Phase Transitions in Rotating Neutron Stars

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As rotating neutron stars slow down, the pressure and the density in the core region increase due to the decreasing centrifugal forces and phase transitions may occur in the center. We extract the analytic behavior near the critical angular velocity Ω_0, where the phase transitions occur in the center of a neutron star, and calculate the moment of inertia, angular velocity, rate of slow down, braking index, etc. For a first order phase transition these quantities have a characteristic behavior, e.g., the braking index diverges as $\sim(\Omega_0 - \Omega)^{-1/2}$. Observational consequences for first, second, and other phase transitions are discussed. [S0031-9007(98)06463-1]

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The physical state of matter in the interiors of neutron stars at densities above a few times normal nuclear matter densities is essentially unknown. Interesting phase transitions in nuclear matter to quark matter [1], mixed phases of quark and nuclear matter [2,3], kaon [4] or pion condensates [5,6], neutron and proton superfluidity [7], hyperonic matter [1,2], crystalline nuclear matter [5], magnetized matter, etc., have been considered. Recently, Glendenning *et al.* [8] considered rapidly rotating neutron stars and what happens as they slow down when the decreasing centrifugal force leads to increasing core pressures. They find that a drastic softening of the equation of state, e.g., by a phase transition to quark matter, can lead to a sudden contraction of the neutron star at a critical angular velocity and shows up in a backbending moment of inertia as a function of frequency. Here we consider another interesting phenomenon, namely, how the star and, in particular, its moment of inertia behave near the critical angular velocity where the core pressure just exceeds that needed to make a phase transition. We calculate the moment of inertia, angular velocities, braking index, etc., near the critical angular velocity and discuss observational consequences for first and second order phase transitions.

The general relativistic equations for slowly rotating stars were described by Hartle [9]. We shall also make the standard approximation of slowly rotating stars, i.e., the rotational angular velocity is $\Omega^2 \ll GM/R^3$. For neutron stars with mass $M = 1.4M_\odot$ and radius $R \sim 10$ km their period should thus be larger than a few milliseconds, a fact which applies to all measured pulsars so far. Hartle's equations are quite elaborate to solve as they consist of six coupled differential equations as compared to the single Tolman-Oppenheimer-Volkoff equation [10] in the nonrotating case. In order to be able to analytically extract the qualitative behavior near the critical angular velocity Ω_0, where a phase transition occurs in the center, we will first solve the Newtonian equations for a simple equation of state. This will allow us to make general predictions on properties of rotating neutrons stars when phase transitions occur in the interior of a star. The corrections from general relativity are typically of order $GM/R \simeq 10\%$ for neutron stars of mass $M \simeq 1.4M_\odot$. The extracted analytical properties of a rotating star are then checked below by actually solving Hartle's equations numerically for a realistic equation of state.

The simple Newtonian equation of motion expresses the balance between the pressure gradient and the gravitational and centrifugal forces

$$\nabla P = -\rho(\nabla V + \mathbf{\Omega} \times \mathbf{\Omega} \times \mathbf{r}) . \quad (1)$$

Here, $V(\mathbf{r})$ is the gravitational potential for the deformed star and ρ the energy ($\sim$mass) density. We assume that friction in the (nonsuperfluid) matter ensures that the star is uniformly rotating. Since cold neutron stars are barotropes, i.e., the pressure is a function of density, the pressure, density, and effective gravitational potential, $\Phi = V - \frac{1}{2}(\mathbf{\Omega} \times \mathbf{r})^2$, are all constants on the *same* isobaric surfaces for a uniformly rotating star [9]. We denote these surfaces by the effective radius a, and for slowly rotating stars it is related to the distance r from the center and the polar angle θ from the rotation axis along $\mathbf{\Omega}$ by [9]

$$r(a, \theta) = a[1 - \epsilon(a)P_2(\cos\theta)] , \quad (2)$$

where $P_2(\cos\theta)$ is the second Legendre polynomial and $\epsilon(a)$ is the deformation of the star from spherical symmetry.

Inserting Eq. (2) into Eq. (1), one obtains for small deformations [9] the $l = 0$ Newtonian hydrostatic equation

$$\frac{1}{\rho}\frac{dP}{da} = -G\,\frac{m(a)}{a^2} + \frac{2}{3}\,\Omega^2 a , \quad (3)$$

where $m(a) = 4\pi \int_0^a \rho(a')a'^2 da'$ is the mass contained inside the mean radius a. The factor $2/3$ in the centrifugal force arises because it only acts in two of the three directions. The equation ($l = 2$) for the deformation $\epsilon(a)$ is given in, e.g., Ref. [11]. The deformation generally increases with decreasing density, i.e., the star is more deformed in its outer layers.

In order to discuss the qualitative behavior near critical angular velocities we first consider a simple equation of state (EOS) with phase transitions for which Eq. (3) can

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be solved analytically, namely, that of two incompressible fluids with a first order phase transition between energy density ρ_1 and ρ_2 ($\rho_1 < \rho_2$) coexisting at a pressure P_0. The mass function $m(a)$ is very simple in the Newtonian limit and the boundary condition $m(R) = M$ relates the star radius R to the radius of the dense core R_0 as

$$R = \left[\bar{R}^3 - \left(\frac{\rho_2}{\rho_1} - 1\right)R_0^3\right]^{1/3}, \tag{4}$$

where $\bar{R} = (3M/4\pi\rho_1)^{1/3}$ is the star radius in the absence of a dense core. Solving Eq. (3) gives the pressure

$$P(a) = P_0 + \frac{1}{2}(R_0^2 - a^2)\rho_1\left(\frac{4\pi}{3}G\rho_1 - \frac{2}{3}\Omega^2\right) + \frac{4\pi}{3}GR_0^2(\rho_2 - \rho_1)\rho_1\left(1 - \frac{R_0}{a}\right), \tag{5}$$

for $R_0 \leq a \leq R$. The boundary condition at the surface $P(R) = 0$ in Eq. (5) gives

$$\omega^2 \equiv \frac{\Omega^2}{2\pi G\rho_1} = 1 - 2\left[\frac{3}{4\pi}\frac{P_0}{G\rho_1^2R^2} + \left(\frac{\rho_2}{\rho_1} - 1\right)\frac{R_0^2}{R^2}(1 - R_0/R)\right] \times (1 - R_0^2/R^2)^{-1}. \tag{6}$$

The phase transition occurs right at the center when $R_0 = 0$ corresponding to the *critical angular velocity* $\Omega_0 = \omega_0\sqrt{2\pi G\rho_1}$, where

$$\omega_0^2 = 1 - 2\frac{P_0\bar{R}}{GM}. \tag{7}$$

Generally, for any EOS the critical angular velocity depends on P_0, M, and ρ_1 but not on ρ_2.

For angular velocities just below ω_0 very little of the high density phase exists and $R_0 \ll R$. Expanding (6) we obtain

$$\frac{R_0}{\bar{R}} \simeq \sqrt{\frac{\omega_0^2 - \omega^2}{2\rho_2/\rho_1 - 1 - \omega_0^2}}. \tag{8}$$

For $\omega \geq \omega_0$ the dense phase disappears and $R_0 = 0$. Generally, one can interpret R_0 as an order parameter in analogy to, e.g., magnetization, the BCS gap, or the Higgs field in the standard model, however, as a function of angular velocity instead of temperature.

The corresponding moment of inertia is for $R_0 \ll R$,

$$I = \frac{4\pi}{5}[\rho_2R_0^5 + \rho_1(R^5 - R_0^5)]\left(1 + \frac{2}{5}\epsilon\right) \simeq \frac{2}{5}M\bar{R}^2\left[1 - \frac{5}{3}\left(\frac{\rho_2}{\rho_1} - 1\right)\frac{R_0^3}{\bar{R}^3}\right]\left(1 + \frac{1}{2}\omega^2\right), \tag{9}$$

where we used the deformation $\epsilon = (5/4)\omega^2$ in the low density phase [11]. However, for the qualitative behavior near Ω_0 only the contraction of the star radius R with the appearance of the dense core R_0 is important, whereas the deformations can be ignored. The contraction is responsible for the term in the moment of inertia and is proportional to $R_0^3 \propto (\omega_0^2 - \omega)^{3/2}$ near the critical angular velocity. Consequently, the derivative $dI/d\omega^2$ displays the same nonanalytic square root dependence as R_0 [see Eq. (8)].

Latent heat is generated in the phase transition and can be ignored because of rapid neutrino cooling which will be even faster than in supernova explosions. Thus temperatures will drop below $\sim$1 MeV in seconds. Such temperatures are negligible compared to typical Fermi energies of nucleons or quarks and the time scales are also much smaller than t_0.

Let us subsequently consider a more realistic EOS for dense nuclear matter at high densities such as the Bethe-Johnson EOS [12]. At high densities it can be approximated by a polytropic relation between the pressure and energy density: $P = K_1\rho^{2.54}$, where $K_1 = 0.021\rho_0^{-1.54}$ and $\rho_0 = m_n 0.15\ \mathrm{fm}^{-3}$ is normal nuclear matter mass density. As we are only interested in the dense core we will for simplicity employ this Bethe-Johnson polytrope (BJP) EOS. The central density of a nonrotating $1.4M_\odot$ mass neutron star with the BJP is $\sim 3.4\rho_0$. Furthermore, we assume that a first order phase transition occurs at density $\rho_1 = 3.2\rho_0$ to a high density phase of density $\rho_2 = 4\rho_0$ with a similar polytropic EOS $P = K_2\rho^{2.54}$. From the Maxwell construction the pressure is the same at the interface P_0 which determines $K_2 = K_1(\rho_1/\rho_2)^{2.54}$.

We now generalize Eq. (3) by including effects of general relativity. From Einstein's field equations for the metric we obtain from the $l = 0$ part

$$\frac{1}{\rho + P}\frac{dP}{da} = -G\frac{m + 4\pi a^3P}{a^2(1 - 2Gm/a)} + \frac{2}{3}\Omega^2 a, \tag{10}$$

where $m(a) = 4\pi\int_0^a \rho(a')a'^2 da'$. In the centrifugal force term we have ignored frame dragging and other corrections of order $\Omega^2GM/R \sim 0.1\Omega^2$ for simplicity and because they have only minor effects in our case. By expanding the pressure, mass function, and gravitational potential in the difference between the rotating and nonrotating case, Eq. (10) reduces to the $l = 0$ part of Hartle's equations [cf. Eq. (100) in [9]]. Note also that Hartle's full equations cannot be used in our case because the first order phase transition causes discontinuities in densities so that changes are not small locally. This shows up, for example, in the divergent thermodynamic derivative $d\rho/dP$.

The rotating version of the Tolman-Oppenheimer-Volkoff equation (10) is now solved for a rotating neutron star of mass $M = 1.4M_\odot$ with the BJP EOS, including a first order phase transition. In Fig. 1 we show the central density, moment of inertia, braking index, star radius, and radius of the interface (R_0) as a function of the scaled angular velocity. It is important to note that $R_0 \propto \sqrt{\Omega_0^2 - \Omega^2}$ for angular velocities just below the critical value Ω_0. The qualitative behavior of the neutron star with the BJP EOS and a first order phase transition

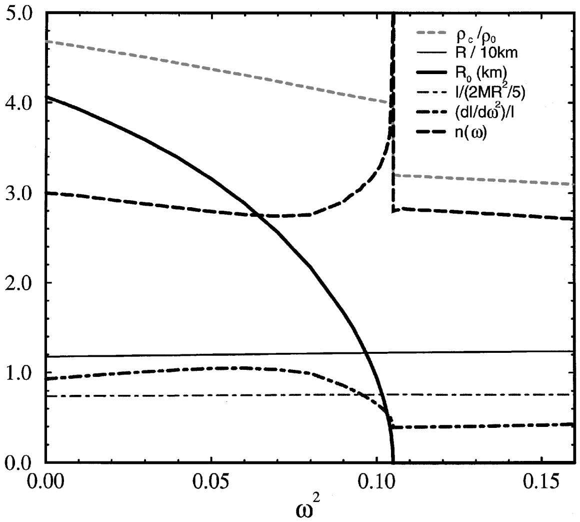

FIG. 1. Central density (in units of ρ_0), radii of the neutron star R and its dense core R_0, moment of inertia, its derivative $I'/I = dI/d\omega^2/I$, and the braking index are shown as a function of the scaled angular velocity $\omega^2 = \Omega^2/2\pi G\rho_1$. The rotating neutron star has mass $1.4M_\odot$ and a Bethe-Johnson-like polytropic equation of state with a first order phase transition taking place at density $\rho_1 = 3.2\rho_0$ to $\rho_2 = 4\rho_0$.

is the same as for our simple analytic example of two incompressible fluids examined above. Generally, it is the finite density difference between the phases that is important and leads to a term in the moment of inertia proportional to $(\Omega_0^2 - \Omega^2)^{3/2}$ as in Eq. (9).

The moment of inertia increases with angular velocity. Generally, for a first order phase transition we find for $\Omega \lesssim \Omega_0$ [see also Eq. (9) and Fig. 1],

$$I = I_0\left[1 + \frac{1}{2}c_1\frac{\Omega^2}{\Omega_0^2} - \frac{2}{3}c_2\left(1 - \frac{\Omega^2}{\Omega_0^2}\right)^{3/2} + \ldots\right]. \tag{11}$$

For the two incompressible fluids with momentum of inertia given by Eq. (9), the small expansion parameters are $c_1 = \omega_0^2$ and $c_2 = (5/2)\omega_0^3(\rho_2/\rho_1 - 1)/(2\rho_2/\rho_1 - 1 - \omega_0^2)^{3/2}$; for $\Omega > \Omega_0$ the c_2 term is absent. For the BJP we find from Fig. 1 that $c_2 \simeq 0.07 \simeq 2.2\omega_0^3$. Generally, we find that the coefficient c_2 is proportional to the density difference between the two coexisting phases and to the critical angular velocity to the third power, $c_2 \sim (\rho_2/\rho_1 - 1)\omega_0^3$. The scaled critical angular velocity ω_0 can at most reach unity for submillisecond pulsars.

To make contact with observation we consider the temporal behavior of angular velocities of pulsars. The pulsars slow down at a rate given by the loss of rotational energy which we shall assume is proportional to the rotational angular velocity to some power (for dipole radiation $n = 3$)

$$\frac{d}{dt}\left(\frac{1}{2}I\Omega^2\right) = -C\Omega^{n+1}. \tag{12}$$

With the moment of inertia given by Eq. (11)the angular velocity will then decrease with time as

$$\frac{\dot\Omega}{\Omega} = -\frac{C\Omega^{n-1}}{I_0}\left(1 - c_1\frac{\Omega^2}{\Omega_0^2} - c_2\sqrt{1 - \frac{\Omega^2}{\Omega_0^2}}\right) \simeq -\frac{1}{(n-1)t}\left[1 - c_2\sqrt{1 - \left(\frac{t_0}{t}\right)^{2/(n-1)}} + \ldots\right], \tag{13}$$

for $t \geq t_0$. Here, the time after formation of the pulsar, using Eq. (12), is related to the angular velocity as $t \simeq t_0(\Omega_0/\Omega)^{n-1}$, and $t_0 = I_0/[(n-1)C\Omega_0^{n-1}]$ for $n > 1$ is the critical time where a phase transition occurs in the center. For earlier times $t \leq t_0$ there is no dense core and Eq. (13) applies when setting $c_2 = 0$ The critical angular velocity is $\Omega_0 = \omega_0\sqrt{2\pi\rho_1} \simeq 6$ kHz for the BJP EOS, i.e., comparable to a millisecond binary pulsar. Applying these numbers to, for example, the Crab pulsar we find that it would have been spinning with critical angular velocity approximately a decade after the Crab supernova explosion, i.e., $t_0 \sim 10$ years for the Crab. Generally, $t_0 \propto \Omega_0^{1-n}$ and the time scale for the transients in $\dot\Omega$ as given by Eq. (13) may be months or centuries. In any case it would not require continuous monitoring which would help a dedicated observational program.

The braking index depends on the second derivative $I'' = dI/d^2\Omega$ of the moment of inertia and thus diverges (see Fig. 1) as Ω approaches Ω_0 from below

$$n(\Omega) \equiv \frac{\ddot\Omega\Omega}{\dot\Omega^2} \simeq n - 2c_1\frac{\Omega^2}{\Omega_0^2} + c_2\frac{\Omega^4/\Omega_0^4}{\sqrt{1 - \Omega^2/\Omega_0^2}}. \tag{14}$$

For $\Omega \geq \Omega_0$ the term with c_2 is absent. The *observational* braking index $n(\Omega)$ should be distinguished from the *theoretical* exponent n appearing in Eq. (12). Although the results in Eqs. (13) and (14) were derived for the pulsar slowdown assumed in Eq. (12), both $\dot\Omega$ and $n(\Omega)$ will generally display the $\sqrt{t - t_0}$ behavior for $t \gtrsim t_0$ as long as the rotational energy loss is a smooth function of Ω. The singular behavior will, however, be smeared on the pulsar glitch "healing" time which in the case of the Crab pulsar is of order of weeks only.

We now discuss possible phase transitions in interiors of neutron stars. The quark and nuclear matter mixed phase described in [2] has continuous pressures and densities. There are no first order phase transitions but, at most, two second order phase transitions, namely, at a lower density, where quark matter first appears in nuclear matter, and at a very high density (if gravitationally stable), where all nucleons are finally dissolved into quark matter. In second order phase transitions the pressure is a continuous function of density and we find a continuous braking index. This mixed phase does not, however, include local surface and Coulomb energies of the quark and nuclear matter structures. As shown in

[3,13] there can be an appreciable surface and Coulomb energy associated with forming these structures and, if the interface tension between quark and nuclear matter is too large, the mixed phase is not favored energetically. The neutron star will then have a core of pure quark matter with a mantle of nuclear matter surrounding it and the two phases are coexisting by a first order phase transition. For a small or moderate interface tension the quarks are confined in droplet, rodlike and platelike structures [3,13] as found in the inner crust of neutron stars [14]. Because of the finite Coulomb and surface energies associated with forming these structures, the transitions change from second order to first order at each topological change in structure. If a kaon condensate appears it may also have such structures [15]. Pion condensates [5], crystalline nuclear matter [6], hyperonic or magnetized matter, etc. may provide other first order phase transitions.

There may also be other transitions in neutron stars. The glitches observed in the Crab, Vela, and a few other pulsars are probably due to quakes occurring in solid structures such as the crust, superfluid vortices or possibly the quark matter lattice in the core [13]. These glitches are very small $\Delta\Omega/\Omega \sim 10^{-8}$ and have a characteristic healing time. In [8] a drastic softening of the equation of state by a phase transition to quark matter leads to a backbending moment of inertia as a function of frequency. As a result, the star will become unstable as it slows down, will suddenly decrease its moment of inertia and create a large glitch. A similar phenomenon occurs if supercooling takes place. However, if the cooling is continuous the temperature will decrease with time, and the phase transition boundary will move inwards. The two phases could, e.g., be quark-gluon/nuclear matter or a melted/solid phase. In the latter case the size of the hot (melted) matter in the core is slowly reduced as the temperature drops, freezing the fluid into the solid mantle. Melting temperatures have been estimated in [14,16] for the crust and in [3] for the quark matter mixed phase. When the very core freezes we have a similar situation as when the star slows down to the critical angular velocity, i.e., a first order phase transition occurs right at the center. Consequently, similar behavior of the moment of inertia, angular velocities, and braking index may occur as in Eqs. (11), (13), and (14) replacing $\Omega(t)$ with $T(t)$.

In summary, we have shown that, if a first order phase transition is present at central densities of neutron stars, it will show up in moments of inertia and, consequently, also in angular velocities in a characteristic way. For example, the slowdown of the angular velocity has a characteristic behavior $\dot{\Omega} \sim c_2\sqrt{1 - t/t_0}$ and the braking index diverges as $n(\Omega) \sim c_2/\sqrt{1 - \Omega^2/\Omega_0^2}$ [see Eqs. (13) and (14)]. The magnitude of the signal generally depends on the density difference between the two phases and the critical angular velocity $\omega_0 = \Omega_0/\sqrt{2\pi G\rho_1}$ such that $c_2 \sim (\rho_2/\rho_1 - 1)\omega_0^3$. The observational consequences depend very much on the critical angular velocity Ω_0, which depends on the equation of state employed, at which density the phase transition occurs, and the mass of the neutron star. By studying a range of angular velocities for a sample of different star masses the chance for encountering a critical angular velocity increases. We encourage a dedicated search for the characteristic transients discussed above. Eventually, one may be able to cover the full range of central densities and find all first order phase transitions up to a certain size determined by the experimental resolution. Since the size of the signal scales with Ω_0^3 the transition may be best observed in rapidly rotating pulsars such as binary pulsars or pulsars recently formed in supernova explosion and which are rapidly slowing down. Carefully monitoring such pulsars may reveal the characteristic behavior of the angular velocity or braking index, as described above, which is a signal of a first order phase transition in dense matter.

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