Strong Electroweak Phase Transition up to $m_H \sim 105$ GeV

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(Received 9 April 1998)

Perturbative calculations suggest that the electroweak phase transition in the minimal supersymmetric standard model (MSSM) can be strong enough for baryogenesis for Higgs masses up to $m_H \sim$ 105 GeV, provided that the lightest stop mass is in the range 100–160 GeV. We have performed large-scale lattice Monte Carlo simulations of the MSSM electroweak phase transition. We find that the transition is in fact *stronger* than in perturbation theory. This guarantees that the perturbative mass bounds are conservative ones, and provides a strong motivation for further studies of MSSM baryogenesis. [S0031-9007(98)06396-0]

PACS numbers: 98.80.Cq, 11.10.Wx, 11.15.Ha, 12.60.Jv

It is known from studies of primordial nucleosynthesis that there is a nonvanishing baryon to photon density ratio in the Universe, $\eta \approx 10^{-10}$ (for recent reviews, see [1]). It is one of the main challenges of cosmology to understand how such an asymmetry could come about. Indeed, different scenarios for producing $\eta > 0$ abound.

Among all the scenarios for baryogenesis, one is unique: The *last instance* in the history of the Universe that a baryon asymmetry could have been generated is the electroweak phase transition [2]. As such, this is also the scenario requiring the *least assumptions* beyond established physics. In principle, even the standard model contains the necessary ingredients for baryon number generation: anomalous baryon number violation, *CP* violation, and an electroweak phase transition providing for a nonequilibrium environment (for a review, see [3]). Once an asymmetry has been generated, it must also be preserved, and this gives a strict constraint on how strongly of the first order the transition must be [2]. In fact, the constraint on the strength of the phase transition is the most rigorous of the constraints mentioned, since it concerns a thermodynamical *equilibrium* situation after the transition, and equilibrium physics is much better understood than nonequilibrium physics.

However, it turns out that on a more quantitative level the standard model is too restricted for baryogenesis. The main reason is that the strength of the electroweak phase transition depends on the Higgs mass, and for $m_H \gtrsim 75$ GeV, there is no electroweak phase transition at all [4,5]. Since the experimental lower bound in the standard model is $m_H \sim 88$ GeV [6], the existence of the baryon asymmetry alone requires physics beyond what is currently known.

The simplest extended scenarios that allow for baryon asymmetry generation at the electroweak phase transition have a Higgs sector which differs from that in the standard model. A particularly appealing scenario is the electroweak phase transition in the minimal supersymmetric standard model (MSSM) [7–9]. Indeed, it has recently become clear that the electroweak phase transition can then be much stronger than in the standard model, and strong enough for baryogenesis at least for Higgs masses up to 80 GeV [10–17]. For the lightest stop mass $m_{\tilde{t}_R}$ lighter than the top mass, one can go even up to $\sim$100 GeV [18]: In the most recent analysis [19], the allowed window was estimated at $m_H \sim 75$–105 GeV, $m_{\tilde{t}_R} \sim 100$–160 GeV. In this regime, the transition could even proceed in two stages [18] via an exotic intermediate color breaking minimum. This Higgs and stop mass window is interesting from an experimental point of view, as well, as the whole range will be covered at LEP and the Tevatron [19].

Unfortunately, the statement concerning the strength of the electroweak phase transition in this regime is subject to large uncertainties. The first indication in this direction is that the 2-loop corrections to the Higgs field effective potential are large and strengthen the transition considerably [11]. A further sign is that the gauge parameter and, in particular, the renormalization scale dependence of the physical results derived from the 2-loop potential, which are formally of the 3-loop order, are numerically quite significant [18]. Hence a nonperturbative analysis is needed.

The purpose of this paper is to study the MSSM electroweak phase transition with lattice Monte Carlo simulations, and to extrapolate the results to the infinite volume and continuum limits. Since the MSSM at finite temperature is a multiscale system with widely different scales from $\sim\pi T$ to $\sim g_W^2 T$, and since there are chiral fermions, the only way to do the simulations in practice is to use an effective 3D theory [20]. This approach consists of a perturbative dimensional reduction into a 3D theory with considerably fewer degrees of freedom than in the original theory [22–24], and of lattice simulations in the effective theory. The analytical dimensional reduction step has been performed for the MSSM in [13–15,18].

Lattice simulations in dimensionally reduced 3D theories have been previously used to determine the properties of the electroweak phase transition in the standard model in great detail [25–31].

In the regime considered, the right-handed stop field U plays an important role in addition to the Higgs field. The effective 3D Lagrangian describing the electroweak phase transition in the MSSM is therefore an SU(3) $\times$ SU(2) gauge theory with two scalar fields [13,18]:

$$\begin{aligned}\mathcal{L}^{3D}_{\rm cont} = {} & \frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{4}G^A_{ij}G^A_{ij} + \gamma H^\dagger H U^\dagger U \\ & + (D^w_i H)^\dagger (D^w_i H) + (D^s_i U)^\dagger (D^s_i U) + m^2_{H3} H^\dagger H \\ & + m^2_{U3} U^\dagger U + \lambda_H (H^\dagger H)^2 + \lambda_U (U^\dagger U)^2 . \qquad (1)\end{aligned}$$

Here D^w_i and D^s_i are the SU(2) and SU(3) covariant derivatives, and H is the combination of the Higgs doublets which is "light" at the phase transition point. The U(1) subgroup of the standard model induces only small perturbative contributions [31], and can be neglected.

The complexity of the original 4D Lagrangian is hidden in Eq. (1) in the expressions of the parameters of the 3D theory. A dimensional reduction computation leading to actual expressions for these parameters has been made in [18] for a particularly simple case. Let us stress here that the reduction is a purely perturbative computation and is free of infrared problems. The relative error has been estimated in [13,18], and should be $\lesssim 10\%$.

It is prohibitively time consuming to study the full parameter space of Eq. (1) with Monte Carlo simulations. Thus, we only consider a special parameter choice: We take a large left-handed squark mass parameter $m_Q \sim$ 1 TeV, vanishing squark mixing parameters, and a heavy CP-odd Higgs particle ($m_A \gtrsim 300$ GeV). We fix $\tan\beta = 3$, corresponding to $m_H \sim 95$ GeV. We then study the 3D theory in Eq. (1), parametrized by the temperature T and the right-handed stop mass parameter $\tilde{m}_U$ ($\tilde{m}_U$ determines the zero temperature right-handed stop mass through $m_{\tilde{t}_R} \approx (m^2_{\rm top} - \tilde{m}^2_U)^{1/2}$). The actual expressions used for the dimensional reduction are given in [32].

The philosophy is now that we determine the nonperturbative results for the continuum theory in Eq. (1) through lattice simulations, and compare them with 3D perturbation theory, employing the same 3D parameters. To be more precise, we compare with 2-loop 3D perturbation theory in the Landau gauge $\xi = 0$ and for the $\overline{\rm MS}$ scale parameter $\overline{\mu} = T$ the values which have been used in [19], as well. This allows one to find out whether there are any nonperturbative effects in the system. Once this has been done, one can go back to a more complicated situation and study it perturbatively, adding to the perturbative results the nonperturbative effects found here. As the reduction step is purely perturbative, the nonperturbative effects found with the 3D approach apply also to the effective potential computed in 4D [11,17,19].

To perform lattice simulations, we discretize the theory in Eq. (1) with standard methods (see [32]). The lattice parameters are expressed in terms of the lattice spacing a and the continuum parameters through 2-loop relations [33] which become exact in the continuum limit.

Well-controlled *infinite volume* and *continuum* limits are essential in order to obtain reliable results. Thus, for each point in the parameter space, we always perform simulations with several lattice volumes and extrapolate to the infinite volume. We use the lattice spacings obtained through

$$\beta_S \equiv \frac{6}{g^2_{S3} a} = 12, 20 , \qquad (2)$$

where $g^2_{S3} = g^2_S T \sim T$ is the 3D SU(3) gauge coupling and a is the lattice spacing. The fact that we use just two values of β_S allows only a linear extrapolation to the continuum limit $\beta_S = \infty$. However, it is understood analytically that the dominant corrections are linear [34], and, moreover, linear extrapolations work extremely well for the case of the standard model [26,31].

All in all, we have performed 42 different Monte Carlo runs: combinations of lattice sizes and parameters. The total CPU time was ~7.5 node years on a Cray T3E.

The physical quantities we discuss here are the critical temperature T_c, the scalar field expectation values, and the latent heat. Quantities such as the latent heat enter, for instance, the estimates for the nucleation and reheating temperatures (see, e.g., [35]), which are needed to decide whether the scalar field expectation values relevant for cosmology should be taken at T_c or some lower temperature.

The phase diagram and the critical temperatures.— The general phase structure of the theory is expected to be the following [18]. The system has a first order transition at $T_c \sim 100$ GeV for $\tilde{m}_U \lesssim 65$ GeV. This transition is strong even though m_H is large, due to the stop loops. As $\tilde{m}_U$ becomes larger ($m_{\tilde{t}_R}$ smaller), the transition gets even stronger, and then at some point one may get a two-stage transition. The existence of a two-stage transition depends on the parameters of the theory, and for large squark mixing parameters the two-stage region is not reached [19].

Our numerical results are shown in Fig. 1. It is seen that the phase diagram is qualitatively the same as in perturbation theory, although the critical temperatures and the triple point have been displaced by a few GeV. We have data at $\beta_S = 20$ only at $\tilde{m}_U = 50$, 65 GeV, and the continuum extrapolation is possible only at these points. Nevertheless, we expect similar (small) effects at the other points. As of now, we have no clear theoretical explanation for the discrepancy between the lattice results and perturbation theory: The reason might be, e.g., a 3-loop perturbative effect, or a genuine nonperturbative contribution.

Latent heat.—The main result of this paper is shown in Fig. 2, which shows the latent heat. It is the most

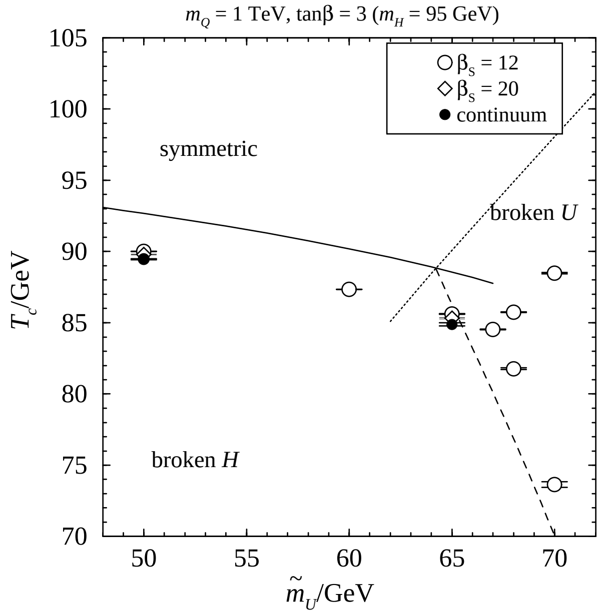

FIG. 1. The phase diagram and the critical temperatures. The continuous lines are from the 2-loop perturbative effective potential in the Landau gauge. Open symbols correspond to infinite volume extrapolations and filled symbols correspond to continuum extrapolations.

important gauge-invariant physical characterization of the strength of a first order transition. We observe that the nonperturbative transition to the standard electroweak minimum at $\tilde{m}_U \lesssim 67$ GeV is significantly (up to 45%) *stronger* than the perturbative transition. In the regime $\tilde{m}_U \gtrsim 67$ GeV, where there is a two-stage transition, a comparison with perturbation theory is more difficult as the whole pattern is shifted to the right, but the qualitative behavior is the same.

Scalar field expectation values.—The Higgs field vacuum expectation value v_H is the object by which one usually characterizes whether the phase transition is strong enough for baryogenesis [2,3], the requirement being $v_H/T \gtrsim 1$. As such, v_H is, however, a gauge dependent quantity. If one computes it in the Landau gauge (v_H^L), as is usual, then in terms of gauge-invariant operators the same expression would be nonlocal. On the other hand, there is a simple local gauge-invariant quantity closely related to v_H, namely, $H^\dagger H \sim v_H^2/2$. The problem with $H^\dagger H$ is that, being a composite operator, it is a scale dependent quantity in, say, the modified minimal subtraction ($\overline{\text{MS}}$) scheme. We hence define on the lattice

$$\frac{v_H}{T} \equiv \left(2\left\langle \frac{H^\dagger H_{\overline{\text{MS}}}(g_{S3}^2)}{T}\right\rangle\right)^{1/2}, \qquad (3)$$

which is a natural gauge-invariant generalization of v_H^L/T, and can be measured in simulations. Note that, with respect to 4D units, there is a trivial rescaling by T in the $H^\dagger H$ appearing in Eq. (3).

The numerical results for $v_H/T, v_U/T$ are shown in Fig. 3. Again, we observe a value larger than in perturbation theory in the regime $\tilde{m}_U \lesssim 67$ GeV. Moreover, in qualitative accordance with perturbation theory, there is a rapid increase in v_H/T_c in the regime of the two-stage transition, $\tilde{m}_U \gtrsim 67$ GeV. The relative nonperturbative strengthening effect is smaller than for the latent heat, which is easy to understand since $L \propto \Delta(H^\dagger H) \sim \Delta v_H^2$ [26], implying $\delta L/L \sim 2\delta v_H/v_H$.

In conclusion, at least for the parameter values studied ($m_H \sim 95$ GeV, $m_{\tilde{t}_R} \sim 150$–160 GeV), the electroweak

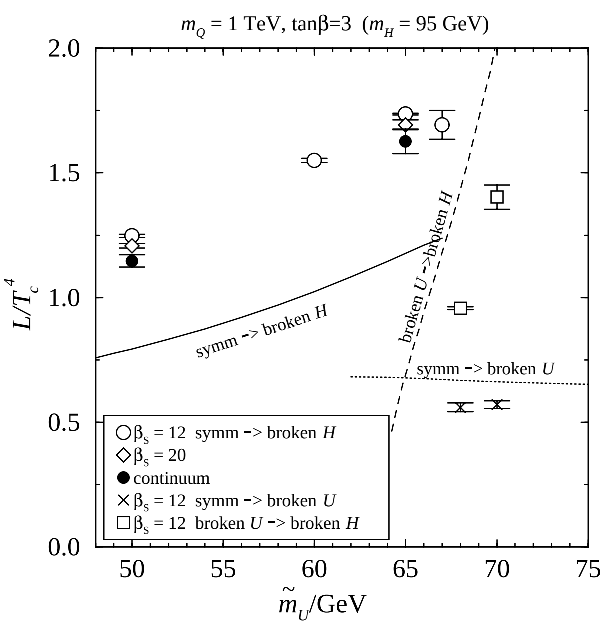

FIG. 2. The latent heat.

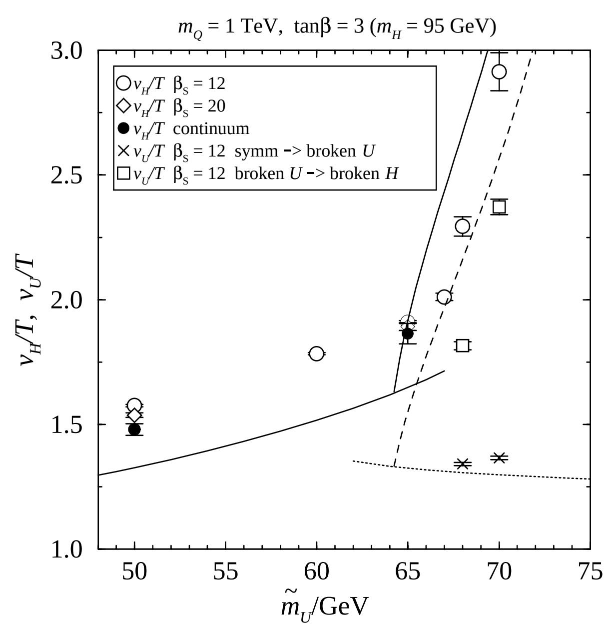

FIG. 3. The scalar field expectation values in the broken phases at T_c.

phase transition is significantly stronger than indicated by 2-loop perturbation theory. This implies that the previous perturbative Higgs and stop mass bounds for electroweak baryogenesis are conservative estimates. In particular, the electroweak phase transition could be strong enough for baryogenesis for *all allowed Higgs masses* in this regime ($m_H \lesssim 105$ GeV) [19]. Because of the nonperturbative strengthening effect seen, the stop mass could be slightly larger than the perturbative value, up to, say, $m_{\tilde{t}_R} \sim 165$ GeV. For the smallest stop masses, on the other hand, there is the possibility of a two-stage transition, in which the Higgs field gets an extremely large vacuum expectation value.

These results provide a strong motivation for precise studies of the nonequilibrium *CP*-violating real time dynamics and baryon number generation at the MSSM electroweak phase transition.

The simulations were made with a Cray T3E at the Center for Scientific Computing, Finland. We acknowledge useful discussions with K. Kajantie, G. D. Moore, M. Shaposhnikov, and C. Wagner. This work was partly supported by the TMR network *Finite Temperature Phase Transitions in Particle Physics,* EU Contract No. FMRX-CT97-0122.

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