

## Reentrant Solid-Liquid Transition in Ionic Colloidal Dispersions by Varying Particle Charge Density

Junpei Yamanaka,<sup>1,\*</sup> Hiroshi Yoshida,<sup>1</sup> Tadanori Koga,<sup>1</sup> Norio Ise,<sup>2</sup> and Takeji Hashimoto<sup>1,3</sup>

<sup>1</sup>Hashimoto Polymer Phasing Project, ERATO, Japan Science and Technology Corporation,  
15 Morimoto-cho, Shimogamo, Sakyo-ku, Kyoto 606-0805, Japan

<sup>2</sup>Central Laboratory, Rengo Co., Ltd., 186-1-4 Ohhiraki, Fukushima, Osaka 553-0007, Japan

<sup>3</sup>Department of Polymer Chemistry, Graduate School of Engineering, Kyoto University, Kyoto 606-8501, Japan

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The influence of the particle surface charge density on the solid-liquid phase transition in electrostatically stabilized colloidal silica and polymer latex dispersions is examined. Both systems show a reentrant transition with increasing charge density. This is not explainable in terms of the Yukawa potential and the charge-renormalization model. [S0031-9007(98)06414-X]

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The ionic colloidal systems have substantial experimental advantages to study the solid-liquid phase transition, since the interparticle interaction can be tuned in a wide range by varying experimental variables such as the effective surface charge density of the particles  $\sigma_e$ , the particle volume fraction  $\phi$ , and the salt concentration  $C_s$  [1]. The phase diagram of the transition has extensively been studied [1–4].  $\phi$  and  $C_s$  have usually been adopted as variables, and, as far as we know, the charge dependence was examined only in recent studies by Palberg *et al.* [3] and by ourselves [4]. In both studies, the  $C_s$  value at the transition point, which was determined at a fixed  $\phi$  and at relatively small  $\sigma_e$ 's, increased with  $\sigma_e$ .

In the present Letter, we report for the first time a three dimensional phase diagram of the solid-liquid transition as a function of  $\sigma_e$ ,  $\phi$ , and  $C_s$ . Here, we investigated much broader charge conditions than in the previous studies, and found that the liquid state became ordered to give the solid state and thereafter reentered into the liquid state with increasing  $\sigma_e$ . Aqueous dispersions of colloidal silica particles were used, whose  $\sigma_e$  value could be varied continuously by changing the amount of added NaOH [4,5]. The phase diagram was determined by observing iridescence due to a Bragg diffraction from the solid structure, and further by applying an ultra-small-angle x-ray scattering (USAXS) method. The reentrant transition was also confirmed for ionic polymer latex systems having various charge densities by direct observations of their internal structures with a laser scanning confocal microscope (LSCM).

Colloidal silica samples, KE-P10W (Nippon Shokubai Co., Ltd., Tokyo, Japan; diameter =  $0.12 \pm 0.01 \mu\text{m}$ ) and SI-80P (Catalyst and Chemicals Co., Ltd., Osaka, Japan;  $0.11 \pm 0.01 \mu\text{m}$ ), were purified by dialysis and the ion-exchange method. They had  $\sigma_e$ 's of 0.07 and  $0.23 \mu\text{C cm}^{-2}$  due to partial dissociation of weakly acidic silanol groups on their surfaces. The degree of dissociation of the silanol groups, and thus the  $\sigma_e$  value, was increased by adding NaOH at  $p\text{H} < 8$ , that is, under

conditions where the concentrations of excess NaOH were less than  $1 \mu\text{M}$ .

The particles thus prepared (hereafter designated as  $\text{Na}^+$ -type silica) had Na ions as counterions, which are partly condensed near the particle surface, reducing the charge number [1]. Consequently,  $\sigma_e$  is quite different from the *analytical (bare)* charge density  $\sigma_a$ , which could be calculated from a concentration of added NaOH. We determined  $\sigma_e$  at various  $\sigma_a$ 's from the conductometry. The electrical conductivity of the silica dispersion,  $K$ , is given by  $K = 10^{-3}(\lambda_C C_C + \lambda_p \phi) + K_b$ , where  $C_C$  is the concentration of free counterions,  $\lambda_C$  and  $\lambda_p$  are the equivalent conductivities of the counterions and the particles, and  $K_b$  is the background conductivity. The  $\lambda_p$  value was determined by comparing conductivities of  $\text{Na}^+$ - and  $\text{K}^+$ -type silica dispersions, assuming that the  $C_C$  values were the same in both cases. The transport number of the particle,  $t_p [\equiv \lambda_p \phi / (\lambda_C C_C + \lambda_p \phi)]$ , did not significantly vary with  $\phi$  at  $\phi = (1-3) \times 10^{-2}$ , and was 0.51–0.48 for  $\text{Na}^+$ -type silica. A relationship between  $\sigma_e$  and  $\sigma_a$  will be discussed later [Fig. 4(A) (below)].

The solid-liquid phase transition was examined under various  $\sigma_e$ ,  $\phi$ , and  $C_s$  conditions. Figure 1 is the phase diagram for KE-P10W silica [6], which was constructed by observing iridescence.  $C_s$  values were estimated from the sum of concentrations of coexisting NaCl, ionic species in the water used ( $2 \mu\text{M}$ ), and excess NaOH, if any. The region whose  $C_s$  is smaller than that at the phase boundary (shown by rectangles) corresponds to the solid state. With increasing  $\phi$  at fixed  $\sigma_e$ 's, the  $C_s$  at the boundary increased monotonously as reported previously [1,2]. On the other hand, a characteristic behavior was observed for  $\sigma_e$  dependence: On increasing  $\sigma_e$  for  $\phi \geq 2 \times 10^{-2}$ , the phase boundary first shifted towards higher  $C_s$  due to an augmented electrostatic interaction. However, with a further increase in  $\sigma_e$ , a maximum was observed at around  $\sigma_e = 0.4-0.5 \mu\text{C cm}^{-2}$ , after which the  $C_s$  value at the boundary decreased. Namely, there existed a reentrant liquid state in the high  $\sigma_e$  region.

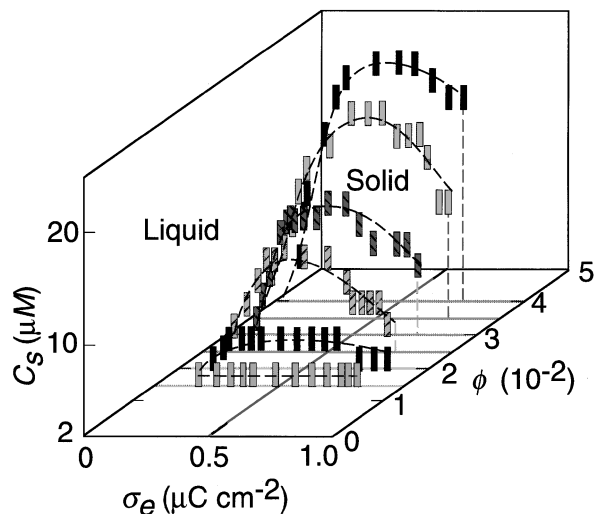


FIG. 1. A phase diagram for the solid-liquid transition of aqueous dispersion of KE-P10W silica particles (diameter =  $0.12 \mu\text{m}$ ) as a function of the effective charge density  $\sigma_e$ , the particle volume fraction  $\phi$ , and the salt concentration  $C_s$ . The solid-liquid phase boundary was shown by rectangles. Dashed lines are guides for the eye.

The phase diagram was further examined by applying the USAXS method [4]. Figure 2 shows the scattering intensity versus scattering vector  $q$  at  $\phi = 3 \times 10^{-2}$ ,  $C_s = 10 \mu\text{M}$ , and at three  $\sigma_e$ 's, which corresponded to the liquid, solid, and the reentrant liquid states in the phase diagram (Fig. 1). In the liquid state ( $\sigma_e = 0.07 \mu\text{C cm}^{-2}$ ),

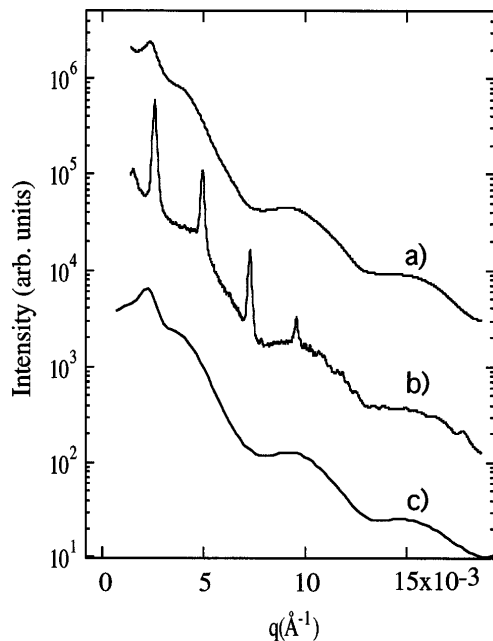


FIG. 2. The USAXS profiles showing the reentrant solid-liquid transition for KE-P10W silica dispersions with varying  $\sigma_e$ . (a)  $\sigma_e = 0.07$ , (b)  $0.36$ , and (c)  $0.72 \mu\text{C cm}^{-2}$ ;  $C_s = 10 \mu\text{M}$ ;  $\phi = 3 \times 10^{-2}$ .

a broad peak due to interparticle interference, resulting from an electrostatic interaction, was observed in addition to the form factor of an isolated particle. In the solid state ( $\sigma_e = 0.36 \mu\text{C cm}^{-2}$ , taken from Fig. 3 of Ref. [4]), sharp peaks were observed, which were attributable to the Bragg diffractions from the (110) plane of a bcc lattice [7]. At  $\sigma_e = 0.72 \mu\text{C cm}^{-2}$ , again a profile due to the liquidlike particle arrangement was observed, which confirmed the presence of the reentrant liquid state.

We further examined ionic polymer latex particles having strongly acidic groups (diameters =  $0.12\text{--}0.13 \mu\text{m}$ ). SS-10, SS-16, and SS-19 ( $\sigma_e = 0.28, 0.42,$  and  $0.70 \mu\text{C cm}^{-2}$ ) were synthesized by an emulsifier-free polymerization method, and N-100 ( $0.50 \mu\text{C cm}^{-2}$ ) was purchased from Sekisui Chemicals Co., Ltd., Osaka, Japan. They were purified as described earlier [8], and the dispersion structures were observed by LSCM as reported before [8]. Figure 3 is a phase diagram determined at  $C_s = 2 \mu\text{M}$  and at relatively dilute conditions. The data for the two kinds of silica in the absence of NaOH ( $\sigma_e = 0.07$  and  $0.23 \mu\text{C cm}^{-2}$ ) were also shown. Coexistence of the solid and liquid states was observed in the solid state near the phase boundary. Clearly, there exists the liquid state at high  $\sigma_e$ 's, not only when  $\phi$  and/or  $\sigma_e$  are small.

We compare now the effective charge and the phase diagram determined here with theoretical studies. The interaction between charged colloidal particles has often been claimed to be explainable in terms of a pair potential of the Yukawa form  $U_Y(r) = A[(Ze)^2/4\pi\epsilon]\exp(-\kappa r)/r$ , where  $r$  is the interparticle distance,  $Z$  is the charge number,  $e$  is the elementary charge,  $1/\kappa$  is the Debye screening length defined as  $\kappa^2 = 4\pi e^2[C_C + 2C_s]/\epsilon k_B T$ ,  $A = [\exp(\kappa a)/(1 + \kappa a)]^2$  (a geometrical factor), and  $a$  is the particle radius [1]. A numerical simulation study on the phase diagram for the Yukawa system has been reported by Robbins *et al.* [9].

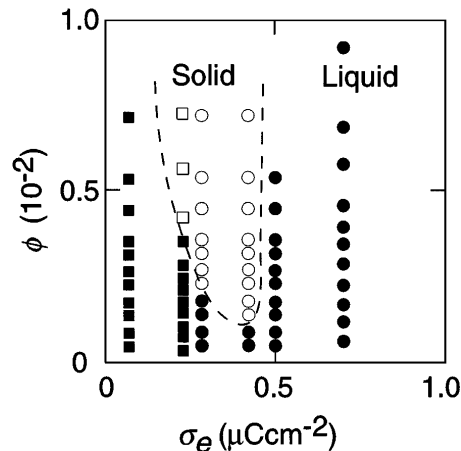


FIG. 3. A phase diagram determined by confocal laser scanning microscopy for polymer latex (circle) and colloidal silica (square) dispersions.  $C_s = 2 \mu\text{M}$ . Open symbols, solid state; filled symbols, liquid state.

$U_Y(r)$  is derived from the linearized Poisson-Boltzmann (P-B) equation assuming the potential energy at the surface  $Ze^2/[a\epsilon(1 + \kappa a)] < k_B T$  (Debye-Hückel (D-H) approximation) [1]. In practice, especially at low-salt conditions, the D-H approximation is not valid. Even then, however, the use of  $U_Y(r)$  was considered to be justified, if one introduces a renormalized charge density  $\sigma^*$ , which is obtained by solving the full P-B equation in a spherical Wigner-Seitz cell and matching the resulting electrostatic potential to the solution of the linearized P-B equation at the cell boundary [10,11]. Several authors [2,3] have reported that the experimental phase diagrams showed close agreement with that by Robbins *et al.* by using  $\sigma^*$ . Furthermore, Palberg *et al.* [3] reported that the renormalized charge was close to the effective charge determined by their conductivity measurements. It should be noted that these conclusions have been drawn for relatively small charge particles ( $\sigma_a < 0.63 \mu\text{C cm}^{-2}$  in Ref. [3]).

In Fig. 4(A),  $\sigma_e$  obtained in the present study (KE-P10W silica,  $\phi = 2 \times 10^{-2}$ ) and  $\sigma^*$  calculated by the method in Ref. [8] (broken curve) are shown as a function of  $\sigma_a$ . The data for the polymer lattices [5] were

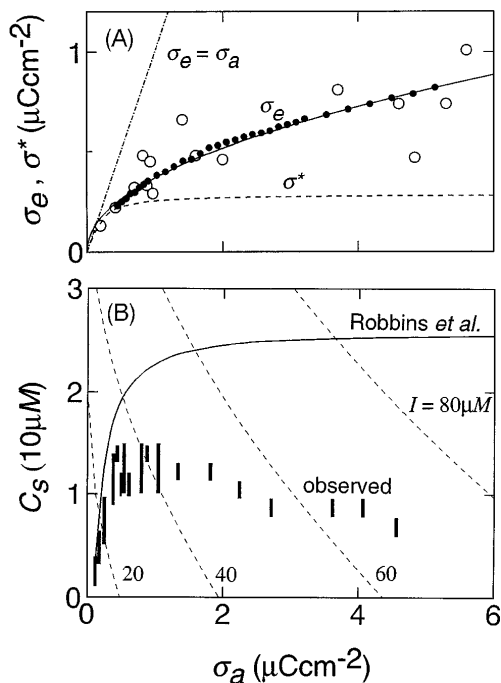


FIG. 4. (A) Relationship between  $\sigma_e$  and  $\sigma_a$  for KE-P10W silica dispersions ( $\phi = 2 \times 10^{-2}$ , filled circles). The full curve is an empirical relationship,  $\ln \sigma_e = 0.49 \ln \sigma_a - 1.0$ , determined from the data for lattices (open circles; extrapolated value to  $\phi = 0$ ) [5]. The renormalized charge density,  $\sigma^*$  (broken curve) was shown for comparison. The dot-dashed curve represents  $\sigma_e = \sigma_a$ . (B) Comparison between observed phase boundary (rectangles) and the numerical simulation results by Robbins *et al.* with  $\sigma^*$ ,  $\phi = 3 \times 10^{-2}$ . Conditions where the ionic strength of the dispersion,  $I (\equiv C_s + \frac{1}{2}C_C)$ , were 20, 40, 60, and 80  $\mu\text{M}$  are also shown.

also demonstrated. At small  $\sigma_a$ 's,  $\sigma_e$  was close to  $\sigma^*$ . However, at higher  $\sigma_a$ 's, the  $\sigma^*$  value tended to level off, while  $\sigma_e$  kept increasing. The observed  $\sigma_e$  was well described by an empirical relationship determined for lattices,  $\ln \sigma_e = 0.49 \ln \sigma_a - 1.0$  (full curve) [5]. Figure 4(B) shows a comparison between the experimental phase boundary and that determined by the method of Robbins *et al.* by using  $\sigma^*$  ( $\phi = 3 \times 10^{-2}$ ). For all of the data points, the D-H approximation was not valid. At small  $\sigma_a$ 's, the predicted boundary (full curve) was close to the observed one, as reported previously [12]. At higher  $\sigma_a$ 's, however,  $C_s$  at the boundary became constant, because of the saturation of  $\sigma^*$ . Thus the reentrant transition was not in accord with the theoretical treatment based on the Yukawa potential and the charge-renormalization concept.

When  $\sigma_e$  was used instead of  $\sigma^*$ ,  $C_s$  of the boundary increased with  $\sigma_a$  under the conditions examined here, and showed a maximum at around  $\sigma_a = 6 \mu\text{C cm}^{-2}$ , as a result of increases in both  $\sigma_e$  and  $C_C$  with increasing  $\sigma_a$ : The ionic strength of the dispersion,  $I (\equiv C_s + \frac{1}{2}C_C)$  for monovalent extraneous ions and counterions [13]), increases with  $\sigma_a$ . Since  $\kappa$  is proportional to  $I^{1/2}$ , the screening effect becomes more pronounced at higher  $\sigma_a$ 's, resulting in a reentrant solid to liquid transition. Conditions where  $I$  values were constant (*isoionic lines*), which are measures of the screening effect, are also shown in Fig. 4(B) for reference. Thus, when  $\sigma_e$  was coupled with  $U_Y(r)$ , the reentrant transition is expected because of the screening effect. However, the predicted maximum position was much higher than the observed one ( $\sigma_a \sim 1 \mu\text{C cm}^{-2}$ ).

The above-mentioned discrepancy might be partly attributed to, e.g., neglect of the many-particle interaction in the theory, which appears to be more significant in strongly interacting systems such as the present one. In our opinion, the Yukawa potential, in which only Coulombic repulsion between two particles is assumed, does not duly account for the phase behavior in low-salt dispersions: Recent experimental results suggest the presence of interparticle *net* attraction in ionic colloidal systems in low-salt conditions [1]. For example, spontaneous formation of void structures [14], where vapor and liquid states are co-existing, seems to be difficult to explain without assuming a net attraction. We point out that the void structure was favored for highly charged particles [15], for which the reentrant disordered states were observed here. In order to describe the interparticle attraction, the Sogami potential  $U_s(r) = [\sinh(\kappa a)/\kappa a]^2 [1 + \kappa a \coth(\kappa a) - \kappa r/2] \times [(Ze)^2/4\pi\epsilon] \exp(-\kappa r)/r$ , which has both electrostatic attractive and repulsive terms, has been used [1]. Although performed under limited conditions, simulation studies with  $U_s(r)$  [16,17] predicted the reentrant behavior.

Further studies are in progress to discuss the reentrant transition in more detail. However, in the present stage, we may reasonably conclude that the argument in terms of the

Yukawa potential and the charge-renormalization concept is not applicable for highly charged colloidal systems.

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\*Author to whom correspondence should be addressed.

Present address: Faculty of Pharmaceutical Sciences, Nagoya City University, 1-3 Tanabe-dori, Nagoya 467-8603, Japan.

Email address: yamanaka@phar.nagoya-cu.ac.jp

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