Transport Optimization and MHD Stability of a Small Aspect Ratio Toroidal Hybrid Stellarator

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A class of small aspect ratio (A) stellarators is found that does not rely on quasisymmetrization to achieve good confinement. These systems depart from canonical stellarators by allowing a small net plasma current. An optimization procedure with bounce-averaged omnigeneity and other desirable physical properties as target criteria is described and applied to show the existence of a compact plasma device having $A \lesssim 3$, high β (ratio of thermal energy to magnetic field energy), and low plasma current. The added design flexibility afforded by the plasma current leads to an attractive low-A hybrid device which is stable to magnetohydrodynamic ballooning modes for $\langle\beta\rangle < 6\%$. [S0031-9007(97)05027-8]

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The tokamak, a toroidally symmetric plasma trap that uses a large plasma current to produce a confining poloidal magnetic field, has been the most successful plasma confinement device to date, simultaneously achieving high temperature ($T_i \geq 10$ keV) and high β ($< 10\%$) plasmas. However, the difficulty and expense of driving a large steady-state current, along with the complexity of protecting against current disruptions, is a disadvantage in a fusion reactor. Low aspect ratio ($A < 3$) stellarators [1–4] offer the attractive feature of a compact steady-state fusion power system with high volume utilization and reduced current-drive requirements. Stellarators, which are nonsymmetric plasma traps relying on external coils to produce the internal transform needed for confinement and stability, also have less current disruption potential compared with tokamaks.

Compact (low-A) stellarators have been previously considered unattractive for several reasons. They suffered from poor neoclassical (collisional) transport due to lack of symmetry, low stability β limits due to localized helical wells in bad curvature regions of the plasma, and fragility of magnetic surfaces due to low order resonances. Recently, progress has been made in substantially improving their collisional confinement by designing systems with "quasisymmetry." (A quasisymmetric confinement system is one in which the $|B|$ Fourier spectra, in Boozer magnetic coordinates [5], has spatial symmetry, but in which the metric tensor is generally *not* symmetric. Since the drift equations in Boozer coordinates depend only on $|B|$ and its derivatives, but not on any metric elements, the resulting particle orbits behave as though in a symmetric field.) Of the two quasisymmetric approaches considered, only quasitoroidal optimizations have been successful [6] at low A, but at unattractively low values [7] of $\beta < 2\%$. The quasihelical approach [8] is expected to be applicable only at higher aspect ratios [9].

The rather large rotational transform (ι) values ($\iota \geq 0.5$) associated with quasisymmetric configurations also make them necessarily low-shear devices, which can be susceptible to magnetic surface breakup. Low edge transform values $\iota \approx 0.1$ reduce the fragility of the magnetic configuration. We have found that one way to maintain this low pedestal value for ι as the pressure increases is to add a small, net toroidal current. This current also increases the design space for optimization of stability and transport, as discussed below.

Quasisymmetry restricts the nonzero components of the $|B|$ spectra to be multiples of a fixed helicity (m/n value, where m is the poloidal, and n is the toroidal, Fourier-mode number), since the pressure in stellarators is limited by helically increasing the allowable β of compact stellarators by easing the quasisymmetry constraint. We have therefore considered alternative techniques for reducing the particle orbit excursions away from magnetic flux surfaces. A general approach has been implemented which uses the alignment of contours of the approximate second adiabatic invariant [10] $J = \oint v_\parallel dl$ with magnetic flux surfaces $\psi = \text{const}$ (ψ denotes the enclosed toroidal flux). Here, the integral for trapped particles is along a magnetic field line between orbit turning points where $v_\parallel = 0$. The component of the bounce-averaged particle drift, normal to a magnetic surface, satisfies $\langle V_D \cdot \nabla\psi\rangle \propto \partial J/\partial\vartheta$. Here ϑ is the poloidal angle which varies the short way around the flux surface. Thus, a configuration which satisfies $J = J(\psi)$ leads to confinement improvement over the entire trapped particle population and reduces the number of transitional particles (particles near the separatrix between trapped and circulating particles). This criterion is a generalization to arbitrary $|B|$ spectra of the optimization previously proposed [11] for a simple model spectrum consisting of only three Fourier components beyond the axisymmetric ones. This bounce-averaged *omnigeneity* has been interpreted [12–14] in terms of equal spacing of $|B|$ contours on a magnetic flux surface. It has also been used to interpret the good confinement of alpha particles in the large A Wendelstein-7X device [15].

We use the VMEC code, a three-dimensional magnetohydrodynamic (MHD) equilibrium solver [16] which is

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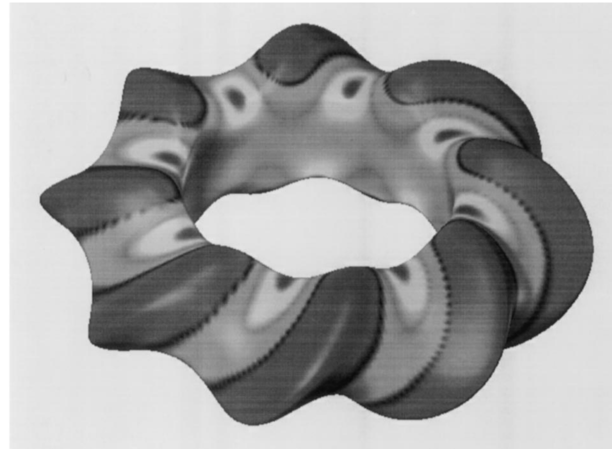
based on nested magnetic surfaces, as the inner physics evaluation loop of a Levenberg-Marquardt optimizer. A sum of squares (χ^2) is minimized, consisting of σ-weighted differences between physics-based *target* values and instantaneous *configuration* values (as computed numerically). The configuration space is defined by the control variables, which are the Fourier harmonics of the cylindrical coordinates R and Z describing the shape of the outermost magnetic flux surface. In the case of stellarator-tokamak hybrids, the control variables also include the radial polynomial expansion coefficients of the plasma current profile. This method is similar to that originally used to design a large-A quasihelical configuration [8]. It differs primarily in the physics optimization targets comprising χ^2, which here consist of the following: (a) alignment of J (and specifically, $B_{\min}$ for deeply trapped particles and $B_{\max}$ to reduce transitional orbit losses) with magnetic flux surfaces, leading to terms of the form $\chi^2 = \langle[\partial J(\psi, \theta, \lambda)/\partial\theta]^2\rangle/\sigma\langle J^2\rangle + \ldots$ (angle brackets denote a flux surface average, the ellipsis arising from $B_{\min}$ and $B_{\max}$, σ is the standard derivation, and $\lambda = \epsilon/\mu$ is the pitch, with ϵ the particle energy and $\mu = mv_\perp^2/2B$ the magnetic moment); (b) matching the rotational transform $\iota(\psi)$ to a specific radial profile (described below); (c) maintenance of a magnetic well $V'' < 0$, needed for interchange stability, over most of the plasma cross section; and (d) $A \equiv R_0/a \approx 3$. To reduce the twists and bends of the coils eventually required to produce the optimal surface, the local flux surface curvature is bounded. The alignment of the $B_{\min}$, $B_{\max}$, and trapped J contours with ψ is performed at three or more radial flux surfaces and, for J, at four values of λ. The initial shape of the outer flux surface and the target values for the ι and V'' profiles are obtained from vacuum equilibria based on a known set of modular (external) coils. In contrast to quasisymmetric optimizations, the present method does *not* directly target the magnetic spectrum $\{B_{mn}\}$. Rather, minimization of the J variation leads to a magnetic spectrum with properties similar to those described in Refs. [12–14].

The optimization technique has been applied to a hybrid stellarator device (with current) with $N = 8$ field periods (number of identical toroidal sections), $A \approx 3$, mean major radius $R = 1.3$ m, $\langle\beta\rangle = 2\%$, and a net toroidal plasma current of 60 kA and a mean on-axis magnetic field of 1 T. The current is small compared with $\approx$1 MA in a tokamak of similar size and magnetic field. We will compare the initial unoptimized device (with $\chi^2 = 100$), whose outer flux surface was determined by a set of N ($= 8$) identical modular, tilted coils, with an optimized configuration based on the alignment of J with ψ (with $\chi^2 = 10$). Figure 1 shows the outer flux surfaces for the two configurations with shading to indicate the constant $|B|$ contours. The unoptimized configuration has $\iota(\psi = 0) = 0.25$, $\iota(\psi = \psi_{\text{edge}}) = 0.15$, and a central region of reversed shear, while the optimized case has

(a)



(b)

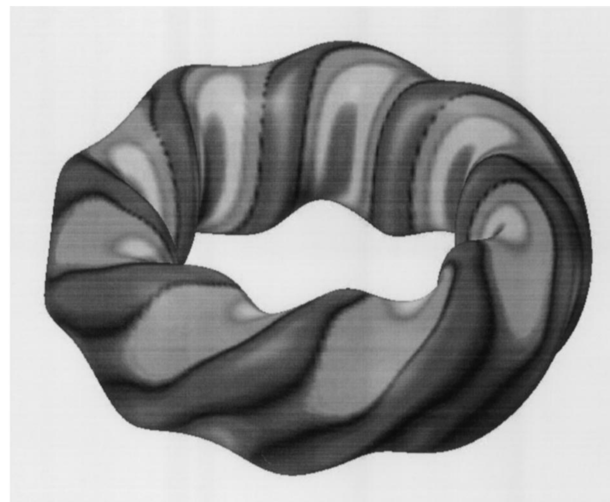


FIG. 1. (a) Outer magnetic flux surface of the unoptimized configuration. (b) Outer magnetic flux surface of optimized configuration.

a monotonically decreasing rotational transform profile, which is tokamaklike (i.e., decreasing toward the plasma edge): $\iota = 0.3 - 0.2(\psi/\psi_{\text{edge}})$. The optimized case also has a lower magnetic ripple, by a factor of 2, over most of the plasma cross section. The $B_{\min}$ contours, depicting the orbits of deeply trapped particles, are shown in Figs. 2(a) and 2(b) for these two cases. They are presented in Boozer coordinate space in which the magnetic surfaces are concentric circles. The unoptimized configuration (a) has completely open $B_{\min}$ contours (i.e., all deeply trapped particles are lost), while the optimized configuration (b) has a large area of closed $B_{\min}$ contours.

To assess the effect of optimization on the thermal confinement properties, we have followed the Monte Carlo evolution [17] of 256 particles started at a single interior radial location with a random distribution in pitch, poloidal, and toroidal angles, and a Maxwellian distribution in energy. The background plasma has a density of 5×10^{13} cm^{-3}, and all particles have a temperature of 1 keV, putting the plasma in a low collisionality regime

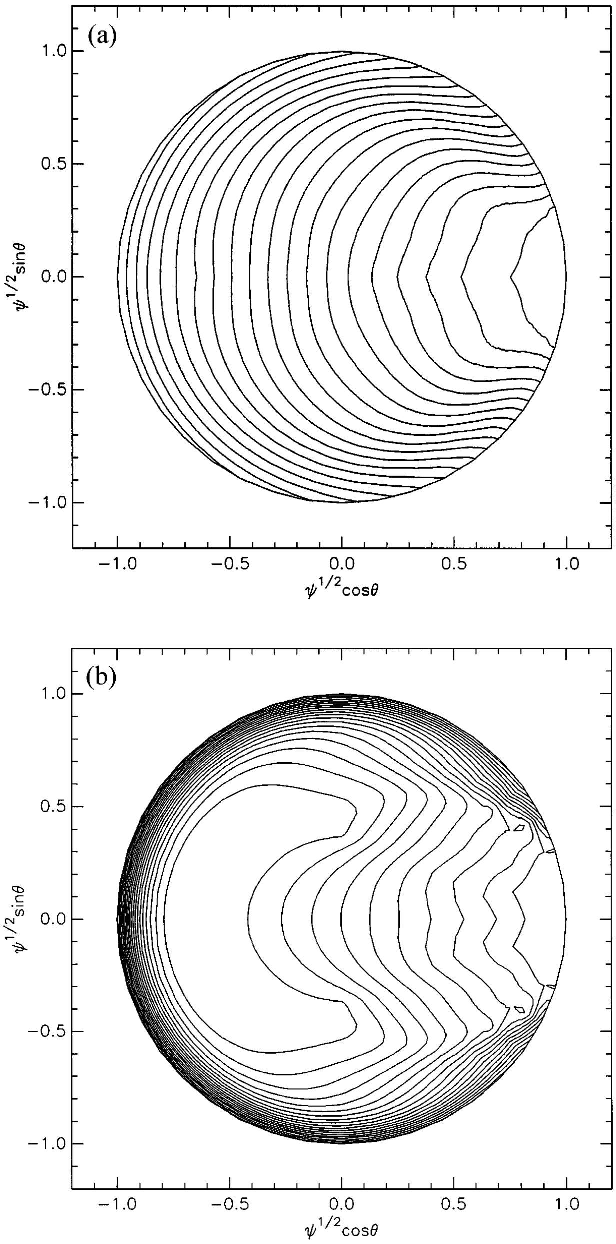


FIG. 2. (a) $B_{\min}$ contours for unoptimized case. (b) $B_{\min}$ contours for optimized case.

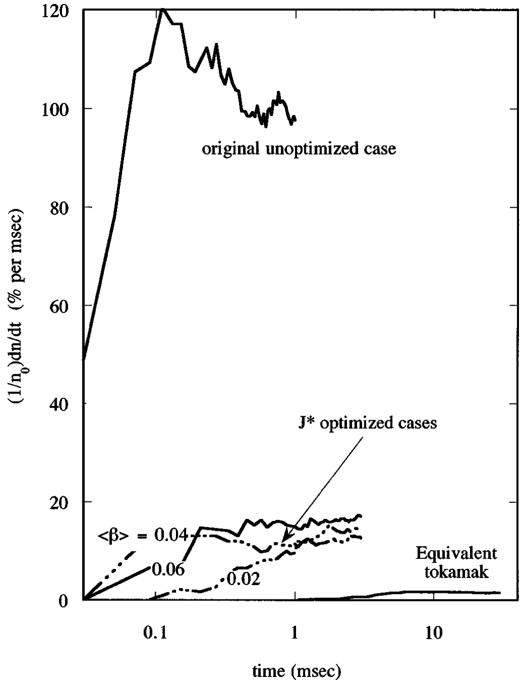


FIG. 3. Comparison of Monte Carlo loss rates of unoptimized and J-optimized configurations for various values of $\langle\beta\rangle$.

where large $1/\nu$ helical ripple losses would occur without optimization. The escape of particles and energy through the outer flux is monitored as a function of time, yielding a loss rate which includes both direct orbit losses and diffusive losses and involves no assumptions regarding localized (small step size) transport. In Figure 3 we show the particle loss rates versus time for the original configuration ($\langle\beta\rangle = 2\%$) and several J-optimized cases ($\langle\beta\rangle = 2, 4,$ and 6%) along with an equivalent tokamak case (obtained by retaining only the axisymmetric $n = 0$ harmonics in the $|B|$ spectrum of the J-optimized cases). These simulations clearly demonstrate that the optimization procedure can substantially reduce loss rates, leading to roughly a factor of 10 confinement improvement compared with the initial configuration. The best J-optimized case shown here has a loss rate within a factor of 8 of the equivalent tokamak when no radial electric field is present, and for $e\Phi/kT_i \sim 1$, the loss rate is reduced to a factor of 2 (of the tokamak).

The confinement of collisionless energetic particles is one of the primary motivations for these optimizations. Plasma heating efficiencies can depend sensitively on the confinement of energetic particle tail populations. As a result of the high level of magnetic ripple in these configurations, a significant loss channel might be expected. In order to quantify this, we followed a small ensemble of orbits at 40 keV that initially pass through the magnetic axis and have a range of pitch angles: initial values of $(v_\parallel/v)$ running from -0.6 to $+1$ [orbits with $-1 < v_\parallel/v) < -0.6$ are confined in both configurations]. We find that although the unoptimized configuration has a significant loss cone over the range $-0.2 < (v_\parallel/v) < 0.4$, the J-optimized configuration has completely healed the loss cone and confines all of the orbits considered. Furthermore, calculations show that even at energies approaching 400 keV, the optimized configuration confines all orbits. Above 400 keV, the deeply trapped orbits become lost because of the breakdown of J conservation. The passing orbits are still confined, but with significant orbit displacements $\Delta a \approx 0.5$.

The present configuration was optimized at a relatively low value of $\langle\beta\rangle = 2\%$. To test its high-pressure ballooning stability properties, we have raised the plasma pressure to $\langle\beta\rangle = 6\%$, keeping the ι profile fixed (flux-conserving), and therefore increasing the net plasma current. It is found that the plasma is stable to ballooning modes over the inner 80% of its cross section. Flattening the pressure near the edge completely stabilizes this configuration. This shows improved stability compared with the unoptimized one, which was unstable to ballooning modes for $\langle\beta\rangle \geq 2.5\%$. The plasma current required to maintain the ι profile (while keeping the plasma-bounding surface fixed) increased modestly, by less than a factor of 2, over this range of pressure, and peaked toward the edge of the plasma as the pressure was raised. Although the current density profile obtained by this flux-conserving optimization had a narrow reversal region at the plasma edge, further optimization seems possible with respect to the current by relaxing the constraint on the iota profile. The reason for the improved stability of the optimized configuration is presently unknown but may be related to its tokamaklike ι profile and the edge-localized current [18]. Further study to analyze current-driven modes is required.

Low aspect ratio stellarator configurations, which are neither quasitoroidal nor quasihelical, have been found that possess improved thermal and energetic particle confinement without sacrificing MHD stability. An optimization procedure has been developed that targets bounce-averaged omnigeneity (i.e., minimization of drift away from the flux surface). This technique uses the shape of the outermost flux surface and the plasma current profile as control parameters. The resulting flexibility in the $|B|$ spectrum, together with the addition of net toroidal plasma current (which seems to be necessary at higher β values to assure sufficient edge rotational transform), have increased the available parameter space at low aspect ratio and resulted in significant improvements (by factors exceeding 10) in confinement of both thermal and energetic particle components. This confinement improvement occurs simultaneously with significant stable $\langle\beta\rangle < 6\%$ over most of the plasma cross section. We anticipate that this optimization technique can be used to improve the performance of other nonsymmetric confinement systems.

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