consistent with the progressive decoupling of the ordered regions over a range of threshold voltages. A "normal" third harmonic nonlinearity (due to hot electron effects [21], for instance) would give $r_3 = c_3|I|^2$, as at high drives; this term is small in the region of the peaks. The nonlinearities decrease at higher temperatures and are negligible above T_f.

Fluctuations are also observed in the electron solid but at high drive amplitudes above the threshold, as shown in Fig. 5 during a field sweep at 0.08 K. The same features are observed as above T_m. The phase fluctuates dramatically but, in each small field range, the fluctuating currents lie on straight lines on the Argand diagram, while measurements at low drive lie close to the normal locus. This again suggests mixed coupled and uncoupled regions (which could still be crystalline [18] with a Hall velocity $v_H > v_1$). Even for low drives in the solid, small fluctuations and deviations from the normal response can occur, suggesting that a small fraction, <3%, may be uncoupled in the crystal, possibly in supercooled grain boundaries [8].

In summary, we have observed (i) fluctuations in the magnetoconductivity above T_m, which increase with decreasing temperature below T_f, (ii) $T_f/T_m = 2.8 \pm 0.3$, which corresponds to $\Gamma_f = 46 \pm 5$, (iii) a lower effective mobility in this region, (iv) novel features in the third harmonic currents due to nonlinearities, and (v) similar fluctuations at high drive levels in the 2D electron solid. The transition at T_m from the solid to the fluctuation region is relatively sharp, while the transition to the normal linear fluid occurs continuously up to T_f.

Previous experiments have also found unusual behavior above T_m. Mehrotra *et al.* [22] found excess scattering in the conductivity above T_m in zero field [23]. Buntar' *et al.* found a sharp change in the escape rate of electrons from the surface at $\Gamma = 47$ [24], explained as a sudden increase in electron correlations. Elliott *et al.* [25] observed an excess viscosity above T_m in the related system of ions below the surface of liquid helium.

One interpretation is that there are metastable ordered domains in the 2D electrons above T_m. Fluidlike and solidlike regions ("patches") are a well-known, but controversial, feature in computer simulations of 2D systems and in some colloidal systems [5,7,8] where the microstructures are complicated and open to differing interpretations. Molecular dynamics simulations [26] exhibit transient patches for $\Gamma > 50$, though the patch lifetime seems to be too short to explain the fluctuations in $\sigma(B)$ by the coherent Bragg-Cerenkov radiation of ripplons. However, the Bragg-Cerenkov coupling depends on crystallite orientation through the resonance condition $\mathbf{v}_H \cdot \mathbf{G}_1/G_1 = v_1$ where $\mathbf{v}_H$ is the Hall velocity and may be sensitive to orientational domains in an hexatic phase.

Domains might also be imposed by external potentials produced by patch potentials on the gold Corbino electrodes or by vibrationally induced ripples on the helium surface. Bedanov and Peeters [27] have shown, in Monte

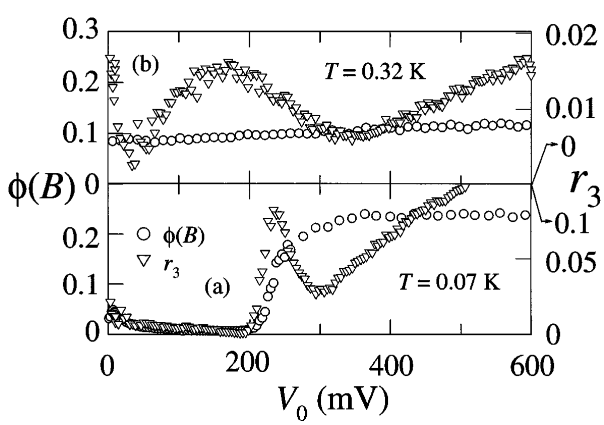

FIG. 4. The phase shift ϕ (○) at 4 kHz, and the third harmonic ratio r_3 (▽), versus V_0 at (a) 0.07 K in the solid and at (b) 0.32 K in the fluctuation region, for $n = 1.44 \times 10^{12}$ m^{-2}.

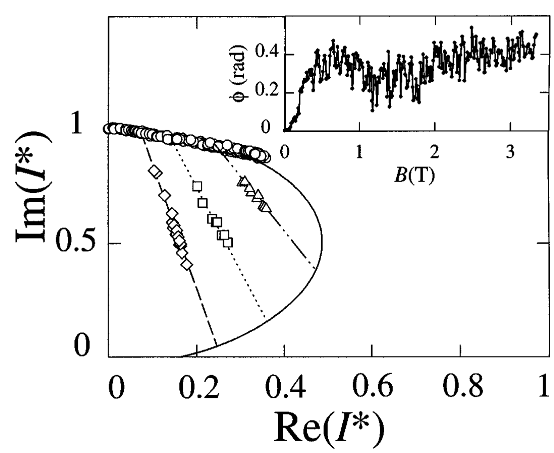

FIG. 5. Argand diagram plot in the 2D solid at $T = 0.08$ K, at low drive ($V_0 = 10$ mV) for $0 < B < 3.3$ T (○), and at high drive (100 mV) for $1.2 < B < 1.3$ T (◇); $2.2 < B < 2.3$ T (□); $3.2 < B < 3.3$ T (△). The solid line is the homogeneous response. [Inset: fluctuations in $\phi(B)$ at high drive].

Carlo simulations, that assemblies of less than 26 electrons in a confining potential, can have melting temperatures over a factor of 2 greater than expected from the mean particle density. However, the dependence of T_f on the electron density suggests that the fluctuations relate to intrinsic electron correlations. Vertical vibrations of the helium surface would modulate the capacitative current amplitude, rather than the phase angle ϕ, and would not scale with T_m.

We thank M. I. Dykman, P. M. Platzman, O. I. Kirichek, Yu. Z. Kovdrya, Yu. G. Rubo, and J. Saunders for useful discussions; the EPSRC (U.K.) for a Research Grant; the EU for support under Contract No. CHRXCT 930374; A. K. Betts, F. Greenough, and J. Taylor for technical assistance; and the Southampton University Microelectronics Centre and the lithography unit of the Rutherford Appleton Laboratory, U.K., for electrode fabrication.

[1] See P.M. Chaikin and T.C. Lubensky, *Principles of Condensed Matter Physics* (Cambridge University Press, Cambridge, 1995) for a general account of 2D order parameters.

[2] M. Kosterlitz and D.J. Thouless, J. Phys. C **6**, 1181 (1973); B.I. Halperin and D.R. Nelson, Phys. Rev. Lett. **41**, 121 (1978); D.R. Nelson and B.I. Halperin, Phys. Rev. B **19**, 2457 (1979); A.P. Young, Phys. Rev. B **19**, 1855 (1979).

[3] S. T. Chui, Phys. Rev. B **28**, 178 (1983).

[4] W. Janke and H. Kleinert, Phys. Rev. B **41**, 6848 (1990).

[5] Y. Saito, Phys. Rev. B **26**, 6239 (1982).

[6] V.N. Ryzhov and E.E. Tareyeva, Phys. Rev. B **51**, 8789 (1995).

[7] K. Strandberg, Rev. Mod. Phys. **60**, 161 (1988).

[8] C. Udink and D. Frenkel, Phys. Rev. B **35**, 6933 (1987); C.A. Murray, W.O. Sprenger, and R.A. Wenk, Phys. Rev. B **42**, 688 (1990); R.E. Kusner, J.A. Mann, and A.J. Dahm, Phys. Rev. B **51**, 5746 (1995).

[9] K. Chen, T. Kaplan, and M. Mostoller, Phys. Rev. Lett. **74**, 4019 (1995); K. Bagchi, H.C. Andersen, and W. Swope, Phys. Rev. Lett. **76**, 255 (1996).

[10] R. Morf, Phys. Rev. Lett. **43**, 931 (1979); V.M. Bedanov, G.V. Gadiyak, and Y.E. Lozovik, J. Exp. Theor. Phys. **61**, 967 (1985).

[11] R.K. Kalia, P. Vashishta, and S.W. de Leeuw, Phys. Rev. B **23**, 4794 (1981); C. Fang-Yen, M.I. Dykman, and M.J. Lea, Phys. Rev. B **55**, 16 272 (1997).

[12] See *Two-Dimensional Electrons on Liquid Helium and Other Substrates*, edited by E.Y. Andrei (Kluwer Academic Press, New York, 1997).

[13] E.Y. Andrei, F.I.B. Williams, D.C. Glattli, and G. Deville, in *The Physics of Low-dimensional Semiconductor Structures*, edited by P. Butcher *et al.* (Plenum Press, New York, 1993), Chap. 14, p. 499.

[14] F. Gallet, G. Deville, A. Valdes, and F.I.B. Williams, Phys. Rev. Lett. **49**, 212 (1982); G. Deville, A. Valdes, E.Y. Andrei, and F.I.B. Williams, Phys. Rev. Lett. **53**, 588 (1984).

[15] M.J. Lea, P. Fozooni, A. Kristensen, P.J. Richardson, K. Djerfi, M.I. Dykman, C. Fand-Yen, and A. Blackburn, Phys. Rev. B **55**, 16 280 (1997).

[16] The phase shift ϕ specifies the losses in the system. The standard expression for a Corbino disk [R. Mehrotra and A.J. Dahm, J. Low Temp. Phys. **67**, 115 (1987)] gives $\sigma(B)$ for a linear, local, and uniform conductor. In other situations, it defines an effective Corbino magnetoconductivity.

[17] M.J. Lea, P. Fozooni, P.J. Richardson, and A. Blackburn, Phys. Rev. Lett. **73**, 1142 (1994).

[18] K. Shirahama and K. Kono, Phys. Rev. Lett. **74**, 781 (1995); J. Low Temp. Phys. **104**, 237 (1996).

[19] A. Kristensen, K. Djerfi, P. Fozooni, M.J. Lea, P.J. Richardson, A. Santrich-Badal, A. Blackburn, and R.W. van der Heijden, Phys. Rev. Lett. **77**, 1350 (1996).

[20] M.I. Dykman and Yu.G. Rubo, Phys. Rev. Lett. **78**, 4813 (1997).

[21] M. Saitoh, J. Phys. Soc. Jpn. **42**, 201 (1977).

[22] R. Mehrotra, C.J. Guo, Y.Z. Ruan, D.B. Mast, and A.J. Dahm, Phys. Rev. B **29**, 5239 (1984).

[23] R. Mehrotra, J. Low Temp. Phys. **68**, 161 (1987).

[24] V.A. Buntar', V.N. Grigoriev, O.I. Kirichek, Yu.Z. Kovdrya, Yu.P. Monarhka, and S.S. Sokolov, J. Low Temp. Phys. **79**, 323 (1990).

[25] P.L. Elliott, C.I. Pakes, L. Skrbek, and W.F. Vinen, Czech. J. Phys. **46**, 335 (1996).

[26] S. Harris, M.Sc. thesis, Royal Holloway, University of London, 1996.

[27] V. Bedanov and F.M. Peeters, Phys. Rev. B **49**, 2667 (1994).